UNIVERSITY OF FORT HARE

MAP 221

DEGREE EXAMINATIONS
November 2024

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Time: 3 HOURS

Subject: Introduction to Analytical Methods

Marks: 100

This question paper consists of 3 pages

Internal examiner(s)

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Instructions

Answer all questions.
Symbols have the usual meanings

Question 1

1.1. What is a solution to a D.E? Give the two types and explain them. (5)

1.2. The number of supermarkets, $C(t)$ through South Africa that are using a computerized checkout System is described by the following initial value problem;

$$\frac{dC}{dt} = C(1 - 0.0005C) \text{ for } C(0) = 1 \;\&\; t > 0.$$

1.2.1. Find the number of supermarkets, $C(t)$ at any time, t. (6)

1.2.2. Compute the number of supermarkets that will be using the computerized method when $t = 10$. (3)

1.3. Show that $(3x^2 + y\cos x)dx + (\sin x - 4y^3)dy = 0$ is an exact differential equation and find its general solution. (5)

1.4. A drink is placed in a room where the ambient temperature is $20^{\circ}C$. A model for the subsequent temperature, T of the drink at time t, in hours is given by Newton's law of cooling. This leads to the differential equation

$$\frac{dT}{dt} = -5(T - 20)$$

1.4.1 Find the general solution to the temperature, T at any time, t. (6)

1.4.2 Find solution of this differential equation, in the case where T=80 when t=0. (4)

1.5. Solve the following differential equation;

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y = 0$$

(6)

[35]

Question 2

2.1 Show that the Laplace transform $\ell\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$ (5)

2.2 Show that $L\{e^{iat}\} = \frac{1}{s-ia}, s > 0.$ (5)

2.3 Show that $\frac{1}{s-ia} = \frac{s+ia}{s^2-a^2}$ (5)

2.4 Find the Laplace transform of the function, $f(t) = \sin 2t \cos 2t$. (10)

2.5 Consider the function, $f(t) = \begin{cases} cost & for\ 0 \leq t \leq \frac{\pi}{2} \\ -1 & for\ t > \frac{\pi}{2} \end{cases}$ and find it's Laplace transform.

(10)

[35]

Question 3

3.1 Consider the following function, $f(x) = sinx$ on $[0, \pi]$

3.1.1 Draw the graph of this function for three periods and (3)

3.1.2 Find the corresponding Fourier cosine series of this function and set $x = \frac{1}{2}\pi$ to

Obtain the value of $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n^2-1}$ (8)

3.2

3.2.1 Show that $\Gamma(n+1) = n\Gamma(n)$ for $n > 0$ (4)

3.2.2 Hence show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ (7)

3.3 Evaluate the following

3.3.1 $\int_0^{\infty} x^2 e^{-5x^2} dx$ (4)

3.3.2 $B(5; \frac{3}{2})$ (4)

[30]

THE END