

UNIVERSITY OF FORT HARE

**MAT 123**

DEGREE EXAMINATIONS

November 2018

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Time: 3 HOURS

Subject: A PRACTICAL APPROACH TO INTEGRAL CALCULUS

Marks:100

This question paper consists of 3 pages

**Internal examiner(s)**

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**Instructions**

Answer **any 4** questions **ONLY**.  
Symbols have the usual meanings

**Question One [25 marks]**

1.1 Evaluate the integral  $I = \int_0^{\pi/2} \frac{\sin 2x}{2 + \cos x} dx$  (6)

1.2 Integrate  $I = \int \frac{x}{\sqrt{x^2 + x + 1}} dx$  (5)

1.3 First make a substitution and then use integration by parts to evaluate the integral:

$$\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2) d\theta \quad (5)$$

1.4 Evaluate  $I = \int_{\pi/4}^{\pi/2} \cot^5 \phi \csc^3 \phi d\phi$  (5)

1.5 Find  $I = \int \frac{x^3}{\sqrt[3]{x^2+1}} dx$  (4)

**Question Two [25 marks]**

2.1 A particle that moves along a straight line has velocity  $v(t) = t^2 e^{-t}$  meters per seconds after  $t$  seconds. How far will it travel during the first  $t$  seconds? (6)

2.2 Find  $I = \int \frac{\cos^5 \alpha}{\sqrt{\sin \alpha}} d\alpha$  (5)

2.3 Uses partial fraction to evaluate the integral  $I = \int \frac{x^2 - 3x + 7}{(x^2 - 4x + 6)^2} dx$  (7)

2.4 Find  $I = \int e^x \sin x dx$  (4)

2.5 Evaluate  $\int_0^3 \frac{dx}{x-1}$  if possible. (3)

**Question Three [25 marks]**

3.1 Let  $x^4 + y^4 + z^4 + 6xyz = 5$ , where  $z = f(x, y)$ . Find  $Z_y$ . (3)

3.2 Find the value of the constant  $C$  for which the integral  $\int_0^{\infty} \left( \frac{1}{\sqrt{x^2+4}} - \frac{C}{x+2} \right) dx$  converges.

Evaluate the integral for this value of  $C$ . (6)

3.3 A vat with 2000L of beer contains 4% alcohol (by volume). Beer with 6% alcohol is pumped into the vat at a rate of 20L/min and the mixture is pumped out at the same rate.

What is the percentage of alcohol after an hour? (7)

3.4 Solve the differential equation  $\frac{du}{dt} = \frac{2t + \sec^2 t}{2u}$ ,  $u(0) = -5$  (5)

3.5 Use Chain Rule to find  $\frac{\partial z}{\partial s}$ , if  $z = x^2 \sin y$ ,  $x = s^2 + t^2$ ,  $y = 2st$  (4)

**Question Four [25 marks]**

4.1 Let  $A$  be the matrix  $A = \begin{bmatrix} 6 & 2 & 0 & 4 \\ 5 & 3 & 2 & 1 \\ 0 & -1 & 1 & -2 \\ 2 & -3 & 1 & 0 \end{bmatrix}$ . Find  $A^{-1}$  using Row Reduction Method. (7)

4.2 Find an equation of the plane through the three given points:

$(-1, 1, -1)$ ,  $(1, -1, 2)$  and  $(4, 0, 3)$  (5)

4.3 For what values of  $c$  is the angle between the vectors  $\langle 1, 2, 1 \rangle$  and  $\langle 1, 0, c \rangle$  equal to  $30^\circ$ ? (5)

4.4 Verify the distributive property  $A(B + C) = AB + AC$

using the matrices  $A = \begin{bmatrix} 4 & 5 & 2 \\ 3 & -2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} -3 & 6 \\ 0 & 7 \\ 1 & 2 \end{bmatrix}$  and  $C = \begin{bmatrix} 5 & -4 \\ 2 & 0 \\ 7 & -6 \end{bmatrix}$ . (5)

4.5 Evaluate the expression  $\frac{3}{4-3i}$  and express your answer in the form  $a + ib$  (3)

**Question Five [25 marks]**

5.1 Find the centroid of the region enclosed by the x-axis and the top half of the ellipse  $9x^2 + 4y^2 = 36$  (6)

5.2 Consider the series  $\sum_{n=1}^{\infty} \frac{n}{(n+1)!}$ .

(a) Find the partial sum  $s_1, s_2, s_3$  and  $s_4$ . Do you recognize the denominators?

Use the pattern to guess a formula for  $s_n$ . (4)

(b) Use mathematical induction to prove your guess. (4)

(c) Show that the given infinite series is convergent, and find its sum. (3)

5.3 Find the polar forms for  $z/w$ , and  $1/z$  by first putting  $z$  and  $w$  into polar form.

$z = 2\sqrt{3} - 2i$ ,  $w = -1 + i$  (5)

5.4 Use the De Moivre's Theorem to solve  $(2\sqrt{3} + 2i)^5$  (3)

END