University of Fort Hare
Together in Excellence

MAT 121F

Supplementary Examinations

January 2019

Subject: **Mathematics**

Time: **3 Hours** Marks: **100**

This question paper consists of 3 pages

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Instructions

Answer all questions

Symbols used have the usual meanings

Question 1

1.1 Evaluate the special trigonometric limit, $\lim_{t\to 0}\frac{\sin 3t}{\sin 8t}$ (6)

1.2 Find $\frac{dy}{dx}$ if:

(a) $y=\cos^2(3x)+\tan 5x$ (3)

(b) $y=\sin x-x\cos x+x^2+4x+3$ (3)

(c) $y=x^2\sin x+2x\cos x-2\sin x$ (3)

1.3 Apply implicit differentiation to compute $\frac{dy}{dx}$ if

$x+\sin y=xy$ (5)

[20]

Question 2

2.1 Solve for x if

$\log_5\sqrt{x}=\log_{\sqrt{x}}5$ (5)

2.2 Differentiate each function below with respect x:

(a) $y=\ln\sqrt{3-x^2}$ (3)

(b) $y=x\ln x-x$ (4)

(c) $y=x^2e^{2x}\cos 3x$ (4)

(d) $y=(1+x^2)\arctan x$ (4)

[20]

Question 3

3.1 A ladder 26ft long rests on horizontal ground and leans against a vertical wall. The foot of the ladder is pulled away from the wall at the speed of 4ft/sec. How fast is the top of the ladder sliding down the wall when the foot is 10ft from the wall? (10)

3.2 A water tank has a shape of a cone, with base radius $2m$ and height $4m$. If water is pumped into the tank at a rate of $2m^3/\min$, find the rate at which the water level is rising when the level is $3m$ deep. (10)

[20]

Question 4

4.1 Find all critical points of the function f defined below

$$f(x) = 2x^3 - 3x^2 + 3$$ (8)

4.2 Use the second derivative test to determine the relative maximum and relative minimum of the function

$$f(x) = x^3 - 3x^2 + 2$$ (12)

[20]

Question 5

5.1 Use L'Hopital's rule to evaluate the indeterminate limits below:

(a) $\lim_{x\to 1}\frac{\ln x}{x-1}$ (5)

(b) $\lim_{x\to 1}\frac{x^2-x}{x^2-1}$ (5)

5.2 Find two positive integers whose sum is 20 and whose product is a maximum. (10)

[20]