UNIVERSITY OF FORT HARE

STS 502

HONOURS DEGREE EXAMINATION

JUNE 2023

Time: 3 HOURS

Subject: APPLIED STATISTICS

Paper: APPLIED TIME SERIES ANALYSIS

Marks: 100

This paper consists of 6 pages including cover page

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INSTRUCTIONS

1. Answer **ALL** Questions from **SECTION A** and **THREE** Questions from **SECTION B.**
2. Calculators may be used.
3. Statistical tables are provided.

SECTION A [52 Marks]

Answer ALL Questions from this section

a) Explain clearly each of the following terms as used in time series analysis:

i) Weakly stationary time series

ii) Random walk

iii) Purely random process

iv) Stochastic process

v) Autocorrelation function [5 marks]

b) Let $X_t = \beta_0 Z_t + \beta_1 Z_{t-1} + \beta_2 Z_{t-2} + \ . \ . \ . \ + \beta_q Z_{t-q}$ be a moving average process of order q and $\{Z_t\}$ a purely random process with mean zero and variance σ_z^2.

Determine : i) the mean of X_t. [2 marks]

ii) the variance of X_t. [4 marks]

iii) the autocovariance function of X_t. [4 marks]

c) Given that $X_1, X_2, \ . \ . \ . \ , X_n$ are observed data from a time series process and that $\hat{X}(n, \ k)$ is the forecast made at time n with lead time k for that process.

If $\hat{X}(n, \ 1) = \lambda_0 X_n + \lambda_1 X_{n-1} + \lambda_2 X_{n-2} + \ . \ . \ .$

where $\lambda_i = \beta_i (1-\beta)^i \qquad i = 0, \ 1, \ 2, \ 3, \ . \ . \ .$

$0 < \beta < 1$

Show that $\hat{X}(n, \ 1)$ gives an infinite number of past observations for the process. [4 marks]

d) A stationary stochastic time series model $X(t)$ has a constant mean μ, variance σ^2, acv.f. $\gamma(\lambda)$, and ac.f. $\rho(\lambda)$. Show that :

i) its ac.f. is an even function. [4 marks]

ii) $|\rho(\lambda)| \leq 1$. [6 marks]

e) An autoregressive process of order one is represented by the model

$$X_t = \alpha X_{t-1} + Z_t$$

Show that the *even* form of its autocorrelation is given by

$$\rho(k) = \alpha^{|k|} \qquad k = 0,\ \pm 1,\ \pm 2,\ . \ . \ .$$

[6 marks]

f) The two expressions below both describe a time series model.

$$A: \quad X_t = Z_t + \alpha Z_{t-1}$$

$$B: \quad X_t = Z_t + \frac{1}{\alpha} Z_{t-1}$$

i) Identify the model described by the two expressions above. [2 marks]

ii) Explain the difference between the two models in relation to your answer in part (i) above. [5 marks]

g) The Box-Jenkins forecast model using the ψ weights is given by

$$X_{n+k} = Z_{n+k} + \psi_1 Z_{n+k-1} + \psi_2 Z_{n+k-2} + \ . \ . \ .$$

Use the equation to determine the forecast error variance for the model. [5 marks]

h) A SARIMA model $(1, 0, 0)(0, 1, 1)_{12}$ is given by the expression:

$$X_t = X_{t-12} + \alpha(X_{t-1} + X_{t-13}) + Z_t + \theta Z_{t-12}$$

If X_{n+1} is known, what would be the forecast for $\hat{X}(n+1, 1)$?

[5 marks]

SECTION B [52 Marks]

Answer THREE Questions from this section

QUESTION TWO [16 Marks]

a) Define the following processes:

i) A moving average process of order q. [1 mark]

ii) Autoregressive process of order p. [1 mark]

iii) Use one of the processes mentioned above to derive the Yule-Walker equations. [6 marks]

b) An observed autoregressive time series process is given by the equation

$$X_t = X_{t-1} - \tfrac{1}{5}X_{t-2} + Z_t$$

i) Is this process stationary? [4 marks]

ii) What is its autocorrelation function? [4 marks]

QUESTION THREE [16 Marks]

a) If π_1 and π_2 are roots of an observed autoregressive time series equation of order 2, AR(2) given by

$$y^2 - \alpha_1 y - \alpha_2 = 0$$

i) Show the solution for the roots of the equation if $|\pi_i| < 1$ [4 marks]

ii) Sketch stationary region satisfied by the solution in part (i) above. [5 marks]

b) A general solution of an AR process of order P is given by:

$$\rho\,(k) = A_1\pi_1^{|k|} + A_2\pi_2^{|k|} + A_3\pi_3^{|k|} + \;.\;\;.\;\;.\; + A_p\pi_p^{|k|}$$

Consider the constants A_1 and A_2 of its real quadratic roots given by the equation in part i (a) above (i. e. $y^2 - \alpha_1 y - \alpha_2 = 0$).

Express the values of A_1 and A_2 in terms of α_1 , α_2, π_1 and π_2.
[*Hint*: solve for $k = 0$ and $k = 1$] [7 marks]

QUESTION FOUR [16 Marks]

a) i) A mixed ARMA model time series process is given by the equation below :

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \;.\;\;.\;\;.\; + \alpha_p X_{t-p} + Z_t + \beta_1 Z_{t-1} + \beta_2 Z_{t-2} + \;.\;\;.\;\;.\; + \beta_q Z_{t-q}$$

Use the backward shift operator B to express the equation above in terms of $\phi(B)$ and $\theta(B)$ which are polynomials of order p and q respectively. [4 marks] ii) Hence express the ARMA model as separate pure MA and pure AR processes. [4 marks]

b) Determine the ψ weights and the π weights of the ARMA $(1, \quad 1)$ process given by:

$$X_t = 0.6X_{t-1} + Z_t - 0.4Z_{t-1}$$ [8 marks]

QUESTION FIVE [16 Marks]

a) State the Wold decomposition theorem and how it is applied to time series. [5 marks]

b) Consider a mixed stochastic model of the form

$$X_t = \cos(\omega_0 t + \theta) + Z_t$$

where ω_0 is a constant in $(0, \quad \pi)$ and θ is a random variable uniformly distributed on $(0, \quad 2\pi)$, and $\{Z_t\}$ a purely random process with mean zero and variance σ_z^2.

i) What is its acv.f ? [4 marks]

ii) What is its overall spectral distribution function? [7 marks]

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