UNIVERSITY OF FORT HARE

MAP 211

SUPPLEMENTARY EXAMINATIONS

July 2025

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Time: **3 HOURS**

Subject: Introduction to Numerical Analysis

Marks:100

This question paper consists of 3 pages.

Internal examiner(s)

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Moderator(s)

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Instructions

Answer All questions.
Symbols have their usual meanings.

Question One

1.1. Prove that for any positive integer k,

$$\Delta^k y_0 = \sum_{i=0}^{k} (-1)^i \binom{k}{i} y_{k-i}$$

1.2.1. $\Delta\left(\frac{\quad}{v_k}\right) = \frac{\quad}{v_{k+1}v_k}$. (5)

1.2.2. $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{1}{4}\delta^2}$ (5)

1.2.3. if $y_k = \sin k$ then $\Delta y_k = 2\sin\frac{1}{2}\cos(k + \frac{1}{2})$. (5)

1.3. Find the second finite difference $\Delta^2 y_k$ if $y_k = k^3$. (5)

1.4. Prove that $\Delta k^{(n)} = nk^{(n-1)}$ for all integers n. (4)

1.5. Find a function y_k for which $\Delta^2 y_k = 9y_k$. (4)

where $\binom{k}{i} = \frac{k!}{i!(k-i)!}$ is the symbol for binomial coefficients. (5)

1.2. Show that,

1.2.1. $\Delta\left(\frac{u_k}{v_k}\right) = \frac{v_k \Delta u_k - u_k \Delta v_k}{\quad}$ (5)

Question Two

2.1. Express $y_k = 2k^{(3)} - k^{(2)} + 4k^{(1)} - 7$ as a conventional polynomial. (5)

2.2. Evaluate the following summation by finite integration $\sum_{i=1}^{n} i^2$. (3)

2.3.

(a) Calculate a fourth divided difference for the following y_k values:

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Question Two

2.4. Find a polynomial of degree four that meets the following conditions.

x_k	0	1	2	4	5
y_k	0	16	48	88	0

(5)

x_k	y_k	y_k'	y_k''
0	1	−1	0
1	2	7	–

(5)

2.5. Find a polynomial $p(x)$ of degree n or less, which together with its first n derivatives takes the values $y_0,\ y_0^{(1)},\ y_0^{(2)},\ \ldots,\ y_o^{(n)}$ for the argument x_0 . (5)

2.6. Find 2 second – degree polynomials, one having $P_1(0) = P_1'(0) = P_1''(0) = 0$, the other having $P_2(4) = 2, P_2'(4) = P_2''(4) = 0$, both passing through (2,1) with $P_1''(2) = P_2''(2) = 0$. (4)

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Question Three

3.1. The distance x of a runner from a fixed point is measured (in meters) at intervals of half a second. The data obtained are

t	0.0	0.5	1.0	1.5	2.0
x	0.00	3.65	6.80	9.90	12.15

Use central differences to approximate the runner's velocity at times $t = 0.5\ s$ and $t = 1.25\ s$.

(6)

3.2. Let $f(x) = \cosh(x)$ and $a = 2$. Let $h = 0.01$ and approximate $f'(a)$ using forward, backward and central differences. Work to 8 decimal places and compare your answers with the exact result, which is sinh (2). (4,4,4)

3.3. The distance x, of a downhill skier from a fixed point is measured at intervals of $0.25\ s$. The data gathered is:

t	0	0.25	0.5	0.75	1	1.25	1.5
x	0	4.3	10.2	17.2	26.2	33.1	39.1

Use a central difference to approximate the skier's velocity and acceleration at the times $t = 0.25\ s, 0.75\ s$ and $1.25\ s$. Give your answers to 1 decimal place. (8)

3.4. It is given that the function $e^{\frac{-x^2}{2}}$ has a second derivative that is never greater than 1 in absolute value.

(a) Use this fact to determine how many subintervals are required for the composite trapezium method to deliver an approximation to

$$\int_0^1 \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx$$

that is guaranteed to have an error less than $\frac{1}{2} \times 10^{-2}$. (3)

$$\int_2 \ln(x)\, dx$$

using Simpson's rule. (4)

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(b) Find an approximation to the integral that is in error by less than $\frac{1}{2} \times 10^{-2}$. (3)

3.5. Determine the minimum number of steps needed to guarantee an error not exceeding ± 0.000001 when numerically evaluating

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END