

UNIVERSITY OF FORT HARE



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**STATISTICAL MECHANICS
PHY504**

HONOURS EXAMINATION

OCT / NOV

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Time: 3 Hours

Subject: PHY 504

Marks: 100

This paper consists of 3 pages including the cover page

Internal Examiners
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External Examiners
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INSTRUCTIONS

Answer all the four question, each question carries 25 marks

QUESTION 1 [25 MARKS]

- 1.1 State the second law of thermodynamics and interpret its significance (3)
- 1.2 Assuming that the internal energy depends only on entropy and volume, show that:
- 1.2.1 $T = \left(\frac{\partial U}{\partial S}\right)_V$ (3)
- 1.2.2 $P = -\left(\frac{\partial U}{\partial V}\right)_S$ (3)
- 1.2.3 Hence deduce that: $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$ (4)
- 1.3 Give the expressions for the four thermodynamic potentials in terms of internal energy, pressure, volume, entropy, temperature. (4)
- 1.4 Consider an isothermal process, show that the Gibbs free energy (G) can be expressed in the form: $G = F - V\left(\frac{\partial F}{\partial V}\right)_T$ (4)
- 1.5 In isochoric process, show that entropy is a measure of the rate of change of Helmholtz free energy with temperature at constant volume, that is: $S = -\left(\frac{\partial F}{\partial T}\right)_V$ (4)

QUESTION 2 [25 MARKS]

- 2.1 Name and describe the three main types of ensembles used in statistical mechanics. (6)
- 2.2 Show that if an ensemble is canonical then the probability P_r of finding the system A in any one particular microstate r of energy E_r is given as: $P_r = \frac{e^{-\beta E_r}}{\sum_r e^{-\beta E_r}}$ (5)
- 2.3 Statistical physics measures mean values of physical quantities, consider a physical quantity F, and find the relative fluctuation of F (5)
- 2.4 A monatomic ideal gas with N molecules is contained in a volume V. The partition function is given as: $Z = \frac{V^N}{h_0^{3N}} \left(\frac{2\pi m}{\beta}\right)^{\frac{3N}{2}}$; Show that the mean energy per molecule is given as: $\langle \epsilon \rangle = \frac{3}{2} kT$ (5)
- 2.5 An external parameter undergoes a quasi-static change from x to $x + \partial x$, show that the corresponding work done is given as: $dW = \frac{1}{\beta} \left(\frac{\partial \ln Z}{\partial x}\right) dx$. (4)

QUESTION 3 [25 MARKS]

3.1 State the *equipartition theorem* of classical physics (3)

3.2 Consider a 3-dimensional harmonic oscillator in equilibrium with a heat reservoir at temperature T and whose energy is given as: $E = \frac{\alpha r^2}{2m} + \frac{\beta \theta^2}{2m} + \frac{\gamma \phi^2}{2m}$ where $r, \theta, \text{ and } \phi$ are the spherical coordinates of the oscillator, use the equipartition theorem to find the mean energy of a harmonic oscillator. (4)

3.3 The energy levels of a harmonic oscillator are equally spaced and satisfy

$$E_n = \left(n + \frac{1}{2}\right) \hbar\omega$$

3.3.1 Write the equipartition function corresponding to this harmonic oscillator (6)

3.3.2 Using the equipartition function in (3.3.1), show that the mean energy is given as: $\bar{E} = \hbar\omega \left[\frac{1}{2} + \frac{1}{\exp(\beta\hbar\omega) - 1} \right]$ (12)

QUESTION 4 [25 MARKS]

4.1 State the properties of:

4.1.1 Maxwell-Boltzmann distribution (2)

4.1.2 Bose-Einstein statistics (2)

4.1.3 Fermi-Dirac statistics (2)

4.2 Show that the mean number of particles for Bose-Einstein statistics is given by:

$$\langle n_s \rangle = \frac{1}{e^{\alpha + \beta \epsilon_s} - 1} \quad (6)$$

4.3 In canonical distribution the energy of the system is seen to change by ϵ_0 such that the energy of each state r is $E_r^* = E_r + \epsilon_0$.

4.3.1 Find the change in the mean energy (3)

4.3.2 Show that the entropy does not change (3)

4.4 Consider a system consisting of N non-interacting atoms in a substance at temperature T and placed in an external magnetic field \vec{H} pointing along the Z-direction. Show that the probability P_m that an atom is in a state labelled m is given as $P_m = C e^{\beta g \mu H m}$. (7)

