University of Fort Hare
Together in Excellence

MAT 123F

Degree Examinations

November 2018

Subject: **Mathematics**

Time: **3 Hours** Marks: **100**

This question paper consists of 4 pages

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Instructions

Answer all questions

Symbols used have the usual meanings

Question 1

1.1 Find

(a) z_1+z_2 if $z_1=2+3i$ and $z_2=4+5i$ (2)

(b) z_1-z_2 if $z_1=4+7i$ and $z_2=-3+9i$ (2)

(c) $\frac{z_1}{z_2}$ if $z_1=1+3i$ and $z_2=2+i$. Use any method (4)

1.2 Let z_1 and z_2 be complex numbers. Prove that

$$|z_1 z_2| = |z_1||z_2|$$ (4)

1.3 convert the complex number

$$z=2-2i$$

into its trigonometric form

$$z=r(\cos\theta+i\sin\theta)$$ (8)

[20]

Question 2

2.1 Let

$$f(x,y)=3x^2y-xy^3$$

Compute the partial derivative $\frac{\partial f}{\partial x}$ from first principles. (5)

2.2 Let

$$f(x,y)=x\cos y+ye^x.$$

Compute f_x, f_{xx}, f_{xy}, f_y, f_{yy} (5)

2.3 Let

$$f(x,y)=x^2-2y^3, \text{ where } x=\sin t \text{ and } y=e^{3t}$$

Use the chain rule to find $\frac{df}{dt}$ (5)

2.4 Solve the differential equation

$$\frac{dy}{dx}=3x^2y \quad (5)$$

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Question 3

3.1 Show that the differential equation

$$\left(x^2+y^2\right)dx+\left(2xy+\cos y\right)dy=0$$

is exact, and the find its general solution.

Use any method of your choice. (10)

3.2 Find the particular solution to the IVP

$$\frac{dy}{dx}+y=x, \qquad y(0)=4 \quad (10)$$

[20]

Question 4

4.1 A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 420.

(a) Find the number of bacteria after 3 hours
(b) When will the population reach 10 000 cells? (10)

4.2 A roast chicken is taken from an oven where its temperature has reached $85^{\circ}C$ and is placed on a table in a kitchen where the temperature is $22^{\circ}C$.

(a) If the temperature of the chicken is $65^{\circ}C$ after 30 minutes, what is the temperature after 45 minutes?
(b) When will the chicken cool to $40^{\circ}C$

Assume Newton's Law of Cooling applies. (10)

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Question 5

5.1 Show that the half-life of a radioactive substance is independent of the original amount. (5)

5.2 A skydiver of mass m jumps from an airplane through the air. The magnitude of the air resistance is

$$R = kv$$

where v is the velocity of the skydiver and k is a constant.

(a) Sketch the free-body diagram of the skydiver. Take the downward

direction as positive.

(b) Applying Newton's second law of motion, obtain a linear first order differential equation

(c) Assuming that initially

$$v(0)=v_0$$

solve the resulting initial value problem for $v(t)$, where t is time

(d) Find the limiting velocity (the velocity after a long time has elapsed) of the skydiver. (15)

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