UNIVERSITY OF FORT HARE

University of Fort Hare
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PHY 502

HONOURS EXAMINATION

DATE	:	JUNE/JULY [redacted] 2023
TIME	:	3 HOURS
SUBJECT	:	PHYSICS 502 Electrodynamics
MARKS	:	100

INTERNAL EXAMINERs: Prof G Makaka

EXTERNAL EXAMINER: Dr D Tinarwo

INSTRUCTIONS: Answer all the four question, each question carries 25 marks

1. (a) Derive the general inhomogeneous wave equations for the scalar and vector potentials in electrodynamics. [13]

(b) Then, specialise to the Lorentz condition and write down the wave equations for this particular condition. [7]

(c) Finally, gauge transform the Lorentz equations just derived. [5]

2. (a) State what is giving rise to the current density $\vec{j}$, which is given at any point $\vec{r}\,'$ and time t, in the usual notation, by $\vec{j}(\vec{r}\,',t) = q\dot{\vec{r}}(t)\delta(\vec{r}\,'-\vec{r}(t))$. [4]

(b) Write the expression for the four-current density j^{μ} in terms of charge density ρ and current density $\vec{j}$. [4]

(c) Write the charge conservation equation in covariant form. [4]

(d) In a certain region of space, the electric scalar potential and the magnetic vector potential are given as: $\Phi(\vec{r},t) = \frac{\Phi_0 e^{i(\omega t-kr)}}{kr}$ and $A(\vec{r},t) = \frac{\vec{A}_0 e^{i(\omega t-kr)}}{kr}$, where Φ_0 and $\vec{A}_0$ are both constant and $r = |\vec{r}|$. Find corresponding expressions for the $\vec{E}$ and $\vec{B}$ fields. [13]

3. (a) (i) Show that the tensor scalar product of the electromagnet field tensor is:

$$F_{\alpha\beta}F^{\beta\alpha} = 2\left(\frac{E^2}{c^2} - B^2\right), \qquad [5]$$

(ii) Hence show that $\tilde{F}_{\alpha\beta}\tilde{F}^{\beta\alpha} = -F_{\alpha\beta}F^{\beta\alpha}$ where $\tilde{F}_{\alpha\beta}$ is the dual of $F_{\alpha\beta}$. [5]

(b) Show That Maxwell's homogeneous equations can be written in the form:

$$\partial^{\alpha}F^{\beta\gamma} + \partial^{\beta}F^{\gamma\alpha} + \partial^{\gamma}F^{\alpha\beta} = 0 \qquad [7]$$

(c) Show that $\partial_{\alpha}\tilde{F}^{\alpha\beta} = \begin{bmatrix} c\nabla\cdot\vec{B} \\ -(\nabla\times\vec{E})_x - \partial_t B_x \\ -(\nabla\times\vec{E})_y - \partial_t B_y \\ -(\nabla\times\vec{E})_z - \partial_t B_z \end{bmatrix}$ [8]

4. (a) The poynting's theorem can be written in the form $\frac{dW}{dt} = -\frac{d}{dt}\int_V U_{em} d\tau - \int_{surface} \vec{s}\cdot d\vec{a}$, show that it can be expressed in the form: $\nabla\cdot\vec{S} = -\partial_t U$, where $U = U_{mech} + U_{em}$ [7]

(b) Starting from the Lorentz force law, show that the Lorentz force density $\vec{f}$ can be expressed in the form: $f_k = \partial_j \vec{T}_{jk} - \varepsilon_0\mu_0 \frac{\partial \vec{S}_k}{\partial t}$, where;

$$\partial_j \vec{T}_{jk} = \varepsilon_0 \left\{ \left(\nabla\cdot\vec{E}\right)\vec{E} + \left(\vec{E}\cdot\nabla\right)\vec{E} - \frac{1}{2}\nabla E^2 \right\}_k + \frac{1}{\mu_0}\left\{ \left(\nabla\cdot\vec{B}\right)\vec{B} + \left(\vec{B}\cdot\nabla\right)\vec{B} - \frac{1}{2}\nabla B^2 \right\}_k$$

[10]

(c) Σ and Σ' are space-time inertial coordinate frames such that at time $t = t' = 0$, the origins coincide. The electromagnetic field tensor $F^{\mu\nu}$ has the following components in Σ:

$$F^{\mu\nu} = \begin{bmatrix} 0 & \frac{-E_x}{c} & \frac{-E_y}{c} & \frac{-E_z}{c} \\ \frac{E_x}{c} & 0 & -B_z & B_y \\ \frac{E_y}{c} & B_z & 0 & -B_x \\ \frac{E_z}{c} & -B_y & B_x & 0 \end{bmatrix}$$

(i) Write the covariant equation that links $F^{\mu\nu}$ to its sources. [4]

(ii) Consider the four-vector $\left(f^0, \vec{f}\right)$ defined by $f^\mu = F^{\mu\nu} j_\nu$. Deduce, in a clear explicit calculation, that $\vec{f}$ is the Lorentz force density form. [4]

END OF PAPER