University of Fort Hare

Together in Excellence

MAT 123 F

MAIN EXAMINATIONS NOVEMBER 2024

Subject: **Mathematics (F)**

Paper: MATHEMATICS (F)

Time: **3 Hours** Marks: **100**

This question paper consists of 3 pages

Internal Examiner

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Moderator

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Instructions

Answer all questions.

Symbols have the usual meanings.

Question 1

1.1 Solve the linear system of equations below

$$\begin{aligned} x_1 - \ 2x_2 + \ 3x_3 &= 11 \\ 4x_1 + \ x_2 - \ x_3 &= 4 \\ 2x_1 - \ x_2 + \ 3x_3 &= 10 \end{aligned}$$

by the Gauss-Jordan Transformation method. (10)

1.2 Use Cramer's rule to solve the linear system.

$$\begin{aligned} 2x + \ y + \ z &= 6 \\ 3x - \ 2y - \ 3z &= 5 \\ 8x + \ 2y + \ 5z &= 11 \end{aligned}$$

(10)

[20]

Question 2

2.1 Discuss and sketch the graph of the curve given by the polar equation

r = 1

in the XY- plane. (10)

2.2 Solve for x if

$$x^2 - 6x + 13 = 0$$

(5)

2.3 Perform the indicated operation.

(a) $z_1 + z_2$ if $z_1 = 2 + 8i$ and $z_2 = 3 + 4i$ (2)

(b) $z_1 - z_2$ if $z_1 = 5 + 3i$ and $z_2 = 4 - 7i$ (3)

[20]

Question 3

3.1 Let

$z_1 = 1 + 3i$ and $z_2 = 2 + i$

Calculate the quotient $\frac{z_1}{z_2}$, by using the multiplicative inverse method. (4)

3.2 Let

$$z = a + bi$$

be a complex number.

<u>Claim:</u> $|z|^2 = z\,\bar{z}$

Prove the claim above. (6)

3.3 Convert the complex number

$$z = -5 - 5i$$

into the polar form (or trigonometric form)

$$z = r(\cos\theta + i\sin\theta),$$

where r and θ are the polar coordinates of z. (10)

[20]

Question 4

4.1 Let

$$f(x, y) = x\cos y + ye^x$$

Compute the partial derivatives f_x, f_{xx}, f_{xy}, f_y, and f_{yy} (in that order) (5)

4.2 Let

$f(x, y) = x^2 - 2y^3$, where $x = \sin t$ and $y = e^{3t}$.

Use the chain rule to find the derivative $\frac{df}{dt}$ (5)

4.3 Solve each differential equation below by separating the variables:

(a) $\frac{dy}{dx} = \frac{xy}{x^2+1}$ (provide an explicit general solution) (5)

(b) $\frac{dy}{dx} = \frac{6x^2}{2y + \cos y}$ (provide an implicit general solution) (5)

[20]

Question 5

5.1 Obtain a particular solution to the initial value problem (IVP)

$$\frac{dy}{dx} + \frac{y}{x} = 4x^2, \qquad y(1) = 2$$ (10)

5.2 Show that the differential equation

$$(3x^2 + y\cos x)dx + (\sin x - 4y^3)dy = 0$$

is exact, and then find its general solution implicitly. (10)

[20]