UNIVERSITY OF FORT HARE

University of Fort Hare
Together in Excellence

PHY 321

DEGREE EXAMINATIONS

NOVEMBER

2024

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Time:	**3 HOURS**
Subject:	**Quantum Mechanics**
Code:	**PHY 321**
Marks:	**100**

This paper consists of 5 pages including the cover page

Internal Examiners

Prof M Simon

External Examiners

Prof. D Tinarwo (Univen)

INSTRUCTIONS

Answer ALL Five (5) Questions.

Question 1 [20]

a) List 4 properties of a wave function (4)

b) Given a de-Broglie wave as:

$$\psi_{(x,t)} = Ae^{i(kx-wt)}$$

Prove and show **ALL** steps that $\frac{\partial^2\psi}{\partial x^2} = -w^2\psi(x,t)$ (6)

c) A free electron has a wave function $\psi_{(x)}$= A sin $(5 \times 10^{10}x)$, where **x** is in meters:

Find:

i) The electron's de-Brogile wavelength (3)
ii) The electron's momentum (3)
iii) The electron's energy in electron volts. (4)

Question 2 [20]

Given the wave function of a particle in a state $\psi_{(x)} = Ne^{\frac{-x^2}{2\alpha}}$, where $N = \left(\frac{1}{\pi\alpha}\right)^{1/4}$

Evaluate **(Δx)(Δp)** and comment on the physical interpretation of the answer.

Hint: To solve (Δx)(Δp), you need the values of $< x >, < x^2 >, < p >, < p^2 >$,. $\psi_{(x)}$ is symmetrical about x = 0; <x> = 0. *Show clearly every stage of the calculation.*

Question 3 [20]

A particle of mass, **m**, trapped in potential **V(x) = 0** for $-a \leq x \leq a$ and **V(x)** = ∞ otherwise.

Evaluate the probability of finding the trapped particle between **x = 0** and **x** = $\left(\frac{a}{n}\right)$ when is in the n^{th}state:

{Hint: Wave function $\psi_{(x)} = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$*}. Show clearly every stage of the calculation.*

Question 4 [20]

Consider an infinite square well. The wave function of a particle trapped in an infinite square well potential of width 2a is found to be:

$$\Psi(x) = C\left\{\cos\left(\frac{\pi x}{2a}\right) + \tfrac{1}{2}\sin\left(\frac{3\pi x}{a}\right) + \tfrac{1}{4}\cos\left(\frac{3\pi x}{2a}\right)\right\} \text{ for } -a \leq x \leq a$$

$$\Psi(x) = 0 \text{ outside}$$

a) Calculate the coefficient C. (8)

b) If the measurement of the total energy is made, what are the possible results of such a measurement? (6)

c) What is the probability of measuring each of them? (6)

Question 5 [20]

a) Consider a particle of mass **m** moving in a one- dimensional potential

$$V(x) = \begin{cases} 0 & -2a < x < 2a \\ \infty & otherwise \end{cases}$$

Find the energy eigenvalues and eigenfunction (10)

b) The normalized wavefunction of a particle is $\psi_{(x,t)} = Ae^{i(ax-bt)}$, where **A**, **a** and **b** are constant: Evaluate the uncertainty in its momentum **(Δp)**? (10)

Hint: $(\Delta p)^2 = \langle p^2 \rangle - \langle p \rangle^2$

*****End.*****

CONSTANTS AND OTHER RELEVANT DATA

Speed of light in vacuum	c	=	2.998×10^{8} m-s^{-1}
Electronic charge	e	=	1.602×10^{-19} C
Boltzmann constant	k	=	1.381×10^{-23} JK^{-1}
		=	8.617×10^{-5} eV K^{-1}
Electron rest mass	m_e	=	9.11×10^{-31} kg
		=	0.511 MeV/c^2
Proton rest mass	m_p	=	938.3 MeV/c^2
Neutron rest mass	m_n	=	939.6 MeV/c^2
Mass unit	u	=	1.661×10^{-27} kg
		=	931.5 MeV/c^2
Planck's constant	h	=	4.136×10^{-15} eV-s
	hc	=	1.24×10^{4} eV-Å
	$\hbar c$	=	1973 eV-Å
Electron-volt	1 eV	=	1.602×10^{-19} J
Angstrom	1 Å	=	10^{-10} m
		=	10 nm
	1 nm	=	10 Å = 10^{-9} m
Stefan-Boltzmann Constant	δ	=	5.670×10^{-8} Wm^{-2} K^{-4}
Bohr radius	a_0	=	5.292×10^{-11} m
		=	$\hbar^2/(ke^2m_e)$
Coulombs Constant	k	=	9×10^{9} Nm^{2}C^{-2}
Bohr magneton	u_B	=	5.788×10^{-5} eV/tesla
	1 tesla (T)	=	1 Wb – m^{-2}
		=	10^4 gauss (G)
1 eV photon has wavelength	λ	=	1.243×10^{-6} m
$ke^2 = 1/4\pi\varepsilon_0$		=	14.40 eV-°A

USEFUL FORMULAE

Relativistic energy	E^2	=	$p^2c^2 + m_0^2c^4$
Compton relations	$\lambda' - \lambda$	=	$\frac{h}{m_e c}(1 - \cos\theta)$
		=	$\frac{hc}{m_e c^2}(1 - \cos\theta)$
Wien's Law	$\lambda_{max}T$	=	2.9×10^{-3} mK
Relativistic kinetic energy	K	=	$(\gamma - 1)\, m_0c^2$
Where	γ	=	$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$
Total energy (relativistic)	E	=	$K + m_0c^2$
		=	γE_0

Values of the integral $I_n = \int_0^\infty x^n e^{-\lambda x^2} dx$ for $n = 0$ to $n = 5$

n	I_n
0	$\frac{1}{2}\sqrt{\frac{\pi}{\lambda}}$
1	$\frac{1}{2\lambda}$
2	$\frac{1}{4}\sqrt{\frac{\pi}{\lambda^3}}$
3	$\frac{1}{2\lambda^2}$
4	$\frac{3}{8}\sqrt{\frac{\pi}{\lambda^5}}$
5	$1/\lambda^3$

If n is even $\int_{-\infty}^{\infty} x^n e^{-\lambda x^2} dx = 2I_n$

If n is odd $\int_{-\infty}^{\infty} x^n e^{-\lambda x^2} dx = 0$