UNIVERSITY OF FORT HARE

Geometry
MAT 225

Degree Examinations

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This paper consists of 3 pages

Internal examiner(s)

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Instructions

Answer all questions.

Symbols used have the usual meaning

Question One

1.1 Give a brief discussion on the Axiomatic System. [6]

1.2 Consider the following Set of axioms:

(A1):There are at least two buildings on campus.
(A2):There is exactly one sidewalk between any two buildings.
(A3):Not all buildings have the same sidewalk between them.

(a)What are the primitive\undefined terms of this system. [3]
(b)Prove that there are at least three buildings on campus. [3]
(c)Are the axioms independent? Explain. [3]
(d) Define a Model. Hence show by a model that it is possible to have exactly three sidewalks and three buildings. [3]

1.3 When are models said to be isomorphic? [2]

1.4 Draw a simple model for the Three-Point geometry. [1]
Hence prove that there exists a set of two lines containing all the points of the geometry. [4]

1.5 State the axioms of the *Four-Point geometry*. Draw a model for this geometry. [5]
Which axioms are also true statements in *Euclidean geometry*? [1]

Question Two

2.1 Explain how a segment and its image might be parallel for a rotation other than the identity. Give an illustration. [3]

2.2 Define a *glide reflection.* [2]
What is the image of the point $(5,7)$ under a glide reflection consisting of a reflection about the $x-axis$ followed by a translation of three units in the positive direction parallel to the $x-axis$. Give an illustration. [4]

2.3Prove that if a transformation is a plane motion, then it is the product of three or fewer reflections and conversely. [5]

2.4(a) Define a plane similarity that is a dilation. [3]
(b) Prove that the image of a circle under a dilation is another circle. [4]

Question Three

3.1 Prove that the internal bisectors of the angles of a triangle meet at the inceter. [4]

3.2 Prove that the external bisector of the angle of a triangle (not iscosceles) divides the opposite side (externally) into two segments proportional to the sides of the triangle adjacent to the angle. [7]

3.3 Give a definition and proof of the *circle of Apollonius*. [6]

3.4 Prove that the product of the lengths of the segment from an exterior point to the points of intersection of a secant with a circle is equal to the square of length of the tangent from the point to the circle. [4]

Question Four

4.1For the *transformation of inversion T*

(a) Give one invariant property. [1]

(b) Explain (with full motivation) how to find the inverse point, with respect to a circle, of a point that lies outside the circle. [4]

(c) Why must each line, not just lines through the centre of inversion, contain the ideal point? [2]

4.2 Prove that the image of a straight line not through the centre of inversion, under the transformation of inversion is a circle passing through the centre of inversion. [5]

4.3 List down four axioms for *Projective Geometry*. [4]

4.4 Draw a *complete quadrangle*, identifying the pairs of *opposite sides* and the *diagonal triangle*. [4]

4.5 Explain the concept of *plane duality* in projective geometry. [2]

4.6 Explain what is meant by *Hyperbolic* and *Elliptic geometry*. Give a brief comparison between them and *Euclidean geometry*. [5]