UNIVERSITY OF FORT HARE

MAT 113

DEGREE EXAMINATIONS

June 2023

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Time: 3 HOURS

Subject: A PRACTICAL APPROACH TO INTEGRAL CALCULUS

Marks: 100

This question paper consists of 3 pages

Internal examiner(s)

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Instructions

Answer **any 4** questions **ONLY**.
Symbols have the usual meanings

Question One [25 marks]

1.1 Integrate $I = \int \frac{x^3+4}{x^2+4}$ (5)

1.2 Derive the formula $\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,dx = \begin{cases} 0 & if \quad m \neq n \\ \pi & if \quad m = n \end{cases}$ (5)

1.3 Evaluate the convergence of $I = \int_0^3 \frac{dx}{x^2-6x+5}$ (5)

1.4 Find $I = \int \cos 3x \cos 4x\, dx$ (4)

1.5 Use integration by parts to prove the reduction formula: $\int cos^n x\, dx = \frac{1}{n} cos^{n-1} x sinx + \frac{n-1}{n} \int cos^{n-2} x\, dx$ (6)

Question Two [25 marks]

2.1 If $I = \frac{V}{R}$, and $V = 250\ volts$ and $R = 50\ ohms$, find the change in I i.e $\frac{\delta I}{\delta V}$ resulting from an increase of 1 $volt$ in V and an increase of $0.5\ ohm$ in R. (6)

2.2 Find $I = \int \frac{\sqrt{9x^2-4}}{x} dx$ (5)

2.3 Evaluate the integral $\int_0^1 \frac{x^3+2x}{x^4+4x^2+3} dx$ (4)

2.4 Calculate the total area bounded by the curve $y = x^2 - 4$, the $x - axis$ and the ordinates at $x = -1$ and $x = 4$ (5)

2.5 Show that $\int_{-\infty}^{\infty} x\, dx$ is divergent. (5)

Question Three [25 marks]

3.1 A simple circuit contains an emf producing a voltage $E(t)$ in $Volts$, a resistor with resistance R in hms , an inductor with inductance L in $Henries$. The current $I(t)$ produced is in $Amperes$. There is also a switch in the circuit. Now suppose $E(t) = 40sin60t\ volts$; $L = 1H$; $R = 20\ Ohms$ and $I(0) = 1A$, find $I(t)$ and $I(0.1)$. (7)

3.2 A constant force $\bar{F} = 10\hat{\imath} + 18\hat{\jmath} - 6\hat{k}$ moves an object along a straight line from the point $(2, \ 3, \ 0)$ to the point $(4, \ 9, \ 15)$. Find the work done if the distance is measured in metres and the force is measured in Newtons. (6)

3.3 Let $x^4 + y^4 + z^4 + 6xyz = 5$, where $z = f(x, y)$. Find Z_y. (3)

3.4 For what values of c is the angle between the vectors $\langle -1, 2, 1 \rangle$ and $\langle 1, 0, c \rangle$ equal to 60°? (5)

3.5 Given that $A = \begin{pmatrix} 4 & 2 & 6 \\ 1 & 8 & 7 \end{pmatrix}$ determine:

(a) A^T (2)

(b) $A \cdot A^T$ (2)

Question Four [25 marks]

4.1 Solve the following differential equation $xy' = y + xe^{y/x}$ (6)

4.2 Use the Row Reduction Method to find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & -1 \end{bmatrix}$. (6)

4.3 If $\frac{2-a-i5}{1-ib} = 3 - i5$, find a and b. (4)

4.4 Use vector methods to prove that the line segment joining the mid-points of two sides of a triangle is parallel to the third side. (5)

4.5 Simplify the following, giving the result in polar form $\frac{10(\cos 126^\circ + i \sin 126^\circ)}{2(\cos 72^\circ + i \sin 72^\circ)}$ (4)

Question Five [25 marks]

5.1 Find the centroid of the region enclosed by the x-axis and the top half of the ellipse $9x^2 + 4y^2 = 36$ (6)

5.2 Consider the series $\sum_{n=1}^{\infty} n/(n+1)!$.

(a) Find the partial sum s_1, s_2, s_3 and s_4. Do you recognize the denominators? Use the pattern to guess a formula for s_n. (4)

(b) Use mathematical induction to prove your guess. (4)

(c) Show that the given infinite series is convergent, and find its sum. (3)

5.3 Find the polar forms for z/w, and $1/z$ by first putting z and w into polar form.

$z = 2\sqrt{3} - 2i, w = -1 + i$ (5)

5.4 Use the De Moivre's Theorem to solve $\left(2\sqrt{3} + 2i\right)^5$ (3)

END