

MAT 121

SUPPLEMENTARY EXAMINATION

November 2024

Subject: Mathematics 1

Paper: A Theoretical Approach to Integral Calculus

Time: 3 Hours

Marks: 100

This question paper consists of 2 pages

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Instructions

Answer all Four questions.
Symbols have the usual meanings

Question One [25 marks]

1.1 Find f if $f'(x) = 5x^4 - 3x^2 + 4$, $f(-1) = 2$ (3)

1.2 Evaluate the following integrals:

(a) $I = \int \sqrt{1 + 2x^4} x^3 dx$ (4)

(b) $I = \int 3^{2x} dx$ (4)

(c) $I = \int_0^2 \frac{x^3 dx}{\sqrt{x^2+4}}$ (5)

(d) $I = \int \frac{dx}{x(x+1)(2x+3)}$ (5)

1.3 Find the area enclosed by the curve and the lines: $y = e^{3x}$, $x = 1$, $x = 2$ and the x -axis. (4)

Question Two [25 marks]

2.1 Discuss the convergence of $I = \int_{-\infty}^0 xe^x dx$ (4)

2.2 Find f_{xyz} if $f(x, y, z) = \sin(3x + yz)$ (3)

2.3 Use the Chain Rule to find $\frac{\partial z}{\partial t}$ if $z = x \arctan(xy)$, $x = t^2$, $y = se^{-t}$ (5)

2.4 Solve the following differential equations

(a) $y' = \frac{xy}{x^2 - y^2}$ (5)

(b) $3x^2 + 2xy + 3y^2 + (x^2 + 6xy)y' = 0$ and $y(1) = 2$ (5)

2.5 Find the unit vector in the direction of vector $\langle -2, 4, 3 \rangle$ (3)

Question Three [25 marks]

3.1 Find the cross product $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ if $\vec{A} = \langle 0, 1, 2 \rangle$, $\vec{B} = \langle 3, 1, 0 \rangle$ (4)

3.2 Find the area of the ΔPQR if $P(1, 0, 0)$, $Q(0, 2, 0)$ and $R(0, 0, 3)$ (4)

3.3 Derive (a) Parametric equations and (b) Symmetric equations of the line parallel to the vector $\langle a, b, c \rangle$ and passing through the point $P_0(x_0, y_0, z_0)$. (6)

3.4 Derive the equation of the plane that passes through $P_0(x_0, y_0, z_0)$ and is perpendicular to the vector $\vec{N} = a\hat{i} + b\hat{j} + c\hat{k}$ (5)

3.5 (a) Find the parametric and symmetric equations of l passing through $P(-3, 2, -3)$ and $Q(1, -1, 4)$

(b) At what point does the line intersect the xy -plane? (6)

Question Four [25 marks]

4.1 (a) Let A be the matrix $A = \begin{bmatrix} 2 & 3 & -4 \\ 1 & 2 & 3 \\ 3 & -1 & -1 \end{bmatrix}$ (i) Show that A is invertible (non-singular) (2)

(ii) Find A^{-1} using Row Reduction. (7)

4.2 Solve the system of equations (use any method):

$$2x + y - z = 2$$

$$x - y + z = 7$$

$$2x + 2y + z = 4$$

(4)

4.3 Let $A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & 1 \\ 3 & 1 & -1 \\ 0 & 1 & 1 \\ -1 & -2 & -1 \end{bmatrix}$. (i) Show that the matrix product AB is

define (ii) Find BA (iii) Find $\frac{1}{3}AB$ (4,2,2)

4.4 Find $Arg\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$ and express $\frac{1}{2} + i\frac{\sqrt{3}}{2}$ in polar form (4)

