

UNIVERSITY OF FORT HARE

**Geometry
MAT 225**

**Degree Examinations
Supplementary
Jan\Feb
2019**

Time: 3 Hrs
Subject: MAT 225

Marks: 100

This question paper consists of 3
pages

Internal examiner(s)

Internal Moderator

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Instructions

Answer all questions.
Symbols used have the usual meaning

Question One

- 1.1 Define an Axiomatic Method. [2]
- 1.2 What does an Axiomatic system consists of? [4]
- 1.3 In an axiomatic system when are a set of axioms *consistent, independent, complete*? [6]
- 1.4 State the axioms for the *Three-Point geometry* G_3 . [4]
 Draw a simple model for this geometry. [1]
- 1.5 Could the three lines in G_3 all contain the same point? Motivate your answer. [2]
- 1.6 State the axioms of the *Four-Line geometry*. Draw a model for this geometry. [5]
 Which axioms are also true statements in *Euclidean geometry*? [1]

Question Two

- 2.1 Define a *glide reflection*. [2]
 When are a segment and its image parallel under a glide reflection? Give illustrations. [5]
- 2.2 Derive the general formula for the image of a point under the product of two translations with vectors $[a,b]$ and $[c,d]$. [2]
- 2.3 Prove that if a transformation is a plane motion, then it is the product of three or fewer reflections and conversely. [5]
- 2.4 Define a *direct* and *opposite* plane motion. [2]
- 2.5(a) Define a plane similarity. [2]
 (b) What is the difference between a dilation and a plane similarity? [2]
 (c) Find the image of $(3,5)$ under a dilation with center $(0,0)$ and with ratio $\frac{3}{4}$. [2]

Question Three

- 3.1 Prove that the internal bisectors of the angles of a triangle meet at the inciter. [5]
- 3.2 Prove that the internal bisectors of the angles of a triangle divides the opposite side into two segments proportional to the sides of the triangle adjacent to the angle. [7]
- 3.3 Give a definition and proof of the *circle of Apollonius*. [7]

3.4 Prove that the product of the lengths of the segment from an exterior point to the points of intersection of a secant with a circle is equal to the square of length of the tangent from the point to the circle. [5]

Question Four

4.1 For the *transformation of inversion* T

- (a) Give one invariant property. [1]
- (b) Explain (with full motivation) how to find the inverse point, with respect to a circle, of a point that lies outside the circle. [4]
- (c) Why must each line, not just lines through the centre of inversion, contain the ideal point? [2]

4.2 Prove that the image of a straight line not through the Centre of inversion, under the transformation of inversion is a circle passing through the Centre of inversion. [4]

4.3 Find the inverse of the point $(6,2)$ with respect to a circle with Centre $(2,3)$ radius of inversion 2. [3]

4.4 List down the axioms for *Projective Geometry*. [4]

4.5 Draw a *complete quadrangle*, identifying the pairs of *opposite sides* and the *diagonal triangle*. [4]

4.6 Explain the concept of *plane duality* in projective geometry. [2]

4.7 Explain what is meant by *Hyperbolic* and *Elliptic geometry*. Give a brief comparison between them and *Euclidean geometry*. [5]