UNIVERSITY OF FORT HARE

MAT 225

SUPPLEMENTARY EXAMINATIONS

December 2024

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Time: 3 HOURS

Subject: Geometry

Marks:100

This question paper consists of 3 pages

Internal examiner(s)

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Instructions

Answer **All 4** Questions.
Symbols have the usual meanings

Question One [25 marks]

1.1 For measurements of distance of $700km$ and difference in angle of elevation of $8.3°$, use the method of Eratosthenes to find the circumference of a planet. (5)

1.2 (a) Draw a model of Fano's geometry consisting of exactly seven points and seven lines. (2)

(b) Using the model in (a), name all the triangles in the geometry of Fano having point 4 as one vertex. (5)

1.3 Prove that there exists a set of two lines containing all the points of the three-point geometry. (6)

1.4 Is the set $\{I, R_1, R_2, R_3\}$ of symmetries for equilateral triangle a subgroup? Motivate your answer. (5)

1.5 Find the perimeter of a right triangle with shorter sides measuring 9 and 12 units. (2)

Question Two [25 marks]

2.1 Find the area of the triangular regions if the sides have the given lengths: $a = 7, b = 12, c = 15$. (5)

2.2 Let T_1, T_2 and T_3 be three transformations. Show that the associative property $(T_3T_2)T_1 = T_3(T_2T_1)$ always holds for transformations. (6)

2.3 Derive the general formula for the image of a point under the product of two translations with vectors $[a, b]$ and $[c, d]$. (5)

2.4 Find the inverse transformation for the transformation whose equations are given:

$$x' = x\cos 30° - y\sin 30°$$
$$y' = x\sin 30° + y\cos 30°$$

(6)

2.5 Describe the inverse of a glide reflection. (3)

Question Three [25 marks]

3.1 Prove synthetically that a segment and its image are parallel under a translation. (6)

3.2 For a given point $y^2 = 4x$, find the image under inversion with respect to the circle $x^2 + y^2 = 9$. (5)

3.3 Is the property of color, defined as $c - (a + b)$ of a set of points $ax + by^2 + cy + d = 0$, invariant under a translation? Justify. (5)

3.4 Find the ratio of similarity if a square region with an area of 12 square units has an image with an area of 17 square units. (4)

3.5 Prove that a quadrilateral is concyclic if and only if one side subtends equal angles at the other two vertices. (5)

Question Four [25 marks]

4.1 Suppose, in the Euclidean sense, that B is the midpoint of $\overline{AC}$. Describe the location of the harmonic conjugate of B with respect to A and C. (5)

4.2 Copy and complete the table below for symmetries of an equilateral triangle: (6)

	I	R_1	R_2	R_3	$R(120)$	$R(240)$
I	I	R_1	R_2	R_3	$R(120)$	$R(240)$
R_1	R_1		$R(240)$		R_3	R_2
R_2	R_2	$R(120)$	I		R_1	
R_3	R_3		$R(120)$	I		R_1
$R(120)$	$R(120)$			R_1	$R(240)$	I
$R(240)$	$R(240)$	R_3		R_2		

4.3 If P and Q are inverse points with respect to S, and if $SP = m$ and the radius of the circle of inversion is n, find the length SQ. (6)

4.4 Define the following terms: (i) Screw Displacement (ii) Dilation (2, 2)

4.5 Find the matrix representing the product of the two transformations: Rotation of $270°$ followed by reflection about $x - axis$. (4)

END