



University of Fort Hare  
*Together in Excellence*

**STM 502**

**BSc HONOURS EXAMINATION PAPER**

**JUNE 2023**

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**Time:** 3Hours  
**Subject:** Multivariate Statistical Analysis  
**Mark:** 100

This paper consists of 3 pages including the cover page

Internal Examiners

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INSTRUCTIONS

Answer all questions  
Statistical tables will be provided

## QUESTION 1 (25 MARKS)

Consider the following pairs of measurements in the table below:

$x_1$	1	2	3	4	5	6	7	8	9	11
$x_2$	18.95	19.00	17.95	15.54	14.00	12.95	8.94	7.49	6.00	3.99

- i. Construct a scatter plot from the data. (4 marks)
- ii. Compute the sample mean array of  $x_1$  and  $x_2$ . (7 marks)
- iii. Compute the sample variance S array. (10 marks)
- iv. Compute the sample correlation R array. (4 marks)

## QUESTION 2 (25 MARKS)

Let A be a matrix given as  $\begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$ . Use the matrix A to answer the following:

- i. Is matrix A symmetric? Hence, define a symmetric matrix. (2 marks)
- ii. Determine the eigenvalues and eigenvectors of matrix A. (8 marks)
- iii. Determine the spectral decomposition of A. (4 marks)
- iv. Find  $A^{-1}$ . (3 marks)
- v. Determine the eigenvalues and eigenvectors of  $A^{-1}$ . (8 marks)

## QUESTION 3 (25 MARKS)

Q3A. Let A(3 4), B(3 -6), and C(-1 2) be the vertices of a triangle, P(x y) be the required circumcentre, and r be the circumradius. Find the values of the following:

- i. The coordinates of the circumcentre i.e. the values of x and y. (5 marks)

- ii. The value of the circumradius,  $r$ . (4 marks)

Q3B. Given the vectors  $x^T = [2 \ 2 \ 1]$  and  $y^T = [3 \ 1 \ -2]$ , find the following:

- i. The length of  $x$ . (5 marks)  
ii. The length of  $y$ . (5 marks)  
iii. The angle between  $x$  and  $y$ . (6 marks)

### QUESTION 4 (25 MARKS)

Given a matrix  $A$  as  $\begin{bmatrix} 2 & 12 \\ 8 & 9 \\ 6 & 9 \\ 8 & 10 \end{bmatrix}$ ,

- i. Evaluate the Hotelling's  $T^2$ , for testing  $H_0: \mu' = [7 \ 11]$ , using the data in matrix  $A$ . (15 marks)
- ii. Specify the distribution of  $T^2$  for the situation in (i). (5 marks)
- iii. Using (i) and (ii), test sampling distribution of  $T^2$  at  $\alpha = 0.05$ . what conclusion do you reach? (5 marks)

.....[100Marks]