University of Fort Hare
Together in Excellence
MAT 123 F

SUPPLEMENTARY EXAMINATIONS
NOVEMBER 2024

Subject: **Mathematics (F)**
Paper: MATHEMATICS (F)

Time: **3 Hours** Marks: **100**

This question paper consists of 3 pages

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Instructions

Answer all questions.
Symbols have the usual meanings.

Question 1

1.1 Solve the system of linear equations below by using the Gauss-Jordan transformation method.

$$\begin{aligned} x_1 + \quad x_2 + \quad 2x_3 &= 8 \\ -x_1 - \quad 2x_2 + \quad 3x_3 &= 1 \\ 3x_1 - \quad 7x_2 + \quad 4x_3 &= 10 \end{aligned}$$

(10)

1.1 Use Cramer's rule to solve the linear system.

$$\begin{aligned} 2x + \quad y + \quad z &= 6 \\ 3x - \quad 2y - \quad 3z &= 5 \\ 8x + \quad 2y + \quad 5z &= 11 \end{aligned}$$

(10)

[20]

Question 2

2.1 Discuss and sketch the graph of the curve given by the polar equation

$$\theta = 45°$$

in the XY- plane. (10)

2.2 Solve for x if

$$x^2 + 6x + 10 = 0$$

(5)

2.3 Perform the indicated operation.

(a) $z_1 + z_2$ if $z_1 = 2 + 8i$ and $z_2 = 3 + 4i$ (2)

(b) $z_1 - z_2$ if $z_1 = 5 + 3i$ and $z_2 = 4 - 7i$ (3)

[20]

Question 3

3.1 Let

$z_1 = 1 + 3i$ and $z_2 = 2 + i$

Calculate the quotient $\frac{z_1}{z_2}$, by using the conjugate method. (4)

3.2 Let

$z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$ be two complex numbers.

Prove that

$$z_1 + z_2 = z_2 + z_1$$ (5)

That is, addition in the set of complex numbers is commutative.

3.3 Convert the complex number

$$z = -1 + \sqrt{3}i$$

into the polar form (or trigonometric form)

$$z = r(\cos\theta + i\sin\theta),$$

where r and θ are the polar coordinates of z. (11)

[20]

Question 4

4.1 Let

$$f(x, y) = 3x^2y - 5x + y^2$$

Find the partial derivative $\frac{\partial f}{\partial y}$ from first principles. (5)

4.2 Let

$$f(x, y) = x\sin y + y\cos x + xe^y$$

Compute the partial derivatives f_x, f_{xx}, f_{xy}, f_y, and f_{yy} (in that order) (5)

4.3 Solve each differential equation below by separating the variables.

(a) $\frac{dy}{dx} = \frac{ye^x}{y^2-1}$ (5)

(b) $\frac{dy}{dx} = e^{3x-2y}$ (5)

[20]

Question 5

5.1 Obtain a particular solution to the initial value problem (IVP)

$$\frac{dy}{dx} + \frac{y}{x} = 4x^2, \qquad y(1) = 2 \qquad (10)$$

5.2 Show that the differential equation

$$\frac{dy}{dx} = \frac{2x^3 + y^3}{3xy^2},$$

is homogeneous, and then find its general solution implicitly. (10)

[20]