University of Fort Hare
Together in Excellence

MAT 123F

Supplementary Examinations

January 2019

Subject: **Mathematics**

Time: **3 Hours** Marks: **100**

This question paper consists of 3 pages

Internal Examiners

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Instructions

Answer all questions

Symbols used have the usual meanings

Question 1

1.1 Solve for x if

$$x^2+6x+10=0$$ (5)

1.2 Compute $\frac{z_1}{z_2}$ if $z_1=3+2i$ and $z_2=5-i$ (4)

1.3 Let z_1 and z_2 be two complex numbers. Prove that

$$z_1+z_2=z_1+z_2$$ (6)

1.4 Write the complex number

$$z=-\sqrt{3}+i$$

into its trigonometric form. (10)

[25]

Question 2

2.1 If

$$f(x,y)=x^3y^2-3xy$$

compute $\frac{\partial f}{\partial y}$ from first principles. (5)

2.2 Let

$$f(x,y)=x^2y+\cos y+y\sin x\,.$$

Compute $f_x,\ f_{xx},\ f_{xy},\ f_y,\ f_{yy}$ (5)

2.3 Use the Chain rule to compute $\frac{df}{dt}$ if

$f(x,y)=xy$ where $x=\cos t$ and $y=\sin t$ (5)

2.4 Find the general solution to the differential equation

$$\frac{dy}{dx}=e^{x-y}$$ (5)

[20]

Question 3

3.1 Obtain a particular solution to the IVP

$$\frac{dy}{dx}+\frac{y}{x}=4x^2, \quad y(1)=2$$ (8)

3.2 Show that the differential equation

$$\frac{dy}{dx}=\frac{x+y}{x-y}$$ (12)

[20]

Question 4

4.1 Show that the differential equation

$$(3xy^2+2y)dx+(2x^2y+x)dy=0$$ (1)

is not exact .

4.2 Find an integrating factor $\mu = \mu(x)$ for (1)

4.3 Multiply the nonexact DE (1) by this integrating factor $\mu(x)$.
You will get an exact DE (2)

4.4 Solve the resulting exact DE (2) by grouping the terms.

(20)

Question 5

5.1 A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 3 hours there are 10 000 bacteria. At the end of 5 hours there are 40 000 bacteria. How many bacteria were present initially? (10)

5.2 A bottle of Coca-Cola at room temperature $(22^{\circ}C)$ is placed in a refrigerator where the temperature is $7^{\circ}C$. After half an hour the Coca-Cola has cooled to $18^{\circ}C$.

(a) What is the temperature of the Coca-Cola after another half hour?
(b) How long does it take for the Coca-Cola to cool to $10^{\circ}C$?

Assume Newton's Law of Cooling of applies. (10)

[20]