



University of Fort Hare
Together in Excellence
MAT 121 F

SUPPLEMENTARY EXAMINATIONS
November 2024

Subject: Mathematics (F)
Paper: MATHEMATICS (F)

Time: 3 Hours

Marks: 100

This question paper consists of 3 pages

Internal examiner

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Moderator

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Instructions

This paper contains seven questions.
Answer all seven questions.
Symbols used have the usual meanings.

Question 1

1.1 Evaluate the special trigonometric limit

$$L = \lim_{x \rightarrow 0} \frac{\sin(7x)}{\sin(3x)} \quad (5)$$

1.2 Let

$$y(x) = \cos(x)$$

Compute the derivative $\frac{dy}{dx}$ from first principles. (5)

1.3 Treating y as a differentiable function of x , use implicit differentiation to find the derivative $\frac{dy}{dx}$ if:

(a) $x \sin y = \cos(x + y)$ (5)

(b) $x + \sin y = xy$ (5)

[20]

Question 2

2.1 Let f be the exponential function defined by

$$f(x) = a^x,$$

where the base a is a positive number that is different from 1.

(a) Find the domain of f (1)

(b) Find the range of f (1)

(c) Sketch the of (3)

$$y = a^x$$

2.2 Solve for x if

$$\log_3(5x + 2) = \log_3(7x - 8) \quad (5)$$

2.3 Use logarithmic differentiation to compute $\frac{dy}{dx}$ if:

(a) $y = \frac{(\sin x)^2 (x^3 + 1)^4}{(x + 3)^8}$ (5)

$$(b) y = \frac{x^4 (\tan x)^3}{\sqrt{x^2+1}} \quad (5)$$

[20]

Question 3

3.1 Let u and v be two differentiable functions of x . Let

$$y = uv$$

Find $\frac{dy}{dx}$ by using logarithmic differentiation. (6)

3.2 Compute $\frac{dy}{dx}$ in each case if:

(a) $y = \ln(3x^2 - 5)$ (2)

(b) $y = e^{-4x}$ (2)

(c) $y = e^{3x} \cos 2x$ (3)

(d) $y = \sin^{-1}(5x)$ (2)

(e) $y = x^2 \tan^{-1} x$ (3)

[18]

Question 4

4.1 Find the interval of x -values where the function

$$f(x) = 2x^3 - 3x^2 + 3$$

is increasing and decreasing. (5)

4.2 Find the absolute maximum and absolute minimum values of the function

$$f(x) = x^3 - 3x + 2$$

on the closed interval $[-2, 3]$ (5)

[10]

Question 5

- 5.1 Gravel is being dumped from a conveyor belt at a rate of $100m^3/min$ forming a conical pile whose base diameter is twice its height. How fast is the height changing when the pile is $12m$ high? (5)

- 5.2 Use L'Hopital's rule to evaluate the indeterminate limit

$$L = \lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x} \quad (5)$$

[10]

Question 6

- 6.1 Use the second derivative test to find the local maximum and local minimum values of the function

$$f(x) = 2x^3 + 3x^2 - 12x \quad (5)$$

- 6.2 Find the equation of a tangent and a normal to the curve

$$y = x^3 - 2x^2 + 4 \text{ at the point of tangency } (2, 4) \quad (5)$$

[10]

Question 7

- 7.1 What is the largest area possible for a right-angled triangle whose hypotenuse is $5cm$ long? (5)

- 7.2 A farmer has $1200m$ of fencing and wants to fence off a rectangular farm that borders a straight river. He needs no fence along a river. What are the dimensions of the field that give the largest area? (7)

[12]