MAT 121

DEGREE EXAMINATIONS

November 2024

Subject: **Mathematics 1**

Paper: A Theoretical Approach to Integral Calculus

Time: **3 Hours**

Marks: **100**

This question paper consists of 2 pages

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Instructions

Answer all Four questions.

Symbols have the usual meanings

Question One [25 marks]

1.1 Find f if $f''(\theta) = \sin\theta + \cos\theta$, $f(0) = 3,\ f'(0) = 4$ (4)

1.2 Evaluate for the following integrals

(a) $I = \int_1^2 \frac{dx}{(3-5x)^2}$ (4)

(b) $I = \int e^{-x} \cos 3x\, dx$ (4)

(c) $I = \int_0^{\pi} cos^4(2t)\, dt$ (4)

1.3 Use integration by parts to prove the reduction formula: $\int \sin^n x\, dx = -\frac{1}{n}\cos x \sin^{n-1} x + \frac{n-1}{n}\int \sin^{n-2} x\, dx$ for $n \geq 2, n \in \mathbb{Z}$. (5)

1.4 Prove the formula, where m and n are positive integers. $\int_{-\pi}^{\pi} \sin mx \cos nx\, dx = 0$ (4)

Question Two [25 marks]

2.1 Discuss the convergence of $I = \int_{-\infty}^{0} xe^x\, dx$ (4)

2.2 Find f_{xxyz} if $f(x, y, z) = \sin(2x + 3yz)$ (4)

2.3 Use the Chain Rule to find $\frac{\partial z}{\partial t}$ if $z = x\arctan(xy)$, $x = t^2$, $y = se^{-t}$ (5)

2.4 Solve the following differential equations

(a) $(x^2 + y^2)\, dy + y^2\, dx = 0$ (4)

(b) $3x^2 + 2xy + 3y^2 + (x^2 + 6xy)y' = 0$ and $y(1) = 2$ (4)

2.5 A body moves in a straight line with a constant acceleration of $a = 10\ m/sec^2$. If the speed $v = 5\ m/sec$ when $t = 0$, show that $v = 10t + 5$. If s is the distance the body moved after t sec, find s as a function of t if $s = 0$ when $t = 0$. (4)

Question Three [25 marks]

3.1 Show that the Differential Equation is not exact but becomes exact when multiplied by the integrating factor μ. Then solve the Differential equation: $y^2 + (1 + xy)y' = 0$ with $\mu = e^{xy}$
(5)

3.2 Find the unit vector in the direction of vector $\langle -2,4,3 \rangle$ (3)

3.3 Plot the triangle whose vertices are $P(1,-1,0)$, $Q(2,1,-1)$, $R(-1,1,2)$ and find the area of the triangle ΔPQR. (6)

3.4 Derive (a) Parametric equations and (b) Symmetric equations of the line parallel to the vector $< a, b, c >$ and passing through the point $P_0(x_0, y_0, z_0)$. (6)

3.5 Derive the equation of the plane that passes through $P_0(x_0\,, y_0\,, z_0)$ and is perpendicular to the vector $\overline{N} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ (5)

Question Four [25 marks]

4.1 Find the cross product $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ if $\vec{A} = \langle 1,2,-3 \rangle$, $\vec{B} = \langle 5,-1,-2 \rangle$ (4)

4.2 (a) Find the parametric and symmetric equations of l passing through $P(-3,2,-3)$ and $Q(1,-1,4)$

(b) At what point does the line intersect the xy-plane? (6)

4.3 (a) Let A be the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & -1 \end{bmatrix}$ (i) Show that A is invertible (non-singular) (1)

(ii) Find A^{-1} using Row Reduction. (6)

4.4 Solve the system of equations using Cramer's Rule:

$2x + y - z = 2$

$x - y + z = 7$

$2x + 2y + z = 4$ (4)

4.5 Find $Arg\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$ and express $\frac{1}{2} + i\frac{\sqrt{3}}{2}$ in polar form (4)