

UNIVERSITY OF FORT HARE



University of Fort Hare
Together in Excellence

ADVANCED CLASSICAL MECHANICS (PHY 501)

HONOURS EXAMINATIONS

DATE: MAY 2023

DURATION: 3 HOURS

MARKS: 100

INTERNAL EXAMINER

Mr N. Shambira

EXTERNAL EXAMINER

Dr. D. Tinarwo

THIS PAPER CONSISTS OF 3 PAGES INCLUDING COVER PAGE

INSTRUCTIONS:

1. Answer all the questions.
2. Write neatly and legibly.
3. Unless otherwise stated, all symbols retain their usual meanings.
4. Values of constants appear at end of the question paper

QUESTION 1 [25 MARKS]

- 1.1 Calculate the inertia tensor for a cone rotating about the center of the base. Take the height of the cone to be h and the base radius of R . Show all working (15)
- 1.2 Consider a thin disk rotating about the z-axis, show that the kinetic energy of the disk is given by $T = \frac{1}{2}I\omega^2$, where I is the moment of inertia of the disk and ω is the angular velocity of the disk. (6)
- 1.3 The moment of momentum \vec{L} is given by $\vec{L} = \vec{r} \times \vec{P}$, where \vec{r} and \vec{P} is the position vector and linear momentum vector respectively. (4)

QUESTION 2 [25 MARKS]

- 2.1 State D' Alembert principle. (4)
- 2.2 Using the D' Alembert principle show that the Lagrange equation is given as $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0$, where L is the Lagrangian equation and q are the generalized coordinates. (13)
- 2.3 Given a Lagrangian equation of a system as $L = \frac{1}{2}m(\dot{\rho}^2 + \rho^2\dot{\theta}^2 + \dot{z}^2) - U(\rho, \theta)$, determine constants of motion and write HJ equation for the system (8)

QUESTION 3 [25 MARKS]

- 3.1 For the canonical transformation, $Q = q^\alpha \cos \beta p$ and $P = q^\alpha \sin \beta p$, it is given that $[Q, P]_{q,p} = 1$. Find the values of α and β . (7)
- 3.2 Prove that $[fg, h] = f[g, h] + [f, h]g$, where f, g and h are functions of q^i, p^j and t . (6)
- 3.3 Prove that (8)
- 3.3.1 $[q_k, F] = \frac{\partial F}{\partial p_k}$ (3)
- 3.3.2 $[p_k, F] = -\frac{\partial F}{\partial q_k}$ (3)
- 3.3.3 and hence show that
- (i) $[q_k, p_i] = \delta_{ik}$ (3)
- (ii) $[q_k, p_i] = \delta_{ik}$ (3)

QUESTION 4 [25 MARKS]

- 4.1 State the condition of canonicity (4)
- 4.2 Show that $F_2 = qP$ act as a generating function for identity transformation, means, $q = Q$ and $p = P$ is obtained from it. (8)
- 4.3 Taking the generating function to be $F_1(q, Q, t)$, show that $K = H + \frac{\partial F_1}{\partial t}$,
 $p_i = \frac{\partial F_1}{\partial q_i}$ and $P_i = -\frac{\partial F_1}{\partial Q_i}$ (6)
- 4.4 Show that $Q = \ln\left(\frac{1}{q} \sin p\right)$ and $P = q \cot p$ is canonical. (7)