UNIVERSITY OF FORT HARE

University of Fort Hare
Together in Excellence

PHY 211 (Classical Mechanics)

HONOURS/DEGREE/DIPLOMA EXAMINATIONS

DATE : June/July 2023

TIME : 3 HOURS

SUBJECT : CLASSICAL MECHANICS (PHY 211)

MARKS : 120

INTERNAL EXAMINER(S)
P.M. KWINANA

MODERATOR:
Dr P. MUKUMBA

INSTRUCTIONS:

Answer any FOUR questions.

Please note that the sheet with useful formulae is attached at the back of the question paper.

Question One *[30 Marks]*

1.1 (a) Convert the following equation written in Cartesian coordinates into an equation in cylindrical coordinates:

$\frac{x-y}{x^2+y^2+1} = xyz$ (3)

(b) Convert the following equation written in Cylindrical coordinates into an equation in Cartesian coordinates:

$zr^3 \cos\theta = 4r + 8$ (3)

(c) Convert the following equation written in Cartesian coordinates into an equation in Spherical coordinates:

$x^2 + y^2 = 4x + z - 2$ (3)

(d) In your own words, how would you describe the relationship between Cartesian, Cylindrical, and Spherical Coordinate systems? State similarities and differences. (1)

[10]

1.2 A train starts from station *A* to reach another station *B*, at a distance *d* from *A*; the motion is at first uniformly accelerated for a given time t^1; the velocity then remains constant for a given time t^2, and this is then uniformly retarded for a time t^3.

(a) Represent the motion in a diagram (label your diagram) (4)

(b) What does the area under a velocity-time graph represent? And, what does the slope of a velocity time graph represent? (2)

(c) Find the acceleration of the train. (3)

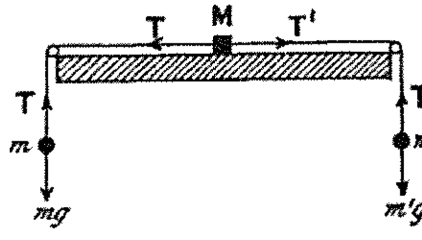
(d) Show that $retardation = \frac{2d}{t^3(t^1+t^3+2t^2)}$. (3)

[10]

1.3 A mass of M kg rests on a smooth horizontal table and is attached by two inelastic strings to masses m, m' kg ($m' > m$), which hang over smooth pulleys at opposite edges of the table. Show that the acceleration of the system and the tension in the strings are given by the following expressions:

$a = \frac{m'-m}{m'+m+M} g$

And $T = mg \left[\frac{2m'+M}{m'+m+M}\right]$



[10]

__Question Two__ *[30 Marks]*

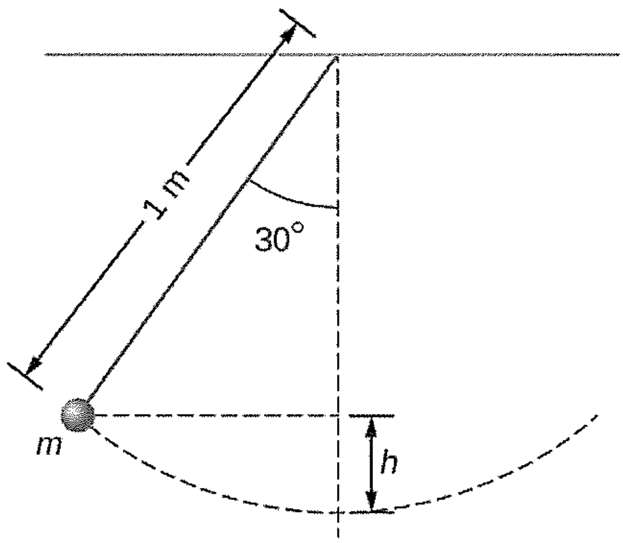
2.1 (a) Derive from first principles and prove that the change in kinetic energy of a body is equal to the work done by an external force acting on the body. (5)

(b) Prove that the mechanical energy of a free-falling object remains constant at every point of motion. Draw an illustrative diagram of your system. (5)

[10]

2.2 A particle of mass m is hung from the ceiling by a massless string of length 1.0 m, as shown in Figure below. The particle is released from rest, when the angle between the string and the downward vertical direction is 30°. Assuming that only the gravitational force is acting on the particle and air resistance is neglected, what is its speed when it reaches the lowest point of its arc?

Hint: Choose a reference point for zero gravitational potential energy to be at the lowest vertical point the particle achieves.



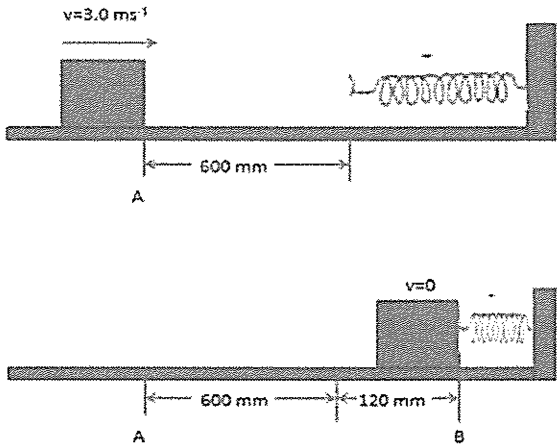
[10]

2.3 A spring is used to stop a crate of mass *50 kg* which is sliding on a horizontal surface. The spring has a spring constant $k = 20kN/m$ and is initially in its equilibrium state. In position A, shown in the top diagram below, the crate has a velocity of *3.0 m/s*. The compression of the spring when the crate is instantaneously at rest (position *B* in the diagram below) is 120 mm.

(i) What is the work done by the spring as the crate is brought to a stop?
(ii) Write an expression for the work done by friction during the stopping of the crate (in terms of the coefficient of kinetic friction).

(iii) Determine the coefficient of friction between the crate and the surface.

(iv) What will be the velocity of the crate as it passes again through position A after rebounding off the spring?

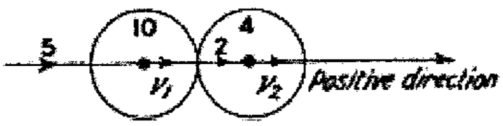


[10]

__Question Three__ ***[30 Marks]***

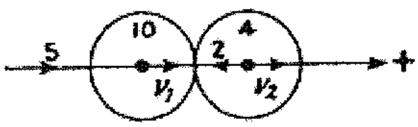
3.1 (a) State Newton's experimental law. (2)

(b) A ball of mass $10kg$, moving at $5m.s^{-1}$, collide with another ball of mass $4kg$, moving at $2m.s^{-1}$ in the same direction. If $e = \frac{1}{2}$, find the velocities after the impact.



(4)

(c) If the $4kg$ ball in previous question is moving in a direction opposite to that of the $10kg$ ball, find the velocities after the impact.



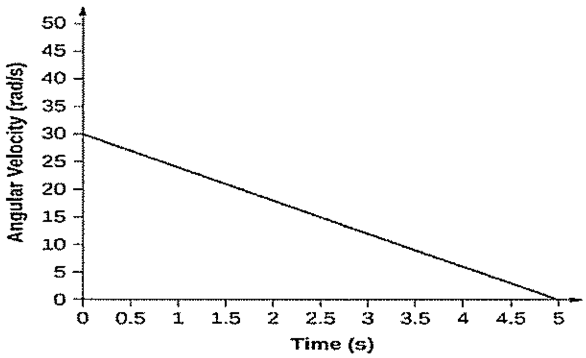
(4)

[10]

3.2 The figure below shows a graph of the angular velocity of a propeller on an aircraft as a function of time. Its angular velocity starts at *30 rad/s* and drops linearly to *0 rad/s* over the course of *5 seconds*.

(a) Find the angular acceleration of the object and verify the result using the kinematic equations. (5)

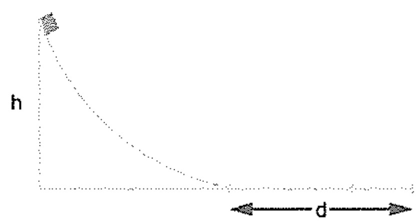
(b) Find the angle through which the propeller rotates during these *5 seconds* and verify your result using the kinematic equations.



(5)

[10]

3.3 (a) A ramp in an amusement park is frictionless. A smooth object slides down the ramp and comes down through a height h, refer to the figure below. What distance *d* is necessary to stop the object on the flat track if the coefficient of friction is μ.



(5)

(b) A carpenter's wheel accelerates from 3 rad/s to 8 rad/s in 6s.

(i) Draw the diagram of this situation.

(ii) Find the acceleration of the wheel.

(iii) The angle through which the wheel turned, in units of revolutions.

(5)

[10]

Question Four ***[30 Marks]***

4.1 (a) Define the moment of inertia of a body rotating about an axis. (3)

(b) Prove that the moment of inertia for a uniform, solid sphere of radius R and mass M rotating about its centre is given by I = $\frac{2}{5}MR^2$. Show all your calculations (7)

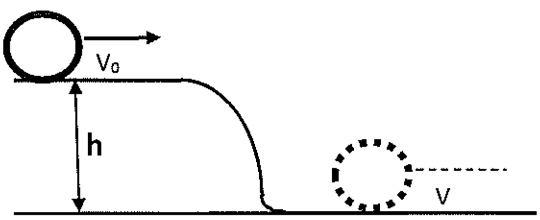
[10]

4.2 A disc rolls **without slipping** along a horizontal surface with velocity V_0. The disc then encounters a **smooth drop** of height h, after which it continues to move with velocity V. At all times the disc remains in a vertical plane as shown in the diagram below.

(a) Show that the moment of inertia of a disc of radius R and mass M about an axis through the centre perpendicular to its plane is I = $\frac{1}{2}MR^2$. Show all your calculations.

(b) Show that the linear velocity, v, of the disc is given by the following expression:

V = $\sqrt{V_0^2 + \frac{4gh}{3}}$



[10]

4.3 (a) State the Parallel Axis Theorem in words and in the form of an equation. (3)

(b) A uniform solid ball rolls down a slope. If the ball has a diameter of 0.5 m and a mass of 0.1 kg, find the following:

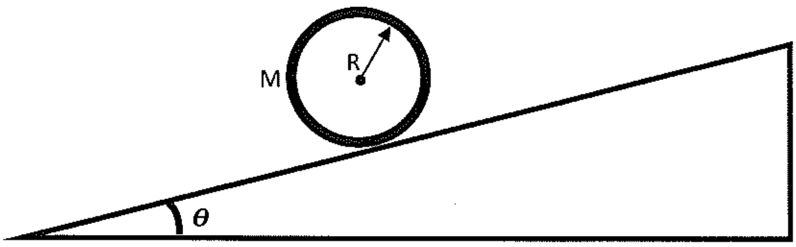
(i) Draw all forces acting on the system and find the acceleration of the ball.

(ii) If the slope has an incline of 30° to the horizontal, what is the speed of the ball after it travels 3 m?

(iii) At this point, what is the angular momentum of the ball?

(iv) If the coefficient of friction between the ball and the slope is 0.26, what is the maximum angle of inclination the slope could have which still allows the ball to roll?

(7)



[10]

Question Five *[30 Marks]*

5.1 (a) Define the following terms:

(i) Simple Harmonic Oscillation

(ii) Damped Harmonic Oscillation

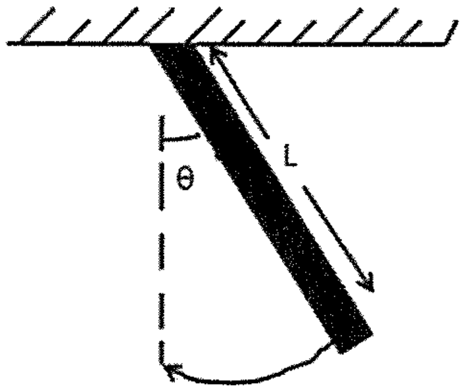
(iii) Forced Oscillation (6)

(b) A solid uniform cylinder of radius r rolls without sliding along the inside surface of a hollow cylinder of radius R, performing small oscillations. Determine the time (period). (8)

5.2 (a) A thin, uniform rod of mass M and length L swings from one of its ends as a physical pendulum as shown in the figure below. Given that the moment of inertia of a uniform rod about one end is I = 1 3 M L2 ,

(a) Obtain an equation for the period of the oscillatory motion for small angles.

(b) What would be the length l of a simple pendulum that has the same period as the swinging rod? Show all your calculations.



(8)

5.3 The equation of motion for a damped oscillator is given by $4d^2x/dt^2 + rdx/dt + 32x = 0$ For what range of values for the damping constant will the motion be

(a) underdamped?

(b) overdamped?

(c) critically damped? Show all your calculations

(8)

[30]

Question Six *[30 Marks]*

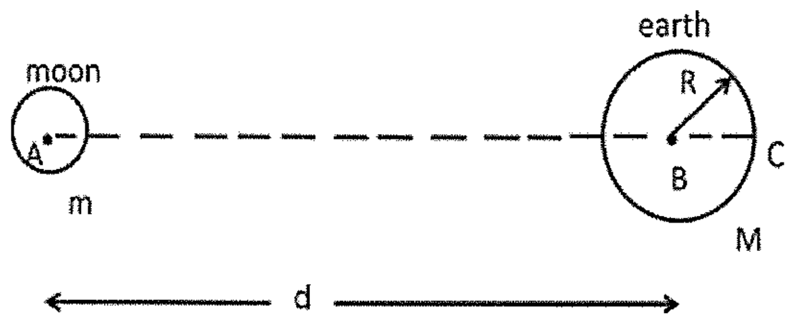
6. 1 (a) State Newton's Law of Universal Gravitation (3)

(b) Show that the gravitational energy of earth assumed to be the uniform sphere of radius R and mass M is 3GM2 /5R. What is the potential energy of earth assuming it to be a uniform sphere of radius $R = 6.4 \times 10^6$m and of mass $M = 6.0 \times 10^{24}$kg. (7)

[10]

6.2 Derive an expression for the gravitational potential V(r) due to a uniform solid sphere of mass M and radius R when r < R. ***[10]***

6.3 A tidal force is exerted on the ocean by the moon. This is estimated by the differential (Δg) which is the difference of the acceleration at B and that at C due to the moon as shown in the figure below. If R is the radius of the earth, d the distance of separation of the centre of earth and moon, M and m the mass of the earth and moon, respectively, show that $\Delta g \approx \frac{2GmR}{d^3}$.

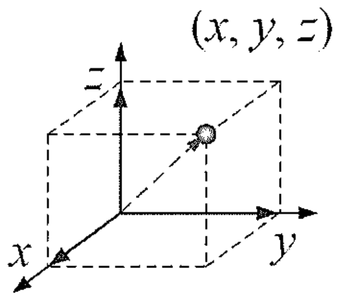


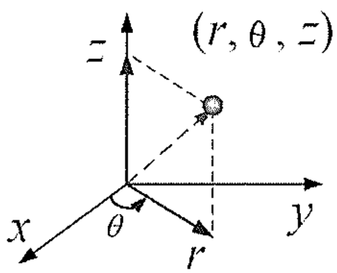
[10]

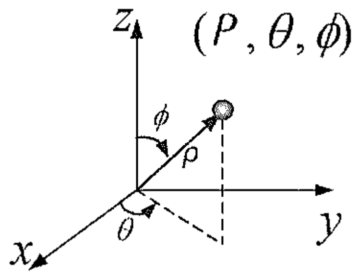
End

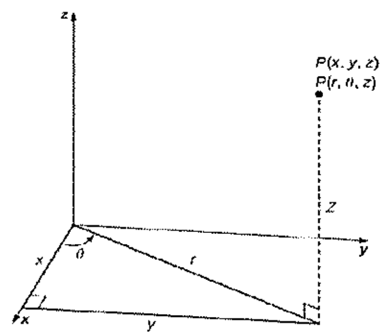
Coordinate Transformation Formulae Sheet

Operations	From **Cartesian (x,y,z) to Cylindrical (r, θ, z) coordinates system**	**From Cylindrical (r, θ, z) to Spherical Coordinates (ρ, θ, φ)**	**From Cartesian (x,y,z) to Spherical Coordinates (ρ, θ, ϕ)**
Definition of coordinates	$x = r\cos\theta$ $y = r\sin\theta$ $z = z$	$r = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}(y/x)$ $z = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$	$\rho = \surd(x^2 + y^2 + z^2)$ $\phi = \cos^{-1}(z/\rho)$ $\theta = \tan^{-1}(y/x)$
	From **Cylindrical (r, θ, z) to Cartesian Coordinates (x,y,z)**	**From Spherical (ρ, θ, φ) to Cylindrical Coordinates (r, θ, z)**	**From Spherical Coordinates (ρ, θ, ϕ) to Cartesian (x,y,z) coordinates**
	$r^2 = x^2 + y^2$ $\theta = \tan^{-1}(y/x)$ $z = z$	$x = \rho \sin\varphi \cos\theta$ $y = \rho \sin\varphi \sin\theta$ $z = \rho \cos\varphi$	$x = \rho \sin\varphi \cos\theta$ $y = \rho \sin\varphi \sin\theta$ $z = \rho \cos\varphi$









Coordinate Transformation Formulae Sheet

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