UNIVERSITY OF FORT HARE
Department of Pure and Applied Mathematics

MAT 122

SUPPLEMENTARY EXAMINATIONS
JANUARY 2019

Time: 3 Hours

Subject: A PRACTICAL APPROACH TO DIFFERENTIAL CALCULUS

Marks: 100

This question paper consists of 2 pages

Internal Examiner(s)

Ms. B. E. Nxala

Mr. Z. Mahlasela

Instructions

Answer ALL Questions
Symbols have the usual meanings.

Question One

1.1 Prove that between any two rational numbers there is a rational number

(5)

1.2 Prove by Mathematical Induction that $\forall n \in \mathbb{Z}^+$, $5^n - 1$ is divisible by 4.

(8)

1.3 Solve the inequality $|x+1| + |x-2| < 5$ and present the solution on a real number line

(4)

1.4 Find the first 4 terms in the expansion of $\frac{1}{\sqrt[3]{1-2x}}$. For which values of x is the expansion valid?

(5)

Question Two

2.1 Find the domain and sketch the graph of the function

$$f(x) = \begin{cases} 1 - x^2 & if \quad x \leq 2 \\ 2x - 7 & if \quad x > 2 \end{cases}$$

(6)

2.2 If $f(x) = \sqrt{1+x}$ and $g(x) = \sqrt{1-x}$ determine the following functions and state the domains.

(a) $f + g$

(b) $\frac{f}{g}$

(6)

2.3 Given that $f(x) = 3x + 1$, $g(x) = 2x^2 + x + 1$ and $h(x) = \frac{x}{\sqrt{x+1}}$. Determine $h \circ g \circ f$

(4)

2.4 For the function $h(x) = 1 - x^2$, sketch, on the same xy-plane, the graphs of

(a) $h(x)$

(b) $h(x) + 2$

(c) $h(x + 2)$

(5)

Question Three

3.1 Prove that $\lim_{x \to 0} \frac{cos(x)-1}{x} = 0$

(4)

3.2 Evaluate

(a) $\lim_{x \to 0} \frac{x + xcos(x)}{sin(x)cos(x)}$

(b) $\lim_{u \to 1} \frac{u^4 - 1}{u^3 - 1}$

(c) $\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$

(10)

3.3 If $1 \leq g(x) \leq x^2 + 2x + 2$ for all x, find $\lim_{x \to 1} g(x)$. State without proof, the Theorem you have used.

(5)

3.4 Determine the value of the constant a, for which the following function is continuous, for all values of x.

$$f(x) = \begin{cases} x^2 - 1 & if \quad x < 3 \\ 2ax & if \quad x \geq 3 \end{cases}$$

(4)

Question Four

4.1 From definition, find $f'(x)$ for the function $f(x) = x^2 - 8x + 9$

(5)

4.2 Find $\frac{dy}{dx}$ if

(a) $y = x^4 - \sqrt[4]{x}$

(b) $y = cos(x) - 2tan(x)$

(c) $y = \frac{1-x}{1+x}$

(d) $y = ln(x^4 + 2x^3 + x^2 + 1)$

(14)

4.3 Find the equation of the tangent to the curve $y = \frac{1}{1+x^2}$ at $x = 1$.

(5)

4.4 Find the minimum and maximum values of the function $y = x^3 - 6x^2 + 9x$ distinguishing between them. Sketch the graph of the function.

(10)