UNIVERSITY OF FORT HARE

WAVES VIBRATIONS AND OPTICS PHY 122

SUPPLEMENTARY EXAMINATIONS

JANUARY

2025

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Time: **3 HOURS**

Subject: **PHY 122**

Marks: **100**

This paper consists of 10_ pages including the cover page and formular sheets

Internal Examiners

P.M KWINANA

Internal Moderator

Dr L. NDANDULENI

INSTRUCTIONS

1. **Answer all questions**
2. NB. ***FORMULAR SHEET*** **is attached at the back of the question paper.**

QUESTION ONE

1.1 Show that the Total energy of Simple Harmonic Motion (SHM) is given by $E_{tot} = \frac{1}{2}kA$ where k = spring constant and A is the amplitude *(maximum displacement of the oscillation)* of the waves. [6]

1.2 A mass m is dropped from a height **h** on to a scale-pan of negligible weight, suspended from a spring of spring constant **k.** The collision maybe considered to be completely inelastic in that the mass sticks to the pan and the pan begins to oscillate. Show that the amplitude of the pan's oscillations as measured from the equilibrium position is given by:

$\sqrt{\frac{m^2g^2}{k^2} + \frac{2mgh}{k}}$. *Let y_0 be the extension of the spring.* [6]

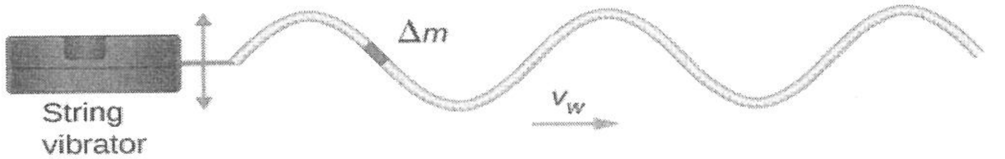
1. 3 Assume that we have a string with a velocity given by v = **λ**T, where T is the Period. Derive and show that the Power (rate of energy transfer) transported by a one-dimensional wave along on a string is given by:

$$\left[\frac{dK}{dt}\right]_{avg} = \frac{1}{2}\mu v \omega^2 A^2 \qquad \textit{assume that } \sin^2(kx - \omega t) \approx \frac{1}{2}$$

Where μ is the linear density of the string, A, the maximum displacement of an oscillator and E_λ is the stored energy in the wave over wavelength $\boldsymbol{\lambda}$. [6]

1.4 Consider a two-meter-long string with a mass of 70.00 g attached to a string vibrator as illustrated in the figure below. The tension in the string is 90.0 N. When the string vibrator is turned on, it oscillates with a frequency of 60 Hz and produces a sinusoidal wave on the string with an amplitude of 4.00 cm and a constant wave speed.

What is the time-averaged power supplied to the wave by the string vibrator?



[3]

1.5 The wave function for a wave on a taut string is $y = 0.35\,(10\pi x - 3\pi t + \pi/4)$, where x and y are in meters and t is in seconds. If the linear mass density of the string is 75.0 g/m,

(a) what is the average rate at which energy is transmitted along the string? [3]

(b) What is the energy contained in each cycle of the wave? [3]

1.6. Consider a sinusoidal wave that the following parameters: f = 120 Hz, ym = 8.5 mm, μ = 525 g/m and τ(Tension) = 45 N. At which average rate does the wave transport energy? [3]

Total Marks [30]

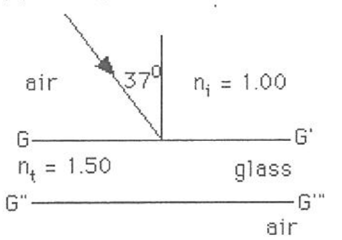
QUESTION TWO

2.1 A light beam is incident upon a parallel glass plate, as shown in Figure below. Find the angle of:

(a) refraction at the first glass surface,

(b) incidence at the lower glass-air surface, (c) refraction as the ray goes from glass to air. Use these angles to trace the path of the ray from air to glass and then

(c) from glass to air.

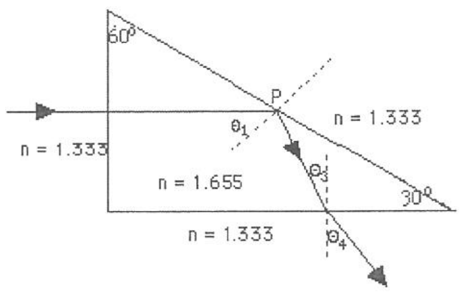


[5 Marks]

2.2 As shown in Figure below, a light ray is incident normally on one face of a 30°-60°-90° dense flint (n = 1.655) prism that is immersed in water (n = 1.333).

(a) Determine the exit angle θ_4 of the ray.

(b) A substance is dissolved in the water to increase the index of refraction. At what value of n of the mixture will total internal reflection cease at point P?



[5 Marks]

2.3. A thin converging lens has a focal length of 10 cm. Find by (i) calculation and (ii) construction of a ray diagram the position of the image of an object for the object distance s_o equal to (a) 30 cm, (b) 15 cm, and (c) 5.0 cm. Also find the magnification for each case. Describe whether the image is real or imaginary, erect or inverted, magnified or diminished.

[5 Marks]

2.4. A thin diverging lens has a focal length of 10 cm. Find by (i) calculation and (ii) construction of a ray diagram the position of the image of an object for object distance equal to (a) 30 cm and (b) 4 cm. Also find the magnification for each case. Describe whether the image is real or imaginary, erect or inverted, magnified or diminished in size.

[5 Marks]

[Total marks 20]

QUESTION THREE

3.1 State at least 3 assumptions for the derivation of a one-dimensional wave equation. (3)

3.2 Show that y(x,t) = cos (4x + 4ct) satisfies the one-dimensional wave equation (6)

3.3 Show that the function $y(x,t) = X^2 + V^2t^2$ is a solution of the general wave equation. (6)

3.4 Show that the one-dimensional wave equation is satisfied by the following function.

$y = A\sqrt{(x + vt)}$. That is, show that the equation satisfies both the left and right side of the one-dimensional equation. (5)

Total marks [20]

QUESTION FOUR

Instruction: write down **ONLY THE LETTER CORRESPONDING TO THE ANSWER OF YOUR CHOICE** below

1. The differential equation of motion of a simple harmonic motion is:

A. $\frac{d^2x}{dt^2} + \omega t = 0$ **B.** $\frac{d^2x}{dt^2} = -\omega x$ **C.** $\frac{d^2t}{dx^2} + \omega^2 x = 0$, **D.** $\frac{d^2t}{dx^2} = -\omega^2 x$ [1]

2. In simple harmonic motion, the particle is

A. always accelerating **B.** always retarding **C.** alternately accelerating and retarding **D.** steady state *(does not change).* [1]

3. *Statement:* Simple harmonic motion is the projection of uniform circular motion on the diameter of the circle in which the latter motion occurs. ***Reason:*** Simple harmonic motion is a uniform motion.

A. Both statement and reason are true, and reason is the correct explanation of statement.

B. Both statement and reason are true, but reason is not the correct explanation of statement.

C. Statement is true but reason is false.

D. Both assertion and reason are false. [1]

4. Which one of the following equations of motion represents simple harmonic motion?

A. Acceleration = F\m

B. Acceleration = k(x+a)

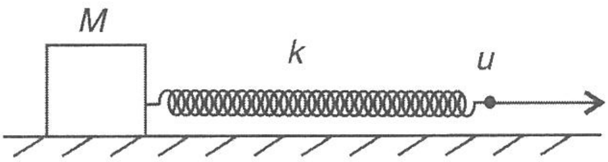
C. Acceleration = −k(x+a)

D. Acceleration = kx [1]

5. The equation of motion of a particle of mass 1g is $\frac{d^2x}{dt^2}$ +π^2x = 0, where x is displacement (in m) from mean position. The frequency of oscillation is (in Hz):

A. (1/5)√10 **B.** 2 **C.** 5√10 **D.** ½ [1]

6. A block of mass M is placed on a horizontal smooth table. It is attached to an ideal spring of force constant k as shown. The free end of the spring is pulled at a constant speed u. Find the maximum extension (x0) in the spring during the subsequent motion. [2]



A. $x_o = \sqrt{\frac{2M}{3k}}u$ **B.** $x_o = \sqrt{\frac{k}{2M}}\,u$ **C.** $x_o = \sqrt{\frac{M}{k}}\,u$ **D.** $x = \sqrt{\frac{2M}{k}}\,u$

7. A sinusoidal wave $f_1(t)$ = 3 sin ω t is superposed on another periodic wave $f_2(t)$ which is also oscillating with the same frequency. If the resultant wave is a sinusoidal wave, which is out of phase with $f_1(t)$ by 53.1° ; then the wave $f_2(t)$ would be-

Hint: tan 53.1° = 1.33

A. f_2(t) = 3 sin(ωt) **B.** f_2(t) = 4 sin(ωt + 53.1°) **C.** f_2(t) = 3 cos(ωt – 53.1°) **D.** f_2(t) = 4cos(ωt) [2]

8. Two waves represented by $y_1 = a\sin\left(\omega t + \frac{\pi}{6}\right), y_2 = a\cos\omega t$, the resultant amplitude will be:

A. a **B.** $a\sqrt{3}$ **C.** $a\sqrt{2}$ **D.** 2a [2]

9. If f (t) = A sin ωt + B cos ωt is written as f (t) = D sin (ωt + φ) then 'φ' is equal to?

tan^{-1}(A/B √(A^2 + B^2) tan^{-1}(B/A) A + B

A. B. C. D.

[2]

A. π/6 B. π C. 2π/3 D. π/3 [2]

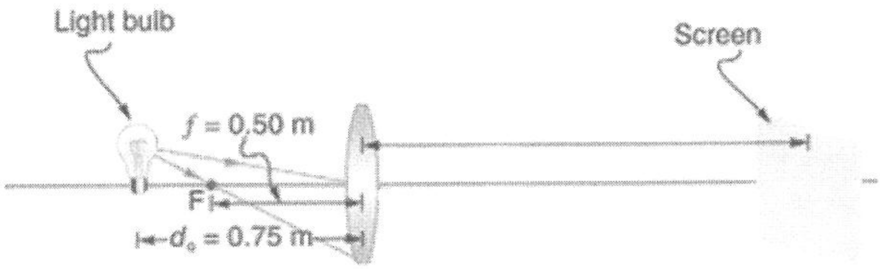
10. Two waves represented as x = a sin(ωt + π/6), x = a cos ωt, then resultant phase difference between them is:

11. The first diffraction minima due to a single slit is observed at 30° for light of wavelength 6500 A. The width of the slit is:

A. 3250 A^0 **B.** 6.5×10^{-4} mm **C.** 2.6×10^{-4} cm **D.** $1.3\text{x } 10^{-6}$ [1]

12. A clear glass light bulb is placed 0.750 m from a convex lens having a 0.500 m focal length, as shown in the Figure. Use ray tracing method to get an approximate location for the image. Then use the thin lens equations to calculate:

(a) the location of the image and (b) its magnification.



A. (a) d_i = 0.75 m, (b) m = + 1 **B.** (a) d_i = 1.5 m, (b) m = -2.

C. (a) d_i = -0.75 m, (b) m = -1. **D.** (a) d_i = -1.5 m, (b) m = +2. [2]

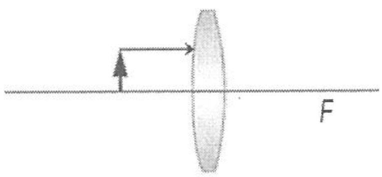
13. Suppose the book page in the Figure below. (a) is held 7.50 cm from a convex lens of focal length 10.0 cm, such as a typical magnifying glass might have. What magnification is produced?

A. (a) d_i = -30 cm, (b) m = 3, (d) image is virtual and upright.

B. (a) d_i = -30 cm, (b) m = 3, (c) image is real and upright.

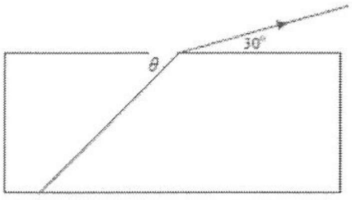
C. (a) d_i = 30 cm, (b) m = -3, (c) image is real and inverted.

D. (a) d_i = 30 cm, (b) m = 3, (d) image is virtual and upright.



[2]

14. The diagram shows a ray of light as it leaves a rectangular block of glass. As the ray of light leaves the block of glass, it makes an angle θ with the inside surface of the glass block and an angle of 30° when it is in the air, as shown in the diagram.



(i) If the refractive index of the glass is 1.5, calculate the value of θ.

(ii) What would be the value of the angle θ so that the ray of light emerges parallel to the side of the glass block?

A. (i) $\theta = 54.^0 74$, (ii) $\theta = 48.^0 2$ **B.** (i) $\theta = 35.^0 26$, (ii) $\theta = 41^0.82$

C. $\theta = 41.^0 82$, (ii) $\theta = 35^0.26$ **D.** $\theta = 48.^0 2$, (ii) $\theta = 54^0.74$ [2]

15. Suppose you have a concave lens of focal length f = 23:0 cm. An object of height 5.0 cm is placed 33.0 cm from the lens. Using the algebraic method, determine: (a) the image distance; (b) the image height; (c) the image magnification; (d) whether the image is real or virtual; and (e) whether the image is upright or inverted.

A. (a) $d_i = -13.5$ cm, (b) $h_i = +2.05$cm, (c) m = +0.4, (d) image is virtual, (e) d_i is upright.

B. (a) $d_i = -13.55$ cm, (b) $h_i = +2.05$cm, (c) m = +0.4, (d) image is real, (e) d_i is upright.

C. (a) $d_i = 13.55$ cm, (b) $h_i = -2.05$cm, (c) m = -0.5, (d) image is virtual, (e) d_i is inverted.

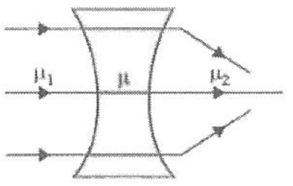
D. (a) $d_i = -13.55$ cm, (b) $h_i = -2.05$cm, (c) m = +0.5, (d) image is real, (e) d_i is inverted. [2]

16. The critical angle (for total internal reflection) for a certain liquid-air surface is $47^0.7$. What is the index of refraction of the liquid?

A. 1.551 **C.** 1.351 **B.** 1.451 **D.** 1.251 [2]

17. The behaviour of light rays is as shown in the figure below. The relation between refractive indices μ, μ_1 and μ_2 is:

A $\mu > \mu_2 > \mu_1$ **B** $\mu < \mu_2 < \mu_1$ **C.** $\mu < \mu_2, \mu = \mu_1$ **D** $\mu_2 < \mu_1, \mu = \mu_2$

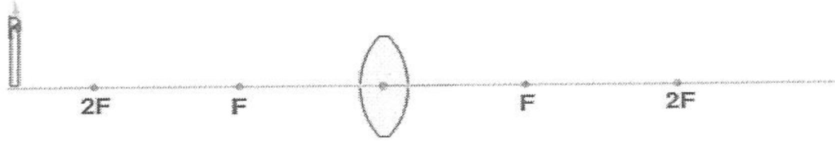


[1]

18. A boy of height 1m stands in front of a convex mirror. His distance from the mirror is equal to its focal length. The height of his image is

A. 0.25m **B.** 0.33m **C.** 0.5m **D.** 0.67m [1]

19. An object is placed in front of a converging lens as shown in the figure at a distance greater than 2F. The image produced by the lens is:



A. Real, inverted and magnified

B. Virtual, upright and magnified

C. Virtual, upright and demagnified

D. Real, inverted and demagnified

E. Virtual, inverted and magnified [2]

TOTAL [30]

Formulae Sheet - Waves Vibrations and Optics

1 Waves Motion

General equation of wave: $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$.

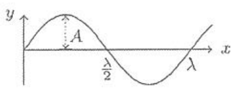
Notation: Amplitude A, Frequency ν, Wavelength λ, Period T, Angular Frequency ω, Wave Number k,

$$T = \frac{1}{\nu} = \frac{2\pi}{\omega}, \quad v = \nu\lambda, \quad k = \frac{2\pi}{\lambda}$$

Progressive wave travelling with speed v:

$$y = f(t - x/v), \rightsquigarrow +x; \quad y = f(t + x/v), \rightsquigarrow -x$$

Progressive sine wave:



$$y = A\sin(kx - \omega t) = A\sin(2\pi\,(x/\lambda - t/T))$$

2 Waves on a String

Speed of waves on a string with mass per unit length μ and tension T: $v = \sqrt{T/\mu}$

Transmitted power: $P_{\text{av}} = 2\pi^2\mu v A^2\nu^2$

Interference:

$$y_1 = A_1\sin(kx - \omega t), \quad y_2 = A_2\sin(kx - \omega t + \delta)$$

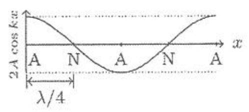
$$\boxed{y = y_1 + y_2 = A\sin(kx - \omega t + \epsilon)}$$

$$A = \sqrt{{A_1}^2 + {A_2}^2 + 2A_1A_2\cos\delta}$$

$$\tan\epsilon = \frac{A_2\sin\delta}{A_1 + A_2\cos\delta}$$

$$\delta = \begin{cases} 2n\pi, & \text{constructive;} \\ (2n+1)\pi, & \text{destructive.} \end{cases}$$

Standing Waves:

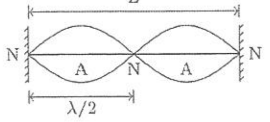


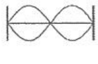
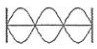
$$y_1 = A_1\sin(kx - \omega t), \quad y_2 = A_2\sin(kx + \omega t)$$

$$\boxed{y = y_1 + y_2 = (2A\cos kx)\sin\omega t}$$

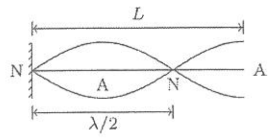
$$x = \begin{cases} \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, & \text{nodes; } n = 0, 1, 2, \ldots \\ n\frac{\lambda}{2}, & \text{antinodes. } n = 0, 1, 2, \ldots \end{cases}$$

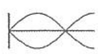
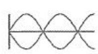
String fixed at both ends:



1. Boundary conditions: $y = 0$ at $x = 0$ and at $x = L$
2. Allowed Freq.: $L = n\frac{\lambda}{2}$, $\nu = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$, $n = 1, 2, 3, \ldots$.
3. Fundamental/1$^{\text{st}}$ harmonics: $\nu_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ 
4. 1$^{\text{st}}$ overtone/2$^{\text{nd}}$ harmonics: $\nu_1 = \frac{2}{2L}\sqrt{\frac{T}{\mu}}$ 
5. 2$^{\text{nd}}$ overtone/3$^{\text{rd}}$ harmonics: $\nu_2 = \frac{3}{2L}\sqrt{\frac{T}{\mu}}$ 
6. All harmonics are present.

String fixed at one end:



1. Boundary conditions: $y = 0$ at $x = 0$
2. Allowed Freq.: $L = (2n+1)\frac{\lambda}{4}$, $\nu = \frac{2n+1}{4L}\sqrt{\frac{T}{\mu}}$, $n = 0, 1, 2, \ldots$.
3. Fundamental/1$^{\text{st}}$ harmonics: $\nu_0 = \frac{1}{4L}\sqrt{\frac{T}{\mu}}$ 
4. 1$^{\text{st}}$ overtone/3$^{\text{rd}}$ harmonics: $\nu_1 = \frac{3}{4L}\sqrt{\frac{T}{\mu}}$ 
5. 2$^{\text{nd}}$ overtone/5$^{\text{th}}$ harmonics: $\nu_2 = \frac{5}{4L}\sqrt{\frac{T}{\mu}}$ 
6. Only odd harmonics are present.

Sonometer: $\nu \propto \frac{1}{L}$, $\nu \propto \sqrt{T}$, $\nu \propto \frac{1}{\sqrt{\mu}}$. $\nu = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

3 Sound Waves

Displacement wave: $s = s_0\sin\omega(t - x/v)$

Pressure wave: $p = p_0\cos\omega(t - x/v)$, $p_0 = (B\omega/v)s_0$

Speed of sound waves:

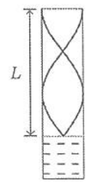
$$v_{\text{liquid}} = \sqrt{\frac{B}{\rho}}, \quad v_{\text{solid}} = \sqrt{\frac{Y}{\rho}}, \quad v_{\text{gas}} = \sqrt{\frac{\gamma P}{\rho}}$$

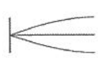
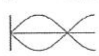
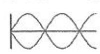
Intensity: $I = \frac{2\pi^2 B}{v}{s_0}^2\nu^2 = \frac{{p_0}^2 v}{2B} = \frac{{p_0}^2}{2\rho v}$

Standing longitudinal waves:

$$p_1 = p_0\sin\omega(t - x/v), \quad p_2 = p_0\sin\omega(t + x/v)$$
$$p = p_1 + p_2 = 2p_0\cos kx\sin\omega t$$

Closed organ pipe:

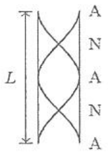


1. Boundary condition: $y = 0$ at $x = 0$
2. Allowed freq.: $L = (2n+1)\frac{\lambda}{4}$, $\nu = (2n+1)\frac{v}{4L}$, $n = 0, 1, 2, \ldots$
3. Fundamental/1$^{\text{st}}$ harmonics: $\nu_0 = \frac{v}{4L}$ 
4. 1$^{\text{st}}$ overtone/3$^{\text{rd}}$ harmonics: $\nu_1 = 3\nu_0 = \frac{3v}{4L}$ 
5. 2$^{\text{nd}}$ overtone/5$^{\text{th}}$ harmonics: $\nu_2 = 5\nu_0 = \frac{5v}{4L}$ 

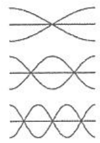
Formulae Sheet - Waves Vibrations and Optics

6. Only odd harmonics are present.

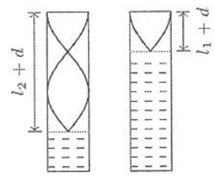
Open organ pipe:



1. Boundary condition: $y = 0$ at $x = 0$
   Allowed freq.: $L = n\frac{\lambda}{2}$, $\nu = n\frac{v}{4L}$, $n = 1, 2, \ldots$
2. Fundamental/1[st] harmonics: $\nu_0 = \frac{v}{2L}$
3. 1[st] overtone/2[nd] harmonics: $\nu_1 = 2\nu_0 = \frac{2v}{2L}$
4. 2[nd] overtone/3[rd] harmonics: $\nu_2 = 3\nu_0 = \frac{3v}{2L}$
5. All harmonics are present.



Resonance column:



$$l_1 + d = \frac{\lambda}{2}, \quad l_2 + d = \frac{3\lambda}{4}, \quad v = 2(l_2 - l_1)\nu$$

Beats: two waves of almost equal frequencies $\omega_1 \approx \omega_2$

$$p_1 = p_0 \sin\omega_1(t - x/v), \quad p_2 = p_0 \sin\omega_2(t - x/v)$$
$$p = p_1 + p_2 = 2p_0 \cos\Delta\omega(t - x/v)\sin\omega(t - x/v)$$
$$\omega = (\omega_1 + \omega_2)/2, \quad \Delta\omega = \omega_1 - \omega_2 \quad \text{(beats freq.)}$$

Doppler Effect:

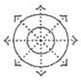
$$\nu = \frac{v + u_o}{v - u_s}\nu_0$$

where, v is the speed of sound in the medium, u_0 is the speed of the observer w.r.t. the medium, considered positive when it moves towards the source and negative when it moves away from the source, and u_s is the speed of the source w.r.t. the medium, considered positive when it moves towards the observer and negative when it moves away from the observer.

4 Light Waves

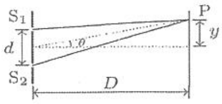
Plane Wave: $E = E_0 \sin\omega(t - \frac{x}{v})$, $I = I_0$

Spherical Wave: $E = \frac{aE_0}{r}\sin\omega(t - \frac{r}{v})$, $I = \frac{I_0}{r^2}$



Young's double slit experiment

Path difference: $\Delta x = \frac{dy}{D}$



Phase difference: $\delta = \frac{2\pi}{\lambda}\Delta x$

Interference Conditions: for integer n,

$$\delta = \begin{cases} 2n\pi, & \text{constructive;} \\ (2n+1)\pi, & \text{destructive,} \end{cases}$$

$$\Delta x = \begin{cases} n\lambda, & \text{constructive;} \\ \left(n + \frac{1}{2}\right)\lambda, & \text{destructive} \end{cases}$$

Intensity:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta,$$
$$I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2, \; I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$
$$I_1 = I_2 : I = 4I_0\cos^2\frac{\delta}{2}, \; I_{\max} = 4I_0, \; I_{\min} = 0$$

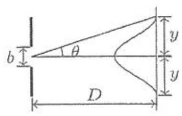
Fringe width: $w = \frac{\lambda D}{d}$

Optical path: $\Delta x' = \mu\Delta x$

Interference of waves transmitted through thin film:

$$\Delta x = 2\mu d = \begin{cases} n\lambda, & \text{constructive;} \\ \left(n + \frac{1}{2}\right)\lambda, & \text{destructive.} \end{cases}$$

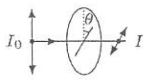
Diffraction from a single slit:



For Minima: $n\lambda = b\sin\theta \approx b(y/D)$

Resolution: $\sin\theta = \frac{1.22\lambda}{b}$

Law of Malus: $I = I_0\cos^2\theta$



Formula Sheet for Wave Properties and Light

WAVE MOTION

A. Descriptive Variables

1. **Types:** Transverse, longitudinal, traveling, standing, harmonic
   a. General form for transverse traveling wave: $y = f(x - vt)$ (to the right) or $y = f(x + vt)$ (to the left)
   b. General form of **harmonic wave**: $y = A\sin(kx - \omega t)$ or $y = A\cos(kx - \omega t)$
   c. **Standing wave:** Integral multiples of $\frac{\lambda}{2}$ fit the length of the oscillating material
   d. **General wave equation:** $\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$
   e. **Superposition Principle:** Overlapping waves interact => constructive and destructive interference

Harmonic Wave Properties

Wavelength	λ (m)	Distance between peaks
Period	T (sec)	Time to travel one λ
Frequency	f (Hz)	$f = \frac{1}{T}$
Angular Frequency	ω (rad/s)	$\omega = \frac{2\pi}{T} = 2\pi f$
Wave Amplitude	A	Height of wave
Speed	v (m/s)	$v = \lambda f$
Wave number	k (m^{-1})	$k = \frac{2\pi}{\lambda}$

2. **Sample:** Determine the velocity and period of a wave with $\lambda = 5.2$ m and $f = 50.0$ Hz
   Equations: $v = \lambda f$ $\quad T = \frac{1}{f}$
   Data: $\lambda = 5.20$ m; $f = 50.0$ Hz
   Calculations: $v = \lambda f = 5.20 \text{ m} \times 50.0 = 260$ m/s
   $T = \frac{1}{f} = \frac{1}{50}$ Hz $= 0.02$ s

B. Sound Waves

1. **Wave nature of sound:** Compression wave displaces the medium carrying the wave
2. **General speed of sound:** $v = \sqrt{\frac{B}{\rho}}$;
   note: B = **Bulk Modulus** (measure of volume compressibility)
   For a gas: $v = \sqrt{\frac{\gamma RT}{M}}$; note: $\gamma = \frac{C_p}{C_v}$ (ratio of gas heat capacities)
   Sample: Calculate speed of sound in Helium at 273 K
   Helium: Ideal gas, $\gamma = 1.66$; $M = 0.004$ kg/mole
   $$v = \sqrt{\frac{\gamma RT}{M}}$$
   $$= \sqrt{\frac{1.66 \times 8.314 \text{ kg m}^2/\text{s}^2 \times 273\text{K}}{0.004\text{kg}}}$$
   $= \sqrt{941{,}900 \text{ m}^2/\text{s}^2} = 971$ **m/s** **note:** $\sqrt{\ }$ applies to the units
3. **Loudness as intensity and relative intensity**
   a. **Absolute Intensity** (I = Power/Area) is an inconvenient measure of loudness
   b. Relative loudness: **Decibel scale (dB):** $\beta = 10 \log \frac{I}{I_0}$; I_0 is the threshold of hearing; $\beta(I_0) = 0$
   c. **Samples:** Jet plane: 150 dB; Conversation: 50 dB; a change in 10 dB represents a 10–fold increase in I
4. **Doppler effect:** The sound frequency shifts $\frac{f'}{f}$ due to relative motion of source and listener;
   v_0 - listener speed; v_s - source speed; v - speed of sound
   $$\frac{f'}{f} = \frac{v + v_0}{v - v_s} \qquad \frac{f'}{f} = \frac{v - v_0}{v + v_s}$$

Key: Identify relative speed of source and listener

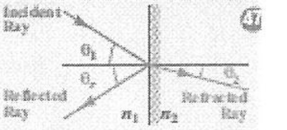
BEHAVIOR OF LIGHT

A. Basic Properties of Light

1. **Goal:** Examine light and its interaction with matter
2. **Key variables:**
   a. c: speed of light in a vacuum
   b. **Index of refraction:** n; $\frac{c}{n}$ = speed of light in medium
   c. **Light as electromagnetic wave:** $\lambda f = c$
   Light characterized by "color" or wavelength
   d. **Light as particle:** $e = hf$; energy of photon
3. **Reflection and Refraction of Light** fig 47
   a. **Law of Reflection:** $\theta_i = \theta_r$ fig 48
   b. **Refraction:** Bending of light ray as it passes from n_1 to n_2
   ***Snell's Law:** $n_1\sin\theta_1 = n_2\sin\theta_2$, n_1, n_2 are the indices of refraction of two materials fig 49
   c. **Internal Reflectance:** $\sin\theta_c = \frac{n_2}{n_1}$
   Light passing from material of higher n to a lower n may be trapped in the material
4. **Polarized light:** E field is not spherically symmetric
   a. **Examples:** Plane/linear polarized, circularly polarized
   b. Polarization by reflection from a dielectric surface at angle θ_c;
   Brewster's Law: $\tan\theta_c = \frac{n_2}{n_1}$

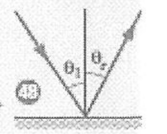
Reflection and Refraction

47



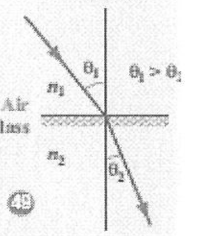
Normal

48



Normal

49



B. Lenses and Optical Instruments

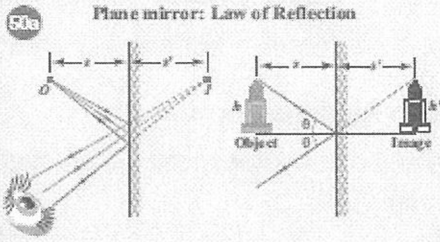
1. **Goal:** Lenses and mirrors generate images of objects
2. **Key concepts and variables**
   a. Radius of curvature: $R = 2f$
   b. **Optic axis:** Line from base of object through center of lens or mirror
   c. **Magnification:** $M = \frac{s'}{s}$
   d. **Laws of Geometric Optics:**
   $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$; $\frac{s}{s'} = -\frac{h}{h'}$
   e. Combination of 2 thin lenses:
   $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ or $f = \frac{f_1 f_2}{f_1 + f_2}$

Lens and Mirror Properties

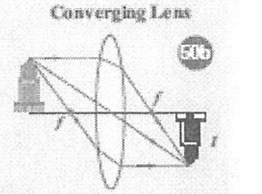
Parameters	+ sign	- sign
f focal length	converging lens concave mirror	diverging lens convex mirror
s object dist.	real object	virtual object
s' image dist.	real image	virtual image
h object size	erect	inverted
h' image size	erect	inverted

3. **Sample Guidelines for ray tracing:**
   a. Rays that parallel optic axis pass through "f"
   b. Rays pass through center of the lens unchanged
   c. Image forms at convergence of ray tracings
   Sample ray tracings: fig 50, a,b,c

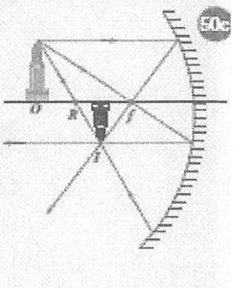
50a Plane mirror: Law of Reflection



Converging Lens 50b



Spherical Concave Mirror 50c



C. Interference of Light Waves

1. **Goal:** Examine constructive and destructive interference of light waves
2. **Key Variables and Concepts:**
   a. **Constructive interference:** fig 51
   b. **Destructive interference:** fig 52
   c. **Huygens' Principle:** Each portion of wave front acts as a source of new waves
3. Diffraction of light, from a **grating** with spacing d, produces an interference pattern; $d\sin\theta = m\lambda$; $(m = 0, 1, 2, 3,...)$
4. **Single-slit experiment**, slit width a; destructive interference for $\sin\theta = \frac{m\lambda}{a}$; $(m = 0, \pm1, \pm2...)$
5. **X-ray diffraction** from a crystal with atomic spacing d; $2d\sin\theta = m\lambda$; $(m = 0, 1, 2, 3,...)$

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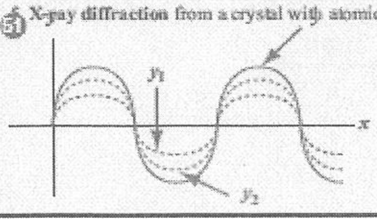
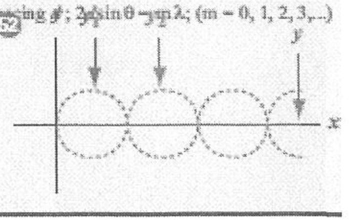


52



Formula Sheet for Wave Properties and Light

Formulas and Identities

Tangent and Cotangent Identities

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Reciprocal Identities

$$\csc(\theta) = \frac{1}{\sin(\theta)} \qquad \sin(\theta) = \frac{1}{\csc(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)} \qquad \cos(\theta) = \frac{1}{\sec(\theta)}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)} \qquad \tan(\theta) = \frac{1}{\cot(\theta)}$$

Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin(\theta) \qquad \csc(-\theta) = -\csc(\theta)$$

$$\cos(-\theta) = \cos(\theta) \qquad \sec(-\theta) = \sec(\theta)$$

$$\tan(-\theta) = -\tan(\theta) \qquad \cot(-\theta) = -\cot(\theta)$$

Periodic Formulas

If n is an integer then,

$$\sin(\theta + 2\pi n) = \sin(\theta) \qquad \csc(\theta + 2\pi n) = \csc(\theta)$$

$$\cos(\theta + 2\pi n) = \cos(\theta) \qquad \sec(\theta + 2\pi n) = \sec(\theta)$$

$$\tan(\theta + \pi n) = \tan(\theta) \qquad \cot(\theta + \pi n) = \cot(\theta)$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\begin{aligned}\cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta)\\ &= 2\cos^2(\theta) - 1\\ &= 1 - 2\sin^2(\theta)\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

Half Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}}$$

Half Angle Formulas (alternate form)

$$\sin^2(\theta) = \tfrac{1}{2}(1 - \cos(2\theta)) \qquad \tan^2(\theta) = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

$$\cos^2(\theta) = \tfrac{1}{2}(1 + \cos(2\theta))$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$$

Product to Sum Formulas

$$\sin(\alpha)\sin(\beta) = \tfrac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos(\alpha)\cos(\beta) = \tfrac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin(\alpha)\cos(\beta) = \tfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos(\alpha)\sin(\beta) = \tfrac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

Sum to Product Formulas

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin(\alpha) - \sin(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta) \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec(\theta) \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc(\theta)$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot(\theta) \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan(\theta)$$

Formula Sheet for Wave Properties and Light

Refractive index of various substances

Common Transparent Materials

Eye, Aqueous humor 1.33
Eye, Cornea 1.38
Eye, Lens 1.41
Eye, Vitreous humor 1.34
Glass, Arsenic trisulfide 2.04
Glass, Borosilicate glass (BSG) 1.523 – 1.526
Glass, Crown (common) 1.52
Glass, Flint, 29% lead 1.569
Glass, Flint, 55% lead 1.669
Glass, Flint, 71% lead 1.805
Glass, phosphosilicate glass (PSG) 1.47
Glass, Fused Silica 1.459
Glass, Pyrex 1.474
Lucite 1.495
Nylon 1.53
Obsidian 1.50
Plastic 1.460 - 1.55
Plexiglas 1.488
Salt 1.516

Vacuum

Vacuum 1.00000

Gases

Air 1.00027

Common Liquids

Acetone 1.36
Alcohol, Ethyl (grain) 1.36
Alcohol, Methyl (wood) 1.329
Beer 1.345
Carbonated Beverages 1.34 - 1.356
Fruit Juice
Chlorine (liq) 1.385
Cranberry Juice (25%) 1.351
Glycerin 1.473
Honey, 13% water content 1.504
Honey, 17% water content 1.494
Honey, 21% water content 1.484
Ice 1.309
Milk 1.35
Oil, Clove 1.535
Oil, Lemon 1.481
Oil, Neroli 1.482
Oil, Orange 1.473
Oil, Safflower 1.466
Oil, vegetable (50° C) 1.47
Oil of Wintergreen 1.536
Rum, White 1.361
Shampoo 1.362
Sugar Solution 30% 1.38
Sugar Solution 80% 1.49
Turpentine 1.472
Vodka 1.363
Water (0° C) 1.33346
Water (100° C) 1.31766
Water (20° C) 1.33283
Whisky 1.356

Metals

Aluminum 1.39
Copper 2.43
Gold 0.166
Mylar 1.65
Nickel 1.08
Platinum 2.33
Silver 1.35
Titanium 2.16