MAT 111

DEGREE EXAMINATIONS

June 2023

Subject: **Mathematics 1**
Paper: A Theoretical Approach to Differential Calculus

Time: **3 Hours** Marks: **100**

This question paper consists of 3 pages , including the Cover page

Internal examiner

Dr O Ndiweni

Internal examiner

Mr Z Mahlasela

Instructions

Answer all questions.
Symbols have the usual meanings

Question One

1.1 Prove by Mathematical Induction that for every positive integers, n, n^3+2n is divisible by 3. (3)

1.2Solve the inequality and represent your solution on a number line.

$$|1-2x|<5$$ (3)

1.2 State and prove the Binomial Theorem. (6)

1.3 State without proof the Binomial Series. Hence find the series expansion of $\frac{1}{\sqrt{1+x}}$.For which values of x is the expansion valid?

Use this expansion to estimate $\frac{1}{\sqrt{1.1}}$ correct to three decimal places. (5)

1.5 Define a one-to-one function. Find the inverse of $f(x)=\frac{2x+1}{x+2}$ (3)

1.6 Given the function $f(x)=\frac{1}{x-3}$.

Write down its domain and on the same $xy-plane$ sketch the graphs of the function (a) $f(x)$ (b) $f(x)+2$ (c) $f(x+2)$.Describe fully the transformations (b) and (c). (4)

1.7Find $fogoh$ if $f(x)=\tan x, g(x)=\frac{x}{x-1}$ and $h(x)=\sqrt[3]{x}$ (3)

[30]

Question Two

2.1 State the $\varepsilon-\delta$ definition of a limit and use this definition to prove

$$\lim_{x\to 10}\left(3-\frac{4}{5}x\right)=-5$$ (7)

2.2State without proof 1'Hospital Rule and hence evaluate $\lim_{x\to 0}\frac{\sin 3}{\sin 4x}$. (5)

2.3 Evaluate $\lim_{x\to 0}\left(\frac{4\tan x}{x}\right)$ State the limit theorem(s) you used. (4)

2.4State the three conditions to be satisfied for a function f to be continuous at a point $x=x_0$. Hence determine whether the f given below is continuous at $x=0$.

$$f(x)=\begin{cases}\frac{x-6}{x-3} & if \quad x<0\\ 2 & if \quad x=0\\ \sqrt{4+x^2} & if \quad x>0\end{cases}$$ (7)

[23]

Question Three

3.1From definition find $f'(x)$ for the function $f(x)=\dfrac{x}{x-1}$ (3)

3.2Find $\dfrac{dy}{dx}$ by implicit differentiation for $x^2y+xy+xy^2=8$ (3)

3.3Find $\dfrac{dy}{dx}$ if (a) $y=e^x\ln x$ (3)

(b) $x^y=y^x$ (4)

(c) $y=5^{\tan x}+(\sin x)^x$ (5)

(d) $y=\sqrt{\ln x}$ (4)

3.4 Find $\dfrac{d^9}{dx^9}\left(x^8\ln x\right)$ (5)

[27]

Question Four

4.1Find the maximum and minimum values of $4x^3+15x^2-18x+7$, distinguishing between them. Sketch the graph of the function. (5)

4.2Find the equation of the tangent line to the curve $y=10^x$ at the point $x=1$. (5)

4.3Given that $z=\sqrt{(9+x^2)}$, find $\dfrac{dz}{dx}$.Use your answer to calculate an approximation for the change in z when x decreases from 4 to 3.99. (5)

4.4 Find $f(x)$ if $f''(x)=x^4-4x^2+3x-2$, $f(0)=0$ and $f(1)=1$ (5)

[20]