UNIVERSITY OF FORT HARE

A Practical approach to Integral Calculus
MAT 123

Supplementary Examinations

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Time: 3 Hrs

Subject: Mathematics 1

Marks: 100

This question paper consists of 3 pages

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Instructions

Answer All Questions

Symbols have the usual meanings.

Question. 1

Choose one correct answer from those given in each question.

1.1 Find the value of the integral, $\int_0^1 e^{-x}dx$

A) 1 - e B) 0 C) 2e D) e - 1

1.2 Find the value of the integral, $\int_0^1 xe^x dx$

A) 2 B) 1 C) e - 1 D) 0

1.3 Find the value of the integral, $\int_0^{\pi} cos^4\, d\, x$

A) 1/3 B) 1/4 C) 3π/4 D) 4π/3

1.4 In partial fractions $\frac{3x-1}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$ find the value of A.

A) -2 B) 1/2 C) 2 D) -1/2

1.5 Find the value of the integral, $\int_0^1 \frac{1}{1+\sqrt{x}}dx$

A) $2(1+\ln 2)$ B) $2(1-\ln 2)$ C) $2\ln 2$ D) $4-\ln 2$ **[15]**

Question. 2

2.1 Determine whether the following statements are true or false, if false correct the statement (5)

2.1.1 The partial fraction form of $\frac{2x-3}{(x-2)(x+2)} = \frac{Ax+C}{x+2} + \frac{Bx+E}{x-2}$.

2.1.2 The equation $3(x^2+2xy)dx + (3x^2+2y)dy = 0$ is exact.

2.1.3 The equation $\frac{dy}{dx} + xy = 0$ is linear.

2.1.4 The equation $(xy+y^2)\frac{dx}{dy} = x^2$ is separable.

2.1.5 The trigonometric substitution for $\int_0^1 \frac{x^2}{\sqrt{16x^2-9}}dx$ is $x = \frac{3}{4}\sin\theta$

2.2.1 If $f(x,y) = x^3y^2 + \frac{x}{y} + \frac{y}{x}$ find (a)$f_x(1,-1)$ (b)$f_y(1,-1)$ (c) $f_{yx}(1,-1)$ (6)

2.3.1 If $xyz = sin(2x+y+3z)$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ (4)

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Question. 3

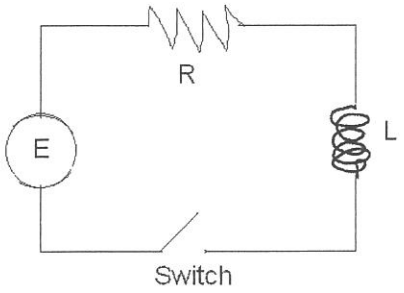
3.1 Consider $x^2\frac{dy}{dx} = \frac{xy+y^2}{1}$

3.1.1 What kind of a differential equation is the above given one (1)

3.1.2 Find the **general solution** of the above **D.E.** (4)

3.1.3 Find the **particular solution** in the above D.E if $y(-2) = -4$ (2)

3.2 In the circuit shown below, a battery supplies a constant voltage of 50 V, the inductance is 5 H, the resistance is 10 ohms and I(0) = 0.



3.2.1 State Kirtchoff's laws mathematically. (2)
3.2.2 Find the current, I(t).at any time, t. (4)
3.2.3 Find the current after 2 seconds. (2)
[15]

Question. 4

4.1 Find the unit vector, $\vec{U}$ that has the same direction as $b = 2\hat{i} - 2\hat{j} + 7\hat{k}$. (3)

4.2 Consider the following two vectors; $\overline{A} =< 1,1,2 >$; $\overline{B} =< 0,2,-3 >$

4.2.1 What is the angle between vector $\overline{A}$ and $\overline{B}$. (3)
4.2.2 Find $\overline{A} \times \overline{B}$ (3)
4.2.3 Find the unit vector, $\vec{U}$ in the direction of vector $\overline{A} \times \overline{B}$. (3)
4.2.4 Show that $\overline{A} \times \overline{B}$ and $\overline{B}$ are orthogonal (perpendicular) (4)

4.3 Find all the values of x such that the following vectors are orthogonal. (4)
$\overline{A}$ = <x, x, -1> and $\overline{B}$ = <1, x, 6>.

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Question. 5

5.1

5.1.1 Find $(1+2i)(3-2i)$ and write your solution in (i) standard complex form (ii) polar form and (iii) Euler form. (6)

5.1.2 Find $\dfrac{1+2i}{3-2i}$ and write your solution in (i) standard complex form (ii) polar form and (iii) Euler form) (6)

5.2 Using De Moivre's formula, find the cube roots of $i+i$. Sketch the roots in the Argand plane. (4)
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Question. 6

6.1 Consider the following matrices;

$$A=\begin{bmatrix}2 & -3 & 1\\ -1 & 4 & 2\\ 2 & -1 & 3\end{bmatrix} \quad B=\begin{bmatrix}2 & -1 & -2\\ -1 & 2 & 1\\ 1 & 0 & -3\end{bmatrix} \quad C=\begin{bmatrix}3 & 1 & -1 & -2\\ -2 & 5 & 4 & 3\\ 1 & 2 & 3 & 1\end{bmatrix}$$

6.1.1 Find A - 3B. (2)

6.1.2 Find AC. (4)

6.1.3 Find the transpose of C. (1)

6.1.4 Find the Determinant of B. (2)

6.1.5 It is not possible to get A+C, give a reason why is this so? (2)

6.2 Find the polar equation given by $x^2 = 4y$ (2)

6.3 Find the Cartesian equation given by $5 = \dfrac{r}{1-\cos\theta}$ (2)

[15]

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