UNIVERSITY OF FORT HARE

University of Fort Hare
Together in Excellence

PHY 502

HONOURS EXAMINATIONS

DATE	:	January/February 2019
TIME	:	3 HOURS
SUBJECT	:	PHYSICS 502 ELECTRODYNAMICS
MARKS	:	100

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EXTERNAL EXAMINER: DR. D. Tinarwo

INSTRUCTIONS

Answer ***ALL FOUR QUESTIONS***

1. (a) Write the expression for the four-current density j^{μ} in terms of charge and current densities ρ and $\vec{j}$. [4]

   (b) Write the expression for four-momentum vector P^{α} in terms of $\vec{E}$, $\vec{p}$ and c. [4]

   (c) Write the charge conservation equation in covariant form. [4]

   (d) Show that $\partial^{\mu}\partial_{\mu}\left(\frac{\Phi}{c},\vec{A}\right)=\mu_0\left(c\rho,\vec{j}\right)$ [4]

   (e) Show that the electrodynamic tensor $F^{\mu\nu}$ is given as:

$$F^{\mu\nu}=\begin{bmatrix} 0 & \frac{-E_x}{c} & \frac{-E_y}{c} & \frac{-E_z}{c} \\ \frac{E_x}{c} & 0 & -B_z & B_y \\ \frac{E_y}{c} & B_z & 0 & -B_x \\ \frac{E_z}{c} & -B_y & B_x & 0 \end{bmatrix}$$ [9]

2. (a) Derive the general inhomogeneous wave equations for the scalar and vector potentials in electrodynamics. [10]

   (b)Then, specialize to the Lorentz condition and write the wave equations for this particular condition. [5]

   (c) Using the Maxwell's equations show that the hertz potential $\vec{\mathrm{H}}_h$ satisfies the wave equation given as: $(\nabla^2-\varepsilon\mu\partial_t^2)\vec{\mathrm{H}}_h=0$ [10]

3. (a) Show that Poynting's theorem can be written in the form :

$$\frac{dW}{dt} = -\frac{d}{dt}\int_V U_{em} d\tau - \int_{surface} \vec{s} \cdot d\vec{a} \,, \qquad [8]$$

(b) Using the solution from (a) show that the differential form for the pointing theorem takes the form: $\nabla \cdot \vec{S} = -\partial_t U$, where $U = U_{mech} + U_{em}$ [4]

(c) Consider a hollow rectangular metallic wave guide, with a cross section of sides *a* and *b*, show that the TE modes satisfy the following wave equation:

$$\left(\nabla_t^2 + (\omega^2 \mu\varepsilon - \kappa^2)\right)\vec{E} = 0 \qquad [8]$$

(d) In relation to (c) find the cut-off frequency. [5]

4. (a) Write an expression for the inhomogeneous Maxwell's equations in covariant from. [4]

(b) Show that Maxwell's homogeneous equations can be written as:

$$\partial^\alpha F^{\beta\gamma} + \partial^\beta F^{\gamma\alpha} + \partial^\gamma F^{\alpha\beta} = 0 \qquad [8]$$

(c) Write Maxwell's equations in the dual form and explain their meaning. [5]

(d) Show that four-force density is given as: $\mathrm{K}_\mu = -\partial_\sigma \eta^\sigma{}_\mu$ [8]