UNIVERSITY OF FORT HARE

Introduction to Analytical Methods
MAP 221

Supplementary Examinations

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Time: 3 Hrs

Subject: Applied Mathematics 3

Marks: 100

This question paper consists of 3 pages

Internal examiner(s)

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Instructions

Answer all questions.
Symbols have the usual meanings

Question. 1

1.1 Solve and find the general solution of the differential equation,

$$\frac{dy}{dx} - 5y = -\frac{5}{2}xy^3.$$ (9)

1.2 If $x(t)$ is the population of Hyenas at any time, t and $y(t)$ the population of Rabbits, the following system describe the predator-prey model of the rabbits against the hyenas,

$$\frac{dx}{dt} = -x + 6y \qquad \text{where } x(0) = 10 \text{ and } y(0) = 100.$$
$$\frac{dy}{dt} = x - 2y$$

1.2.1 Find the population of each species at any time t. (10)

1.2.2 At a stable state $\frac{dy}{dt} = \frac{dx}{dt}$, find time it takes to reach a steady state. (4)

1.2.3 What is the population of each species at the point of reaching the steady state. (1)

1.3 The position of a particle moving along the x-axis is determined by the equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 8x = 20\cos 2t.$$

If the particle starts from rest at $x = 0$, find

1.3.1 x as a function of t, (10)

1.3.2 the amplitude, period and frequency after a long time. (6)

[40]

Question.2

2.1 Using $e^{iat} = \cos at + i\sin at$,

2.1.1 Show that $\mathcal{L}\{e^{iat}\} = \frac{1}{s-ia}, s > 0$. (4)

2.1.2 Show that $\frac{1}{s-ia} = \frac{s+ia}{s^2+a^2}$ (2)

2.1.3 Use the above to derive (without integration by parts) the formulas for $\mathcal{L}\{\cos at\}$ and $\mathcal{L}\{\sin at\}$. (2)

2.2 Find the Laplace transform of the function, $f(t) = \sin 2t\cos 2t$. (5)

2.3 Find the inverse Laplace transform of $F(s) = \dfrac{50}{s^2(s^2+2s+5)}$ (9)

2.4 Solve the following initial value-problem using Laplace transforms

$$\frac{d^3y}{dt^3} + 8y = 32t^3 - 16t \qquad \text{where } y(0) = y'(0) = y''(0) = 0$$ (10)

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Question. 3

3.1 Consider the following function $f(x) = 4x,\ 0 < x < 10,$ period 10.

3.1.1 Draw the graph of this function and find its corresponding Fourier series. (10)

3.2.1 Show that $\Gamma(n+1) = n\Gamma(n)$ for $n > 0$ (3)

3.2.2 Hence show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ (8)

3.3 Evaluate the following

3.3.1 $\int_0^\infty x^2 e^{-2x^2}\, dx$ (4)

3.3.2 $B\left(\frac{3}{2}, 2\right)$ (3)

[28]

-----------------------------**THE END**-----------------------------