

UNIVERSITY OF FORT HARE

**A Practical approach to Differential Calculus
MAT 112**

Degree Examinations

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Time: 3 Hrs

Subject: Applied Mathematics

Marks: 100

This question paper consists of 3 pages

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Instructions

Answer all questions.

Symbols have the usual meanings



Question 1

- 1.1 Prove by means of *mathematical induction* that for all positive integral values n ; $7^n - 1$ is divisible by 6. (6)
 - 1.2 Find the first four terms in the expansion of $\frac{1}{\sqrt{2-3x}}$ and state for which values of x the expansion is valid. (7)
 - 1.3 Find without expanding the binomial, the term containing x^{14} in $\left(\frac{2}{x} - x^2\right)^{10}$ (simplify it). (4)
 - 1.4 Solve for x if (a) $\frac{4}{x} \leq 5 - x$ and (b) $\left|\frac{2x+3}{x-1}\right| \leq 4$ and represent your solution in the form of (i) interval notation (ii) an real number line. (10)
- [27]**

Question 2

- 2.1 Consider the function $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \leq x < 3 \\ (x-3)^2 & \text{if } x > 3 \end{cases}$ and (9)
- 2.1.1 Sketch of its graph
 - 2.1.2 Evaluate each of the following limits; (i) $\lim_{x \rightarrow 0^+} f(x)$ (ii) $\lim_{x \rightarrow 0^-} f(x)$ (iii) $\lim_{x \rightarrow 0} f(x)$
(iv) $\lim_{x \rightarrow 3^+} f(x)$ (v) $\lim_{x \rightarrow 3^-} f(x)$ (vi) $\lim_{x \rightarrow 3} f(x)$
 - 2.1.3 Where is f discontinuous?
- 2.2 Find the number a that makes f to be continuous at $x = -1$. $f(x) = \begin{cases} \frac{x^2+3x+2}{x+1}, & \text{if } x \neq -1 \\ a & \text{if } x = -1, \end{cases}$ (4)
- 2.3 Evaluate the following limit $\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2}}{x}$. (4)
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Question 3

- 3.1 Find $\frac{dy}{dx}$ if (28)
- (a) $y = \frac{3x-7}{x^2+5x-4}$ (b) $y = \sin(\tan(\cos x))$ (c) $xy = \cos(xy)$ (d) $y = 2^{3x}$ (e) $y = e^{(x-\frac{1}{x})}$
(f) $y = x^{\tan x}$ (g) $y = \tan^{-1} \sqrt{x^2 - 1}$
- 3.2 Given $f(x) = x^3 - 3x + 1$, find (10)
- (a) the intervals of increase and decrease, (b) the critical point(s) and state which is (are) local maxima or minima, (c) the points of deflection if any, then (d) sketch the graph of f .
- 3.3 A gardener wishes to enclose a rectangular area with 1200 meters of fence. He intends to divide the area into four equal sub-areas. What is the largest rectangular area that may be enclosed. (5)

[43]

Question 4

Evaluate the following integrals

$$4.1 \int_1^4 (8x^3 - x) dx \quad (2)$$

$$4.2 \int_4^9 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2} \quad (4)$$

$$4.3 \int \sin^2(5x) dx \quad (4)$$

$$4.4 \int (1 - \sin^2 3t) \cos 3t dt \quad (4)$$

[13]

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