UNIVERSITY OF FORT HARE
DEPARTMENT OF ECONOMICS

ALICE AND EAST LONDON CAMPUS

ECO222/E

MAIN EXAMINATION
NOVEMBER 2024

Time: 3 Hours

Subject: Mathematical Economics **Marks: 100**

This paper consists of 3 pages including the cover page

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Instructions

- **This paper consists of two Questions**
- **Answer ALL questions, showing clearly all the calculations.**
- **Use clearly drawn diagrams using a pencil.**
- **Calculators are allowed**
- **Answers must be rounded off to two (2) decimal places.**

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Question 1 (50)

1. A firm facing marginal revenue $MR = 250 - 0.25q$ and marginal costs $MC = 36 + 2.4q$ and fixed costs $FC = 120$.

   a) Derive the Total Revenue function. (4)

   b) Derive the Total Costs function. (4)

   c) Derive the Profit function from a) and b) above, also state the economic equation you relied on in your derivation. (4)

   d) State and derive the first-order condition for the profit function in (c) above. (3)

   e) Calculate the level of output that is consistent with maximize profit. (4)

   f) Verify whether the points in (e) represent a maximum or minimum. (2)

   g) Draw the diagram representing the profit function (show all the intercepts and turning points) (8)

2. Lusanda has a utility function $U = 45House^{0.4}Car^{0.5}$. The prices of the two goods, House and Car, are 2 and 3 (measured in millions of rands) for each, respectively, and Lusanda has an income of R450 Million.

   a) Derive the budget constrain equation (2)

   b) State the economic condition for utility maximisation. (2)

   c) Derive the slope of the indifference and budget constraint curves. (4)

   d) Using the economic condition in (b) and the slopes in (c) above, find the Houses and Cars Lusanda needs to maximise his utility. (5)

   e) Depict your answer in (d) on the diagram showing the number of utils, number of Houses and Cars and intercepts of the budget constrain curve. (8)

Question 2 (50)

1. Given the inverse demand function for grapes in Alice as follows $P_{grapes} = 150 - 2q_{grapes} - 0.2q_{grapes}^2$
   a) What is the market clearing price (P_{grapes}^E) when quantity (Q_{grapes}^E) for grapes is 15 units? (3)
   b) What is the value of the consumer's surplus consistent with the equilibrium in a) above? (4)
   c) Use a clearly drawn diagram showing (P_{grapes}^E) and (Q_{grapes}^E) , intercepts and the consumer's surplus for grapes in Alice. (5)
2. The demand functions for apricots (subscripted a) and blackberries (subscripted b) are inter-connected by the following demand functions: $Q_a^D = 96 - 4P_a + 2P_b$ and $Q_b^D = 40 + 8P_a - 5P_b$. The supply functions for apricots and blackberries (similarly subscripted) are: $Q_a^S = -4 + 28P_a$ and $Q_b^S = -4 + 7P_b$
   a) By imposing the equilibrium condition that quantity supplied equals quantity demanded in both markets, find a pair of simultaneous in the variables P_a and P_b (6)
   b) Show that this pair of simultaneous equations in a) above into matrices of coefficients (A), variables (P) and constants (B) such that $AP = B$ (3)
   c) Use the Cramer's rule to find the equilibrium values of P_a and P_b. Hence find the equilibrium quantities of apricots and blackberries (8)
3. A firm produces two products which are sold in two separate markets with the demand schedules $P_1 = 600 - 0.3Q_1$ and $P_2 = 500 - 0.2Q_2$. Production costs are related, and the firm faces the total cost function $TC = 16 + 1.2Q_1 + 1.5Q_2 + 0.2Q_1Q_2$
   a) Derive the Total Revenue function and state the economic equation you relied on. (3)
   b) Derive the Profit function and state the economic equation you relied on in your derivation. (3)
   c) State and derive the first-order conditions for the profit function in (b) above. (3)
   d) Calculate the level of output that is consistent with maximising profit. (6)
   e) Verify whether the points in (d) represent a maximum or minimum. (6)