



University of Fort Hare
Together in Excellence

Mathematical Economics
ECO 222/ 222E
Main Exam
DEGREE EXAMINATIONS
NOVEMBER 2018

Time: 3 Hours

Subject: Mathematical Economics 2B

Marks: 100

This paper consist of 4 pages including cover page

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INSTRUCTIONS:

- Answer ALL THREE (3) questions, showing clearly all the calculations.
 - Use a PENCIL to draw clear diagrams where necessary
 - Calculators are allowed
 - Answers must be rounded off to two (2) decimal places

Question 1 (40 Marks)

- a) For a production function $Q = 36K + 120L - 3K^2 - 4L^2 - 10 + 2KL$
- (i) By differentiation find MP of labour L and capital K. [5]
 - (ii) Determine the maximum value of q [3]
- b) The demand equation of a good is $P + Q - 30$ and total cost function is $TC = \frac{1}{2}Q^2 + 6Q + 7$
- (i) Find the level of output which maximises total revenue (TR) [3]
 - (ii) Find the level of output which maximises profit. [4]
 - (iii) Calculate MR and MC at the value of Q in (ii) What do you notice. [5]
- c) If a consumer's utility function is given as $U = 45X^{0.4}Y^{0.5}$, where X and Y are two goods worth prices R2 and R3 respectively. Given an individual's income of R450, with the use of **(Economics and Substitution method)** find the quantities of X and Y that maximise utility (U). [7]
- d) The prices of inputs K and L are given as 12 and 3 Rands per unit and respectively, and the firm operates with the production function $Q = 25K^{0.5}L^{0.5}$. **(Use the Lagrangian Multiplier method)**
- (i) What is the minimum cost of producing 1250 units of output? [8]
 - (ii) Calculate and interpret the value of Lambda [5]

Question 2 (28 Marks)

- a) What total revenue a firm will earn if it charges a price of R715 and its $MR = 969 - 0.15q^2$ [8]
- b) If a firm faces the marginal cost schedule $MC = 180 + 0.3q^2$ and marginal revenue schedule $MR = 540 + 0.6q^2$
- (i) And total fixed costs are R65, what is the maximum profit it can make? (Assume that the second-order condition for a maximum is met. [10]
- c) Given the inverse demand function $P^D = 150 - 2q - 0.2q^2$.
- (i) Find the consumer's surplus when quantity purchased is $q=15$. [5]
- (ii) How does consumer's surplus change if q increases to $q=20$? [5]
- d) The demand curve for a product is $P = 60 - q - q^2$ and the supply function is $P = 10 + 4q$.
- (i) Determine the equilibrium price and quantity and the value of the consumer's surplus and producer's surplus at equilibrium. [5]
- (ii) What happens to the values of the consumer's surplus and producer's surplus if a tax of 2 per unit is imposed on the supply of the product? [5]

Question 3 (32 Marks)

a) Given the following 3x3 matrix:

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 3 \\ 4 & 0 & 0 \end{bmatrix}$$

- (i) Find the determinant of matrix A using the expansion by co-factor [2]
(ii) Find the inverse matrix (A^{-1}) [3]

b) Given the following matrices

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix}$$

- (i) Find the determinant $|A|$ [4]
(ii) Find the sum of $B + C$ [3]
- c) The demand functions for apricots (subscripted a) and blackberries (subscripted b) are inter-connected by the following demand functions:

$$Q_a^D = 96 - 4P_a + 2P_b \quad \text{and} \quad Q_b^D = 40 + 8P_a - 5P_b$$

The supply functions for apricots and blackberries (similarly subscripted) are:

$$Q_a^S = -4 + 28P_a \quad \text{and} \quad Q_b^S = -4 + 7P_b$$

- (i) By imposing the equilibrium condition that quantity supplied equals quantity demanded in both markets, find a pair of simultaneous in the variables P_a and P_b [5]
- d) Show that this pair of simultaneous equations can be written in matrix form as $AP=b$ Where $A = \begin{bmatrix} 32 & -2 \\ -8 & 12 \end{bmatrix}$ $P = \begin{bmatrix} P_a \\ P_b \end{bmatrix}$ $b = \begin{bmatrix} 100 \\ 44 \end{bmatrix}$ [10]
- e) By inverting the matrix A, find the equilibrium values of P_a and P_b . Hence find the equilibrium quantities of apricots and blackberries [5]

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