UNIVERSITY OF FORT HARE

WAVES VIBRATIONS AND OPTICS PHY 122

DEGREE EXAMINATIONS

NOVEMBER

2024

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Time: 3 HOURS

Subject: PHY 122

Marks: 100

This paper consists of 10_ pages including the cover page and formular sheets

Internal Examiners

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Internal Moderator

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INSTRUCTIONS

1. Answer all questions
2. NB. *FORMULAR SHEET* is attached at the back of the question paper.

QUESTION ONE

Instruction: write down **ONLY THE LETTER CORRESPONDING TO THE ANSWER OF YOUR CHOICE** below

1. The differential equation of motion of a simple harmonic motion is:

A. $\frac{d^2x}{dt^2} + \omega t = 0$ **B.** $\frac{d^2x}{dt^2} = -\omega x$ **C.** $\frac{d^2t}{dx^2} + \omega^2 x = 0$, **D.** $\frac{d^2t}{dx^2} = -\omega^2 x$ [1]

2. In simple harmonic motion, the particle is

A. always accelerating **B.** always retarding **C.** alternately accelerating and retarding **D.** steady state *(does not change).* [1]

3. ***Statement:*** Simple harmonic motion is the projection of uniform circular motion on the diameter of the circle in which the latter motion occurs.
Reason: Simple harmonic motion is a uniform motion.

A. Both statement and reason are true, and reason is the correct explanation of statement.

B. Both statement and reason are true, but reason is not the correct explanation of statement.

C. Statement is true but reason is false.

D. Both assertion and reason are false. [1]

4. Which one of the following equations of motion represents simple harmonic motion?

A. Acceleration = F\m

B. Acceleration = k(x+a)

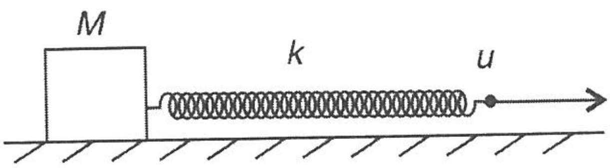
C. Acceleration = −k(x+a)

D. Acceleration = kx [1]

5. The equation of motion of a particle of mass 1g is $\frac{d^2x}{dt^2} + \pi^2 x = 0$, where x is displacement (in m) from mean position. The frequency of oscillation is (in Hz):

A. (1/5)√10 **B.** 2 **C.** 5√10 **D.** ½ [1]

6. A block of mass M is placed on a horizontal smooth table. It is attached to an ideal spring of force constant k as shown. The free end of the spring is pulled at a constant speed u. Find the maximum extension (x0) in the spring during the subsequent motion. [2]



A. $x_o = \sqrt{\frac{2M}{3k}}u$ **B.** $x_o = \sqrt{\frac{k}{2M}}\,u$ **C.** $x_o = \sqrt{\frac{M}{k}}\,u$ **D.** $x = \sqrt{\frac{2M}{k}}\,u$

7. A sinusoidal wave $f_1(t)$ = 3 sin ω t is superposed on another periodic wave $f_2(t)$ which is also oscillating with the same frequency. If the resultant wave is a sinusoidal wave, which is out of phase with $f_1(t)$ by 53.1° ; then the wave $f_2(t)$ would be-

Hint: tan 53.1° = 1.33

A. f_2(t) = 3 sin(ωt) **B.** f_2(t) = 4 sin(ωt + 53.1°) **C.** f_2(t) = 3 cos(ωt – 53.1°) **D.** f_2(t) = 4cos(ωt) [2]

8. Two waves represented by $y_1 = a \sin\left(\omega t + \frac{\pi}{6}\right), y_2 = a \cos \omega t$, the resultant amplitude will be:

A. a **B.** a$\sqrt{3}$ **C.** a$\sqrt{2}$ **D.** 2a [2]

9. If f (t) = A sin ωt + B cos ωt is written as f (t) = D sin (ωt + φ) then 'φ' is equal to?

A. $\tan^{-1}$(A/B **B.** √($A^2 + B^2$) **C.** $\tan^{-1}$(B/A) **D.** A + B [2]

10. Two waves represented as x = a sin(ωt + π/6), x = a cos ωt, then resultant phase difference between them is:

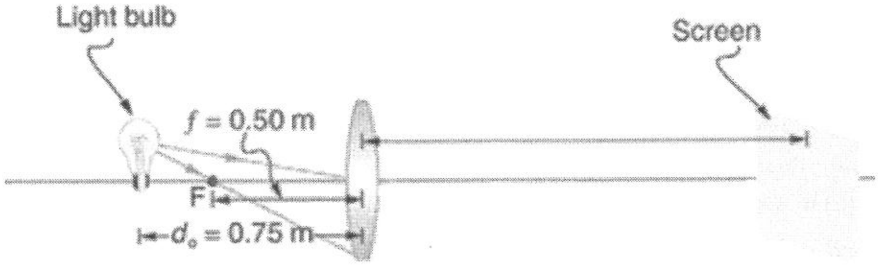
A. π/6 **B.** π C. 2π/3 **D.** π/3 [2]

11. The first diffraction minima due to a single slit is observed at 30° for light of wavelength 6500 A. The width of the slit is:

A. 3250 A^0 **B.** 6.5×10^{-4} mm **C.** 2.6×10^{-4} cm **D.** 1.3x 10^{-6} [1]

12. A clear glass light bulb is placed 0.750 m from a convex lens having a 0.500 m focal length, as shown in the Figure. Use ray tracing method to get an approximate location for the image. Then use the thin lens equations to calculate:

(a) the location of the image and (b) its magnification.



A. (a) d_i = 0.75 m, (b) m = + 1 **B.** (a) d_i = 1.5 m, (b) m = -2.

C. (a) d_i = -0.75 m, (b) m = -1. **D.** (a) d_i = -1.5 m, (b) m = +2. [2]

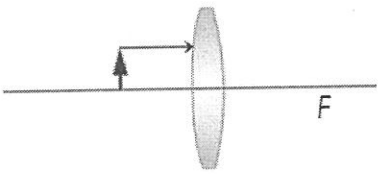
13. Suppose the book page in the Figure below. (a) is held 7.50 cm from a convex lens of focal length 10.0 cm, such as a typical magnifying glass might have. What magnification is produced?

A. (a) d_i = -30 cm, (b) m = 3, (d) image is virtual and upright.

B. (a) d_i = -30 cm, (b) m = 3, (c) image is real and upright.

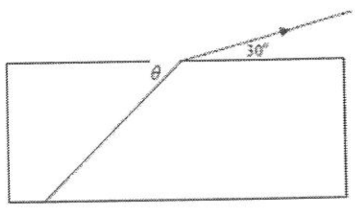
C. (a) d_i = 30 cm, (b) m = -3, (c) image is real and inverted.

D. (a) d_i = 30 cm, (b) m = 3, (d) image is virtual and upright.



[2]

14. The diagram shows a ray of light as it leaves a rectangular block of glass. As the ray of light leaves the block of glass, it makes an angle θ with the inside surface of the glass block and an angle of 30° when it is in the air, as shown in the diagram.



(i) If the refractive index of the glass is 1.5, calculate the value of θ.
(ii) What would be the value of the angle θ so that the ray of light emerges parallel to the side of the glass block?

A. (i) θ = 54.^{0}74, (ii) θ = 48.^{0}2 **B.** (i) θ = 35.^{0}26, (ii) θ = 41^0.82

C. θ = 41.^{0}82, (ii) θ = 35^0.26 **D.** θ = 48.^{0}2, (ii) θ = 54^0.74 [2]

15. Suppose you have a concave lens of focal length f = 23:0 cm. An object of height 5.0 cm is placed 33.0 cm from the lens. Using the algebraic method, determine: (a) the image distance; (b) the image height; (c) the image magnification; (d) whether the image is real or virtual; and (e) whether the image is upright or inverted.

A. (a) d_i = -13.5 cm, (b) h_i = +2.05cm, (c) m = +0.4, (d) image is virtual, (e) d_i is upright.

B. (a) d_i = -13.55 cm, (b) h_i = +2.05cm, (c) m = +0.4, (d) image is real, (e) d_i is upright.

C. (a) d_i = 13.55 cm, (b) h_i = -2.05cm, (c) m = -0.5, (d) image is virtual, (e) d_i is inverted.

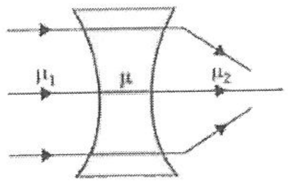
D. (a) d_i = -13.55 cm, (b) h_i = -2.05cm, (c) m = +0.5, (d) image is real, (e) d_i is inverted. [2]

16. The critical angle (for total internal reflection) for a certain liquid-air surface is 47^0.7. What is the index of refraction of the liquid?

A. 1.551 **C.** 1.351 **B.** 1.451 **D.** 1.251 [2]

17. The behaviour of light rays is as shown in the figure below. The relation between refractive indices μ, μ_1 and μ_2 is:

A $\mu>\mu_2>\mu_1$ **B** $\mu<\mu_2<\mu_1$ **C.** $\mu<\mu_2, \mu=\mu_1$ **D** $\mu_2<\mu_1, \mu=\mu_2$

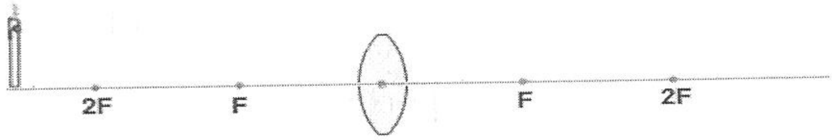


[1]

18. A boy of height 1m stands in front of a convex mirror. His distance from the mirror is equal to its focal length. The height of his image is

A. 0.25m **B.** 0.33m **C.** 0.5m **D.** 0.67m [1]

19. An object is placed in front of a converging lens as shown in the figure at a distance greater than 2F. The image produced by the lens is:



A. Real, inverted and magnified

B. Virtual, upright and magnified

C. Virtual, upright and demagnified

D. Real, inverted and demagnified

E. Virtual, inverted and magnified [2]

TOTAL [30]

QUESTION TWO

2.1 Show that the Total energy of Simple Harmonic Motion (SHM) is given by $E_{tot} = \frac{1}{2}kA^2$ where k = spring constant and A is the amplitude *(maximum displacement of the oscillation)* of the waves. [6]

2.2 A mass m is dropped from a height **h** on to a scale-pan of negligible weight, suspended from a spring of spring constant **k**. The collision maybe considered to be completely inelastic in that the mass sticks to the pan and the pan begins to oscillate. Show that the amplitude of the pan's oscillations as measured from the equilibrium position is given by:

$$\sqrt{\frac{m^2g^2}{k^2} + \frac{2mgh}{k}}$$

trig function

. *Let y_0 be the extension of the spring.* [6]

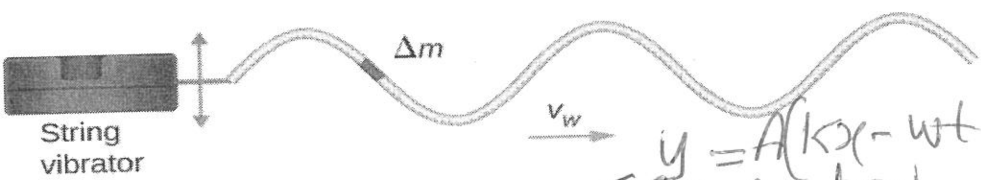
2. 3 Assume that we have a string with a velocity given by v = **λ**T, where T is the Period. Derive and show that the Power (rate of energy transfer) transported by a one-dimensional wave along on a string is given by:

$$\left[\frac{dK}{dt}\right]_{avg} = \frac{1}{2}\mu v\omega^2 A^2 \qquad \textit{assume that } \sin^2(kx - \omega t) \approx \frac{1}{2}$$

Where μ is the linear density of the string, A, the maximum displacement of an oscillator and E_λ is the stored energy in the wave over wavelength **λ**. [6]

2.4 Consider a two-meter-long string with a mass of 70.00 g attached to a string vibrator as illustrated in the figure below. The tension in the string is 90.0 N. When the string vibrator is turned on, it oscillates with a frequency of 60 Hz and produces a sinusoidal wave on the string with an amplitude of 4.00 cm and a constant wave speed.

What is the time-averaged power supplied to the wave by the string vibrator?



$y = A(kx - wt + \phi)$

Trig function [3]

2.5 The wave function for a wave on a taut string is y = 0.35 (10πx − 3πt + π/4), where x and y are in meters and t is in seconds. If the linear mass density of the string is 75.0 g/m,

(a) what is the average rate at which energy is transmitted along the string? [3]

(b) What is the energy contained in each cycle of the wave? [3]

2.6. Consider a sinusoidal wave that the following parameters: f = 120 Hz, ym = 8.5 mm, μ = 525 g/m and τ(Tension) = 45 N. At which average rate does the wave transport energy? [3]

Total Marks [30]

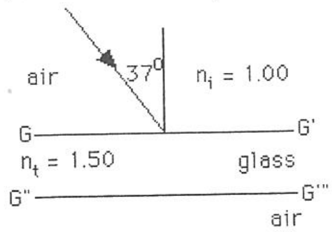
QUESTION THREE

3.1 A light beam is incident upon a parallel glass plate, as shown in Figure below. Find the angle of:

(a) refraction at the first glass surface,

(b) incidence at the lower glass-air surface, (c) refraction as the ray goes from glass to air. Use these angles to trace the path of the ray from air to glass and then

(c) from glass to air.

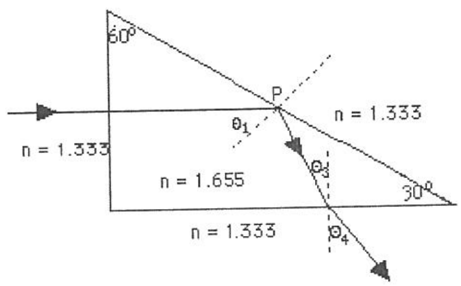


[5 Marks]

3.2 As shown in Figure below, a light ray is incident normally on one face of a 30°-60°-90° dense flint (n = 1.655) prism that is immersed in water (n = 1.333).

(a) Determine the exit angle θ_4 of the ray.

(b) A substance is dissolved in the water to increase the index of refraction. At what value of n of the mixture will total internal reflection cease at point P?



[5 Marks]

3.3. A thin converging lens has a focal length of 10 cm. Find by (i) calculation and (ii) construction of a ray diagram the position of the image of an object for the object distance s_o equal to (a) 30 cm, (b) 15 cm, and (c) 5.0 cm. Also find the magnification for each case. Describe whether the image is real or imaginary, erect or inverted, magnified or diminished.

[5 Marks]

3.4. A thin diverging lens has a focal length of 10 cm. Find by (i) calculation and (ii) construction of a ray diagram the position of the image of an object for object distance equal to (a) 30 cm and (b) 4 cm. Also find the magnification for each case. Describe whether the image is real or imaginary, erect or inverted, magnified or diminished in size.

[5 Marks]

[Total marks 20]

QUESTION FOUR

4.1 State at least 3 assumptions for the derivation of a one-dimensional wave equation. (3)

4.2 Show that $y(x,t) = \cos(4x + 4ct)$ satisfies the one-dimensional wave equation (6)

4.3 Show that the function $y(x,t) = X^2 + V^2t^2$ is a solution of the general wave equation. (6)

4.4 Show that the one-dimensional wave equation is satisfied by the following function.

$y = A\sqrt{(x + vt)}$. That is, show that the equation satisfies both the left and right side of the one-dimensional equation. (5)

Total marks [20]

Formulae Sheet - Waves Vibrations and Optics

1 Waves Motion

General equation of wave: $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$.

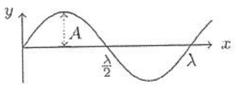
Notation: Amplitude A, Frequency ν, Wavelength λ, Period T, Angular Frequency ω, Wave Number k,

$$T = \frac{1}{\nu} = \frac{2\pi}{\omega}, \quad v = \nu\lambda, \quad k = \frac{2\pi}{\lambda}$$

Progressive wave travelling with speed v:

$$y = f(t - x/v), \rightsquigarrow +x; \quad y = f(t + x/v), \rightsquigarrow -x$$

Progressive sine wave:



$$y = A\sin(kx - \omega t) = A\sin(2\pi\,(x/\lambda - t/T))$$

2 Waves on a String

Speed of waves on a string with mass per unit length μ and tension T: $v = \sqrt{T/\mu}$

Transmitted power: $P_{\text{av}} = 2\pi^2\mu v A^2\nu^2$

Interference:

$$y_1 = A_1\sin(kx - \omega t), \quad y_2 = A_2\sin(kx - \omega t + \delta)$$

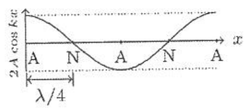
$$\boxed{y = y_1 + y_2 = A\sin(kx - \omega t + \epsilon)}$$

$$A = \sqrt{{A_1}^2 + {A_2}^2 + 2A_1A_2\cos\delta}$$

$$\tan\epsilon = \frac{A_2\sin\delta}{A_1 + A_2\cos\delta}$$

$$\delta = \begin{cases} 2n\pi, & \text{constructive;} \\ (2n+1)\pi, & \text{destructive.} \end{cases}$$

Standing Waves:

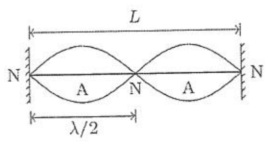


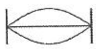
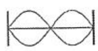
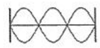
$$y_1 = A_1\sin(kx - \omega t), \quad y_2 = A_2\sin(kx + \omega t)$$

$$\boxed{y = y_1 + y_2 = (2A\cos kx)\sin\omega t}$$

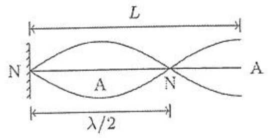
$$x = \begin{cases} \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, & \text{nodes;} \quad n = 0, 1, 2, \ldots \\ n\frac{\lambda}{2}, & \text{antinodes.} \quad n = 0, 1, 2, \ldots \end{cases}$$

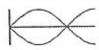
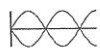
String fixed at both ends:



1. Boundary conditions: $y = 0$ at $x = 0$ and at $x = L$
2. Allowed Freq.: $L = n\frac{\lambda}{2}$, $\nu = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$, $n = 1, 2, 3, \ldots$.
3. Fundamental/1st harmonics: $\nu_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ 
4. 1st overtone/2nd harmonics: $\nu_1 = \frac{2}{2L}\sqrt{\frac{T}{\mu}}$ 
5. 2nd overtone/3rd harmonics: $\nu_2 = \frac{3}{2L}\sqrt{\frac{T}{\mu}}$ 
6. All harmonics are present.

String fixed at one end:



1. Boundary conditions: $y = 0$ at $x = 0$
2. Allowed Freq.: $L = (2n+1)\frac{\lambda}{4}$, $\nu = \frac{2n+1}{4L}\sqrt{\frac{T}{\mu}}$, $n = 0, 1, 2, \ldots$.
3. Fundamental/1st harmonics: $\nu_0 = \frac{1}{4L}\sqrt{\frac{T}{\mu}}$ 
4. 1st overtone/3rd harmonics: $\nu_1 = \frac{3}{4L}\sqrt{\frac{T}{\mu}}$ 
5. 2nd overtone/5th harmonics: $\nu_2 = \frac{5}{4L}\sqrt{\frac{T}{\mu}}$ 
6. Only odd harmonics are present.

Sonometer: $\nu \propto \frac{1}{L}$, $\nu \propto \sqrt{T}$, $\nu \propto \frac{1}{\sqrt{\mu}}$. $\nu = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

3 Sound Waves

Displacement wave: $s = s_0\sin\omega(t - x/v)$

Pressure wave: $p = p_0\cos\omega(t - x/v)$, $p_0 = (B\omega/v)s_0$

Speed of sound waves:

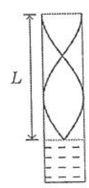
$$v_{\text{liquid}} = \sqrt{\frac{B}{\rho}}, \quad v_{\text{solid}} = \sqrt{\frac{Y}{\rho}}, \quad v_{\text{gas}} = \sqrt{\frac{\gamma P}{\rho}}$$

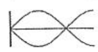
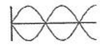
Intensity: $I = \frac{2\pi^2 B}{v}{s_0}^2\nu^2 = \frac{{p_0}^2 v}{2B} = \frac{{p_0}^2}{2\rho v}$

Standing longitudinal waves:

$$p_1 = p_0\sin\omega(t - x/v), \quad p_2 = p_0\sin\omega(t + x/v)$$
$$p = p_1 + p_2 = 2p_0\cos kx\sin\omega t$$

Closed organ pipe:

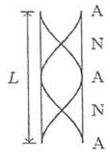


1. Boundary condition: $y = 0$ at $x = 0$
2. Allowed freq.: $L = (2n+1)\frac{\lambda}{4}$, $\nu = (2n+1)\frac{v}{4L}$, $n = 0, 1, 2, \ldots$
3. Fundamental/1st harmonics: $\nu_0 = \frac{v}{4L}$ 
4. 1st overtone/3rd harmonics: $\nu_1 = 3\nu_0 = \frac{3v}{4L}$ 
5. 2nd overtone/5th harmonics: $\nu_2 = 5\nu_0 = \frac{5v}{4L}$ 

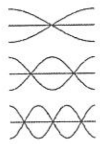
Formulae Sheet - Waves Vibrations and Optics

6. Only odd harmonics are present.

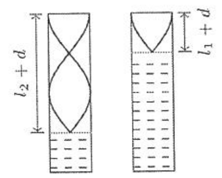
Open organ pipe:



1. Boundary condition: $y = 0$ at $x = 0$
   Allowed freq.: $L = n\frac{\lambda}{2}$, $\nu = n\frac{v}{4L}$, $n = 1, 2, \ldots$
2. Fundamental/1st harmonics: $\nu_0 = \frac{v}{2L}$
3. 1st overtone/2nd harmonics: $\nu_1 = 2\nu_0 = \frac{2v}{2L}$
4. 2nd overtone/3rd harmonics: $\nu_2 = 3\nu_0 = \frac{3v}{2L}$
5. All harmonics are present.



Resonance column:



$$l_1 + d = \frac{\lambda}{2}, \quad l_2 + d = \frac{3\lambda}{4}, \quad v = 2(l_2 - l_1)\nu$$

Beats: two waves of almost equal frequencies $\omega_1 \approx \omega_2$

$$p_1 = p_0 \sin\omega_1(t - x/v), \quad p_2 = p_0 \sin\omega_2(t - x/v)$$
$$p = p_1 + p_2 = 2p_0 \cos\Delta\omega(t - x/v)\sin\omega(t - x/v)$$
$$\omega = (\omega_1 + \omega_2)/2, \quad \Delta\omega = \omega_1 - \omega_2 \quad \text{(beats freq.)}$$

Doppler Effect:

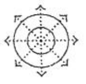
$$\nu = \frac{v + u_o}{v - u_s}\nu_0$$

where, v is the speed of sound in the medium, u_0 is the speed of the observer w.r.t. the medium, considered positive when it moves towards the source and negative when it moves away from the source, and u_s is the speed of the source w.r.t. the medium, considered positive when it moves towards the observer and negative when it moves away from the observer.

4 Light Waves

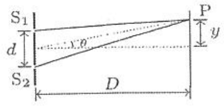
Plane Wave: $E = E_0 \sin\omega(t - \frac{x}{v})$, $I = I_0$

Spherical Wave: $E = \frac{aE_0}{r}\sin\omega(t - \frac{r}{v})$, $I = \frac{I_0}{r^2}$



Young's double slit experiment

Path difference: $\Delta x = \frac{dy}{D}$



Phase difference: $\delta = \frac{2\pi}{\lambda}\Delta x$

Interference Conditions: for integer n,

$$\delta = \begin{cases} 2n\pi, & \text{constructive;} \\ (2n+1)\pi, & \text{destructive,} \end{cases}$$

$$\Delta x = \begin{cases} n\lambda, & \text{constructive;} \\ \left(n + \frac{1}{2}\right)\lambda, & \text{destructive} \end{cases}$$

Intensity:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta,$$
$$I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2, \; I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$
$$I_1 = I_2 : I = 4I_0\cos^2\frac{\delta}{2}, \; I_{\max} = 4I_0, \; I_{\min} = 0$$

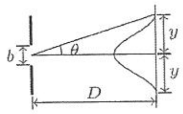
Fringe width: $w = \frac{\lambda D}{d}$

Optical path: $\Delta x' = \mu\Delta x$

Interference of waves transmitted through thin film:

$$\Delta x = 2\mu d = \begin{cases} n\lambda, & \text{constructive;} \\ \left(n + \frac{1}{2}\right)\lambda, & \text{destructive.} \end{cases}$$

Diffraction from a single slit:



For Minima: $n\lambda = b\sin\theta \approx b(y/D)$

Resolution: $\sin\theta = \frac{1.22\lambda}{b}$

Law of Malus: $I = I_0\cos^2\theta$