



University of Fort Hare
Together in Excellence

PHY 322

MATHEMATICAL METHODS OF PHYSICS

SUPPLEMENTARY EXAMINATIONS

JANUARY

YEAR: 2019

TIME : 3 hours
SUBJECT : Mathematical Methods of Physics
MARKS : 100

This paper consists of FOUR pages including the cover page

INTERNAL EXAMINER

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EXTERNAL EXAMINER

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INSTRUCTIONS

ANSWER ANY **FIVE** QUESTIONS

QUESTION 1 [20 marks]

- 1.1. Find the directional derivative of the vector $4\bar{i} + 4\bar{j} - 2\bar{k}$ at the point 1,1,2; [10]
if $\phi(x, y, z) = x^2 + y^2 - z$
- 1.2. Prove that $\nabla \cdot (A + B) = \nabla \cdot A + \nabla \cdot B$ [10]
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QUESTION 2 [20 marks]

- 2.1. Write an expression for **curl** in orthogonal curvilinear coordinates. [4]
- 2.2. Use the divergence theorem to evaluate $\iiint_s \bar{F} \cdot d\bar{S}$ where $\bar{F} = xy\bar{i} - \frac{1}{2}y^2\bar{j} + z\bar{k}$ and the surface consists of the three surfaces, $z = 4 - 3x^2 - 3y^2$, $1 \leq z \leq 4$ on the top, $x^2 + y^2 = 1$, $0 \leq z \leq 1$, on the sides and $z = 0$ on the bottom. [10]
- 2.3. Given $A = 5 \sin \theta a_\theta + 5 \sin \phi a_\phi$, find $\nabla \cdot A$ at $(0.5, \pi/4, \pi/4)$ [6]
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QUESTION 3 [20 marks]

- 3.1. Show that $\partial(g(x)) = \frac{\partial(x - x_1)}{|g'(x_1)|}$ where x_1 are roots of the function $g(x)$ [7]
- 3.2. Use the property in part 3.1 above to expand $\delta(g(x)) = \delta(\sin x)$ [6]
- 3.3. Sketch the solution in part 3.2 above. [4]
- 3.4. Use the property properties of the Dirac delta function to integrate [3]
 $\int_1^4 f(t) \delta(t - 2) dt$
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QUESTION 4 [20 marks]

4.1. Deduce the Legendre equation from spherical coordinates, by using method of separation of variables and assuming azimuthal symmetry. [12]

4.2. Legendre's polynomials are given as : $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$ [2]

Write down the first two polynomials .

4.3. Express the general solution of the given differential equation in terms of the Bessel functions : $4x^2 y'' + 8xy' + (x^4 - 3)y = 0$.

The solution should be in the form $Y(x) = x^\alpha \left[c_1 J_p(kx^\beta) + c_2 J_{-p}(kx^\beta) \right]$

where p, k, α and β have their usual meanings. [6]

QUESTION 5 [20 marks]

5.1. Find the rank of $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 4 \\ -1 & -3 & 1 \end{bmatrix}$ [6]

5.2. Determine the eigenvalues of $B = \begin{bmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{bmatrix}$. [8]

5.3. Find the determinant of $A = \begin{bmatrix} 2 & -6 & 2 \\ 2 & -8 & 3 \\ -3 & 1 & 1 \end{bmatrix}$ [6]

QUESTION 6 [20 marks]

6.1. State the rank of the following;

6.1.1. Scalar

6.1.2. Vector [2]

6.2. Let T be a second order tensor. Show that $(T^m)^T = (T^T)^m$ [3]

6.3. Show that $\mathbf{T}:1 = \text{Tr}(\mathbf{T})$, where T is an arbitrary second-order Tensor [5]

6.4. Using the properties of Kronecker and the permutation tensor, write in tensor notation, the proof of the following identity

$$\bar{C} \times (\bar{A} \times \bar{B}) = \bar{A}(\bar{C} \cdot \bar{B}) - \bar{B}(\bar{C} \cdot \bar{A}) \quad [10]$$

$$\left\{ \text{Note : } \varepsilon^{ijv} \varepsilon_{v\alpha\beta} = \partial_{\alpha}^i \partial_{\beta}^j = (\partial_{\alpha}^i \partial_{\beta}^i - \partial_{\beta}^i \partial_{\alpha}^j) \right\}$$

QUESTION 7 [20 marks]

7.1. Find the Fourier series of

$$f(x) = x^2, \quad -1 < x < 1 \quad [8]$$

7.2. Find the complex Fourier series of [9]

$$f(x) = x, \quad -\pi < x < \pi$$

7.3. Calculate the Fourier transform of h(x) [3]
