

UNIVERSITY OF FORT HARE

STA 111

DEGREE EXAMINATIONS

JUNE 2023

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Time: 3 Hours

Subject: Descriptive Statistics and Differentiation

Marks: 100

This paper consists of 8 pages including the cover page

Examiner

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Internal Examiner

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INSTRUCTIONS

- Answer ALL questions
- Statistical tables are attached
- A list of key formulae is attached
- Programmable calculators may not be used

Section A

Question One

[5 marks]

Please chose what you believe to be the correct answer from the given choices.

- 1.1 Two events, A and B, are mutually exclusive and each has a nonzero probability. If event A is known to occur, the probability of the occurrence of event B is: [1]
- a) One b) Any positive value c) Zero
d) Any value between 0 to 1.
- 1.2 A numerical description of the outcome of an experiment is called a: [1]
- a) Descriptive statistic b) Probability function
c) Variance d) Random variable
- 1.3 If two events are independent, then: [1]
- a) They must be mutually exclusive. b) Their intersection must be zero.
c) The sum of their probabilities must be equal to one. d) None of these alternatives is correct
- 1.4 The average of 5 numbers is 6. The average of 3 of them is 8. What is the average of the remaining two? [1]
- a) 4 b) 5 c) 3 d) 3.5 e) 0.5
- 1.5 Consider n pairs of numbers $(x_1, y_1), (x_2, y_2), \dots,$ and (x_n, y_n) . The mean and standard deviation of the x-values are $\bar{x} = 5$ and $S_x = 4$, respectively. The mean and standard deviation of the y-values are $\bar{y} = 10$ and $S_y = 10$, respectively. Of the following, which could be the least squares regression line? [1]
- a) $\hat{y} = 3x - 5$ b) $\hat{y} = 3x$ c) $\hat{y} = 2.5x + 5$
d) $\hat{y} = 0.3x + 8.5$ e) $\hat{y} = 0.4x + 10$

Question two

[60 marks]

2.1 Figure 1 below relates to the weights in kg of patients who visited a clinic for counseling on their diets.

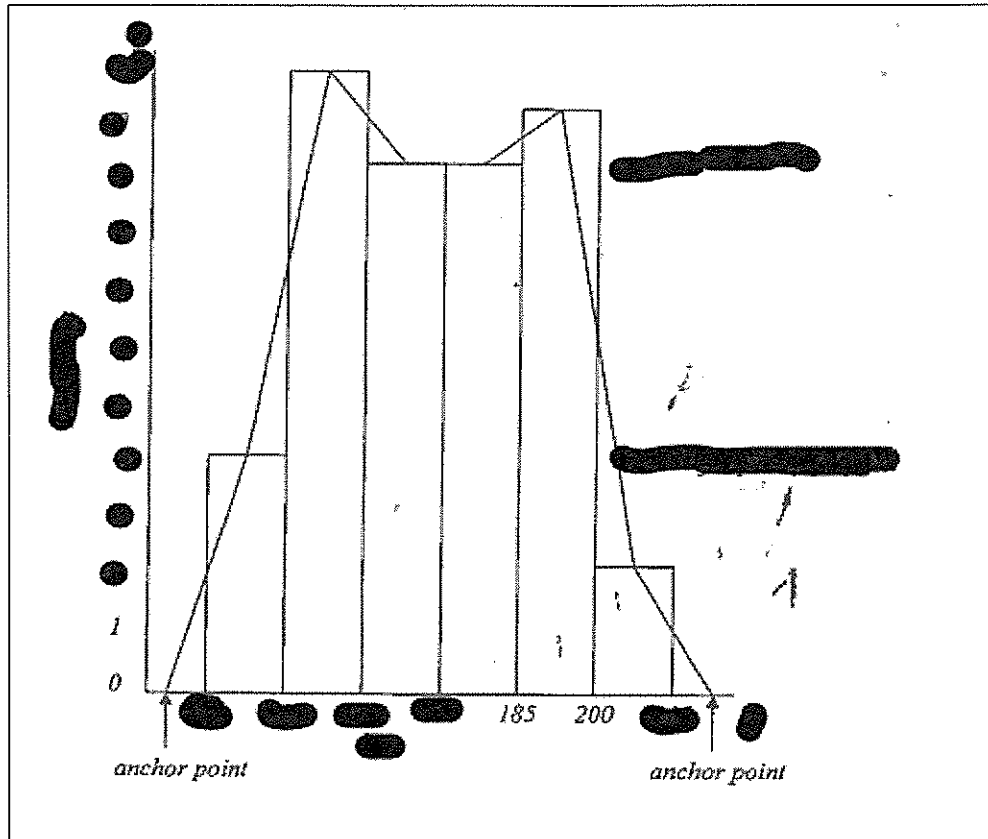


Figure 1: Weights of patients

- i. Arrange the data in a Statistically comprehensive way. [5]
- ii. Calculate the value of:
 - a) The arithmetic mean [3]
 - b) The mode [3]
 - c) The 55th Percentile [3]
 - d) The 6th Decile [3]
 - e) The Inter Quartile Range [5]

f) The Coefficient of Variation [5]

g) The value of the new arithmetic means if:

1) The heaviest person has been replaced by a person who is 10 kg lighter. [4]

2) The lighter patient has been replaced with a person who is 20 kg heavier. [4]

2.2 In a random sample of UKZN students 50% indicated they are business majors, 40% engineering majors, and 10% other majors. Of the business majors, 60% were females; whereas, 30% of engineering majors were females. Finally, 80% of the other majors were male.

i. Represent the situation above graphically (Tree diagram). [5]

ii. Determine the following:

a) The probability of selecting a female student. [3]

b) The probability of selecting a male student who is an engineering major? [3]

c) The probability of selecting a female student given that she is majoring in business? [3]

iii. Are the events majoring in Business and being a Female student independent? (Prove your answer). [1, 2]

iv. Name two mutually exclusive events. (Prove your answer). [1, 2]

2.4 A city installs 2000 electric lamps for street lighting. These lamps have a mean burning life of 1000 hours with a standard deviation of 200 hours. The normal distribution is a close approximation to this case. Determine the value of the third quartile. [5]

Question three

[15 marks]

3.1 Consider a company that markets and repairs small computers. The following table illustrates the relationship between the length of a service call, in minutes, and the number of electronic components in the computer that must be repaired or replaced.

Length of service call	23	29	49	64	74	87	96	97	109	119	149	145	154	166
Number of components	1	2	3	4	4	5	6	6	7	8	9	9	10	10

Determine:

- i.* The linear regression line is to be used when estimating the length of service calls knowing the number of electronic components to be repaired or replaced. [9]
- ii.* The correlation coefficient between the two variables and interpret the results. [3]
- iii.* The coefficient of determination, and interpret the results. [3]

Section B

Question four

[20 marks]

4.1 Consider the functions, $f(t) = t^3 - 3t + 1$, $g(x) = 2x - 3$, and $\Phi(z) = 3z^3 - z + 4$. Determine the value of $\Phi(f(0), g(1))$ [3]

4.2 Evaluate the following:

i. $\lim_{x \rightarrow 0} \left(\frac{1 - e^x}{e^{2x} - 1} \right)$ [3]

ii. $\lim_{x \rightarrow \infty} \left(\frac{20x^4 - 7x^3}{2x + 9x^2 + 5x^4} \right)$ [3]

iii. $\frac{d \left(\frac{\sqrt{7+9x^2}}{1-2x} \right)}{dx}$ [3]

iv. $\frac{d(3x^4 + 2x + 1)}{dx}$, at $x = 3$ [2]

4.3 Given a curve of $y = 2x^3 + 4x + 5$, and a point (1,1). Determine the equation of:

- i.* The Normal to the curve at the given point. [3]
- ii.* The Tangent to the curve at the given point. [3]

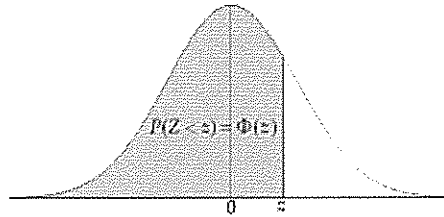
Table 1: Area under the standard normal curve

Normal probabilities to the left of Z.

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-\frac{1}{2}x^2} dx$$

Entries in the table are values of $z \geq 0$

$$\Phi(-z) = 1 - \Phi(z)$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99951	0.99953	0.99956	0.99958	0.99960	0.99962	0.99964	0.99966	0.99968	0.99970
3.4	0.99971	0.99973	0.99975	0.99977	0.99979	0.99981	0.99983	0.99985	0.99987	0.99989
3.5	0.99991	0.99993	0.99995	0.99997	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
3.6	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
3.7	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
3.8	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
3.9	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
4.0	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999

Table 2: Values of the inverse normal distribution

Entries in the table of the inverse function $z = \Phi^{-1}(u)$ for $u \geq 0.5$ where $u = \Phi(z)$ denotes the standard normal distribution function Note that $\Phi(-z) = 1 - \Phi(z) > 0.5$ when $\Phi(z) < 0.5$

$\Phi(z)$	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.50	0.0000	0.0025	0.0050	0.0075	0.0100	0.0125	0.0150	0.0175	0.0201	0.0226
0.51	0.0251	0.0276	0.0301	0.0326	0.0351	0.0376	0.0401	0.0426	0.0451	0.0476
0.52	0.0502	0.0527	0.0552	0.0577	0.0602	0.0627	0.0652	0.0677	0.0702	0.0728
0.53	0.0753	0.0778	0.0803	0.0828	0.0853	0.0878	0.0904	0.0929	0.0954	0.0979
0.54	0.1004	0.1030	0.1055	0.1080	0.1105	0.1130	0.1156	0.1181	0.1206	0.1231
0.55	0.1257	0.1282	0.1307	0.1332	0.1358	0.1383	0.1408	0.1434	0.1459	0.1484
0.56	0.1510	0.1535	0.1560	0.1586	0.1611	0.1637	0.1662	0.1687	0.1713	0.1738
0.57	0.1764	0.1789	0.1815	0.1840	0.1866	0.1891	0.1917	0.1942	0.1968	0.1993
0.58	0.2019	0.2045	0.2070	0.2096	0.2121	0.2147	0.2173	0.2198	0.2224	0.2250
0.59	0.2275	0.2301	0.2327	0.2353	0.2378	0.2404	0.2430	0.2456	0.2482	0.2508
0.60	0.2533	0.2559	0.2585	0.2611	0.2637	0.2663	0.2689	0.2715	0.2741	0.2767
0.61	0.2793	0.2819	0.2845	0.2871	0.2898	0.2924	0.2950	0.2976	0.3002	0.3029
0.62	0.3055	0.3081	0.3107	0.3134	0.3160	0.3186	0.3213	0.3239	0.3266	0.3292
0.63	0.3319	0.3345	0.3372	0.3398	0.3425	0.3451	0.3478	0.3505	0.3531	0.3558
0.64	0.3585	0.3611	0.3638	0.3665	0.3692	0.3719	0.3745	0.3772	0.3799	0.3826
0.65	0.3853	0.3880	0.3907	0.3934	0.3961	0.3989	0.4016	0.4043	0.4070	0.4097
0.66	0.4125	0.4152	0.4179	0.4207	0.4234	0.4261	0.4289	0.4316	0.4344	0.4372
0.67	0.4399	0.4427	0.4454	0.4482	0.4510	0.4538	0.4565	0.4593	0.4621	0.4649
0.68	0.4677	0.4705	0.4733	0.4761	0.4789	0.4817	0.4845	0.4874	0.4902	0.4930
0.69	0.4959	0.4987	0.5015	0.5044	0.5072	0.5101	0.5129	0.5158	0.5187	0.5215
0.70	0.5244	0.5273	0.5302	0.5330	0.5359	0.5388	0.5417	0.5446	0.5476	0.5505
0.71	0.5534	0.5563	0.5592	0.5622	0.5651	0.5681	0.5710	0.5740	0.5769	0.5799
0.72	0.5828	0.5858	0.5888	0.5918	0.5948	0.5978	0.6008	0.6038	0.6068	0.6098
0.73	0.6128	0.6158	0.6189	0.6219	0.6250	0.6280	0.6311	0.6341	0.6372	0.6403
0.74	0.6433	0.6464	0.6495	0.6526	0.6557	0.6588	0.6620	0.6651	0.6682	0.6713
0.75	0.6745	0.6776	0.6808	0.6840	0.6871	0.6903	0.6935	0.6967	0.6999	0.7031
0.76	0.7063	0.7095	0.7128	0.7160	0.7192	0.7225	0.7257	0.7290	0.7323	0.7356
0.77	0.7388	0.7421	0.7454	0.7488	0.7521	0.7554	0.7588	0.7621	0.7655	0.7688
0.78	0.7722	0.7756	0.7790	0.7824	0.7858	0.7892	0.7926	0.7961	0.7995	0.8030
0.79	0.8064	0.8099	0.8134	0.8169	0.8204	0.8239	0.8274	0.8310	0.8345	0.8381
0.80	0.8416	0.8452	0.8488	0.8524	0.8560	0.8596	0.8633	0.8669	0.8705	0.8742
0.81	0.8779	0.8816	0.8853	0.8890	0.8927	0.8965	0.9002	0.9040	0.9078	0.9116
0.82	0.9154	0.9192	0.9230	0.9269	0.9307	0.9346	0.9385	0.9424	0.9463	0.9502
0.83	0.9542	0.9581	0.9621	0.9661	0.9701	0.9741	0.9782	0.9822	0.9863	0.9904
0.84	0.9945	0.9986	1.0027	1.0069	1.0110	1.0152	1.0194	1.0237	1.0279	1.0322
0.85	1.0364	1.0407	1.0450	1.0494	1.0537	1.0581	1.0625	1.0669	1.0714	1.0758
0.86	1.0803	1.0848	1.0893	1.0939	1.0985	1.1031	1.1077	1.1123	1.1170	1.1217
0.87	1.1264	1.1311	1.1359	1.1407	1.1455	1.1503	1.1552	1.1601	1.1650	1.1700
0.88	1.1750	1.1800	1.1850	1.1901	1.1952	1.2004	1.2055	1.2107	1.2160	1.2212
0.89	1.2265	1.2319	1.2372	1.2426	1.2481	1.2536	1.2591	1.2646	1.2702	1.2759
0.90	1.2816	1.2873	1.2930	1.2988	1.3047	1.3106	1.3165	1.3225	1.3285	1.3346
0.91	1.3408	1.3469	1.3532	1.3595	1.3658	1.3722	1.3787	1.3852	1.3917	1.3984
0.92	1.4051	1.4118	1.4187	1.4255	1.4325	1.4395	1.4466	1.4538	1.4611	1.4684
0.93	1.4758	1.4833	1.4909	1.4985	1.5063	1.5141	1.5220	1.5301	1.5382	1.5464
0.94	1.5548	1.5632	1.5718	1.5805	1.5893	1.5982	1.6072	1.6164	1.6258	1.6352
0.95	1.6449	1.6546	1.6646	1.6747	1.6849	1.6954	1.7060	1.7169	1.7279	1.7392
0.96	1.7507	1.7624	1.7744	1.7866	1.7991	1.8119	1.8250	1.8384	1.8522	1.8663
0.97	1.8808	1.8957	1.9110	1.9268	1.9431	1.9600	1.9774	1.9954	2.0141	2.0335
0.98	2.0537	2.0749	2.0969	2.1201	2.1444	2.1701	2.1973	2.2262	2.2571	2.2904
0.99	2.3263	2.3656	2.4089	2.4573	2.5121	2.5758	2.6521	2.7478	2.8782	3.0902

KEY FORMULAE

$$1. \frac{\sum mf}{N}$$

$$2. \frac{\sum mf}{n}$$

$$3. \frac{\sum m^2 f - \frac{(\sum mf)^2}{n}}{n-1}$$

$$4. \frac{\sum m^2 f - \frac{(\sum mf)^2}{N}}{N}$$

$$5. L + \frac{\left[\left(\frac{n}{2} \right) - pcf \right] * i}{f}$$

$$6. L + \frac{(f_1 - f_0) * i}{2f_1 - f_0 - f_2}$$

$$7. L + \frac{\left[\left(\frac{jn}{4} \right) - pcf \right] * i}{f},$$

$j = 1, 2, 3$

$$8. L + \frac{\left[\left(\frac{jn}{100} \right) - pcf \right] * i}{f},$$

$j = 1, 2, 3, \dots, 99$

$$9. \frac{SS_{xy}}{SS_{xx}}$$

$$10. \frac{bSS_{xy}}{SS_{yy}}$$

$$11. \frac{s}{\bar{x}} * 100\%$$

$$12. Q_3 - Q_1$$

$$13. n(n-1)(n-2) \dots 1$$

$$14. \frac{n!}{(n-r)!}$$

$$15. \frac{n!}{(n-r)!r!}$$

$$16. \sum x^2 - \frac{(\sum x)^2}{n}$$

$$17. \sum y^2 - \frac{(\sum y)^2}{n}$$

$$18. \sum xy - \frac{(\sum xy)^2}{n}$$

$$19. \bar{y} - b\bar{x}$$