UNIVERSITY OF FORT HARE

Integral Calculus: A Theoretical Approach
Mat 121

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This question paper consists 4 pages

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Instructions

Answer all questions.

Symbols used have the usual meaning

Question 1

1.1 For the complex number $z = 2 - i\sqrt{7}$. Find:

(a) the conjugate (1)

(b) the modulus (1)

(c) the argument (1)

1.2 Prove that $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$ for complex numbers z_1 and z_2. (3)

1.3 State without proof De Moivre's Theorem.
Hence use this theorem to find $(2\sqrt{3} + 2i)^5$. (5)

1.4 Find the cube root of $8i$ and sketch the roots on the z-plane. (4)

1.5 For $u = x^a yz$ find $\dfrac{\partial^2 u}{\partial x \partial y}$ (2)

Question 2

2.1 First use substitution and then integration by parts to evaluate the integral $\int \sin\sqrt{x}dx$. (4)

2.2 Evaluate the integrals (a) $\int \tan^2 x\sec^4 xdx$ (b) $\int \cos 3x \sin 6xdx$ (6)

2.3 Chose a suitable trigonometric substitution to evaluate the following integrals:

$$\int \frac{\sqrt{9x^2 - 4}}{x} dx$$ (5)

2.4 Write down the partial decomposition of $\dfrac{1}{x(x+1)(2x+3)}$. After evaluating for the constants use this partial decomposition to evaluate $\int \dfrac{1}{x(x+1)(2x+3)} dx$. (4)

2.5 Use an appropriate rationalizing substitution to evaluate the integral

$$\int \frac{1}{\sqrt{x} + \sqrt[4]{x}} dx$$ (4)

2.6 An integrating factor $I(x)$ is used to solve the linear differential equation $\dfrac{dy}{dx} + P(x)y = Q(x)$ .Use this procedure to find the integrating factor for the initial value problem $x^2 \dfrac{dy}{dx} + 2xy = \cos x$, $y(\pi) = 0$. (3)

2.7 Determine whether the equation $y' = \dfrac{xy + y^2}{x^2}$, is homogeneous, and solve it. (4)

2.8 Show that the equation $\sin y + (1 + x\cos y)\dfrac{dy}{dx} = 0$ is exact, and hence solve it. (4)

Question 3

3.1 If A is a $m \times n$ matrix and $B = A^T$, find the size of

(a) B

(b) BB^T

(c) $B^T B$ (3)

3.2 Given the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \\ 2 & 0 & 1 \end{bmatrix}$

(a) Find det(A) by

(i) Cofactor expansion method.

(ii) "Basket-Weave" method. (4)

(b) Find A^{-1} using the Adjoint method. (5)

3.3 (a)Write the system of linear equations $\begin{aligned} x - z &= 3 \\ 2y - 2z &= 2 \\ 2x + z &= 3 \end{aligned}$ in matrix form. (1)

(b)Solve the system using **Cramer's Rule.** (4)

3.4 Find all the solutions of the given linear system using the Gauss method with back substitution, $\begin{aligned} 2x_1 + x_2 + -x_3 &= 2 \\ x_1 - x_2 + x_3 &= 7 \\ 2x_1 + 2x_2 + x_3 &= 4 \end{aligned}$. (4)

Question 4

4.1 The ellipse $\left(\dfrac{x^2}{16}\right) + \left(\dfrac{y^2}{9}\right) = 1$ is shifted 4 units to the right and 3 units up to generate the ellipse $\left(\dfrac{(x-4)^2}{16}\right) + \left(\dfrac{(y-3)^2}{9}\right) = 1$.

(a) Find the foci, vertices, and center of the new ellipse. (3)

(b) Plot the new foci, vertices, and center, and sketch in the new ellipse. (4)

4.2 Given that $P_1(1,-1,3)$ and $P_2(-1,4,5)$ find

(i) The coordinates of the midpoint of the line segment joining P_1 to P_2.

(ii) The unit vector in the direction of $\overrightarrow{P_1P_2}$.

(iii) Express $\overrightarrow{P_1P_2}$ as a product of its length and direction.

(iv) Find the angle between the vectors $\bar{u} = 4i - 3j + k$ and $\bar{v} = 2i - k$.
Are they orthogonal? (6)

Question 5

5.1 Show that $\tanh(\ln x) = \dfrac{x^2 - 1}{x^2 + 1}$. (3)

5.2 Find $\dfrac{dy}{dx}$ for the following $y = x^{\sinh x}$ (3)

5.3 Let $a_n = \dfrac{2n}{3n+1}$ (a) Determine whether $\{a_n\}$ is convergent. (3)

(b) Determine whether $\{a_n\}$ is convergent. (3)

5.4 Test the series $\sum_{n=1}^{\infty} (-1)^n \dfrac{n^3}{3^n}$ for absolute convergence. (3)