

UNIVERSITY OF FORT HARE

MNU 122

SUPPLEMENTARY EXAMINATIONS

November 2024

Time: 3 HOURS

Subject: COMPUTATIONAL METHODS

Marks:100

This question paper consists of 3 pages.

Internal examiner(s)

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Instructions

Answer All questions.
Symbols have their usual meanings.

Question One

1.1. Solve the following system of linear equations using the Gaussian Elimination Method.

$$\begin{cases} x + y - z = -2 \\ 2x - y + z = 5 \\ -x + 2y + 2z = 1 \end{cases} \quad (10)$$

1.2. Show that if,

$$A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \text{ then } (A + B)^2 \neq A^2 + 2AB + B^2 \quad (8)$$

Question Two

2.1. Express the following equations in linear form.

(a) $y = ax + b \ln x$

(b) $y = ae^x$

(c) $y = ax^2 + bx$

(d) $y = ax + by$

(e) $xa^y = b$

(10)

2.2. The torque T required to rotate shafts of different diameters D on a machine is shown below. If the law is $T = aD^n$, determine the best values for the constants a and n using the least squares method.

D	7	10	18	25	40
T	0.974	1.71	4.33	7.28	15.3

(15)

2.3. If $R = a + \frac{b}{d^2}$, find the best values for a and b from the set of corresponding values below:

d	0.1	0.2	0.3	0.5	0.8	1.5
R	5.78	2.26	1.6	1.27	1.53	1.1

(15)

Question Three

3.1. Show that $f(x) = x^3 + 4x^2 - 10$ has a root in $[1,2]$ and use the Bisection Method to determine an approximation to the root, accurate to within $f(x) < 0.01$. (15)

3.2. Use the Newton-Raphson Method to solve the equation $f(x) = x^3 - x - 1$ accurate to $f(x) < 0.001$. Let $x_0 = 1$. (12)

3.3. Let $f(x) = x^2 - a$. Show that the Newton Method leads to the iteration.

$$x_{n+1} = \frac{1}{2} \left(x_n + \frac{a}{x_n} \right) \quad (5)$$

Question Four

4.1. Derive the Trapezoidal Rule for approximating $\int_a^b f(x) dx$. (5)

4.2. Derive the Simpsons Rule for approximating $\int_a^b f(x) dx$. (5)

END

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