University of Fort Hare

Together in Excellence

DEPARTMENT OF COMPUTATIONAL SCIENCES

STATISTICS DESCIPLINE

STM 211 & 212
INTRODUCTION TO MATHEMATICAL STATISTICS
A & B

SUPPLEMENTARY EXAM

MAY/JUNE 2025

TIME: 3 HOURS

MARKS: 100

Internal Examiners

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Moderator

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This paper consists of 4 pages including the cover pages

Instructions

Write your answers

Non-programmable Calculators may be used

QUESTION ONE [20 Marks]

1.1 Which of the following sequences converge and which diverge?

a) $\sum_{n=1}^{\infty} \frac{1}{0.1^{-n}}$ [2]

b) $\lim_{x\to\infty} \frac{6x^2+7x+6}{x+2}$ [2]

c) Which of the following numbers is larger: $\binom{93}{30}$ or $\binom{93}{63}$? Quote the relative theorem. [2]

1.2 Express the following recurring decimal as a fraction: [5]

$$0.\overline{021}$$

1.4 Find the partial derivatives of the following functions:

a) $f(x,y) = \frac{x-y}{x+y}$, hence calculate $f_x = (0,1), \text{and } f_y = (0,1)$ **[5]**

b) $f(x,y) = xln(x,y)$ **[4]**

QUESTION TWO [9 Marks]

2.1

Given the following table

	Sport	
Gender	Play	Not Play
Male	25	18
Female	14	23

Let A = event of playing sport
Let B = male

Find the following probabilities

1. $P(A)$ **[2]**
2. $P(B^c)$ **[2]**
3. $P(A^c \cap B)$ **[2]**
4. $P(A^c \cap B^c)$ **[3]**

QUESTION THREE [34 Marks]

3.1 Given that the random variable X has the following function:

$$f(x) = \begin{cases} k(2-x), & 0 \le x \le 2 \\ 0, & elsewhere \end{cases}$$

a) Find the value of the constant k for the given function to be taken as a *p.d.f* [3]

b) Find the moment generating function, hence use it to calculate: [4]

i. The first moment [6]

ii. The second moment [6]

iii. The variance [5]

3.2 What is the probability density function (p.d.f) of a continuous random variable, whose cumulative distribution function (c.d.f):

$F(x) = \frac{1}{1+e^{-x}}$ for $-\infty < x < \infty$ [4]

3.3 Consider the function

$f(x) = \begin{cases} 2x^{-2}, & 1 < x \le 2 \\ 0, & elsewhere \end{cases}$ Check if the function $f(x)$ can be taken as p.d.f [2]

3.4 The mean score of an Insurance Commission Licensure Examination is 75, with a standard deviation of 5. What percentage of the data set lies between 50 and 100? [4]

QUESTION FOUR [20 Marks]

4.1 Suppose a fair die is tossed 6 times. Let X be the number of times a prime number is obtained.

(a) What type of variable is X? [2]

(b) Find the probability distribution of the random variable X. [4]

(c) Find the following:

(i) The probability that X=0. [2]

(ii) The probability that X=1. [2]

(iii) The probability that X≤1. [2]

(d) Find the following:

(i) The expected value (mean) of the variable X. [2]

(ii) The standard deviation of the variable X. [2]

4.2 What is the probability of getting a score of not less than 5 in a throw of a six-sided die? [4]

QUESTION FIVE [17 Marks]

5.1 Consider the following p.d.f on the z-score of a randomly selected student

Z	-1	1	3	5
$P(Z)$	0.25	0.25	0.25	0.25

(a) Find the following:

(i) $P(Z = 3)$ [2]

(ii) $P(Z = 1)$ [2]

(iii) $P(-1 \leq Z \leq 3)$ [3]

(b) Find the mean [3]

(c) Find the variance and the standard deviation of the discrete random variable Z. **[4]**

(d) Find the moment generating function $M_Z(t)$. **[3]**