



University of Fort Hare
Together in Excellence

Mathematical Economics
ECO 222/ 222E

DEGREE EXAMINATIONS
NOVEMBER 2017

Time: 3 Hours

Subject: Mathematical Economics 2B

Marks: 100

This paper consist of 4 pages including cover page

Internal Examiners:

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INSTRUCTIONS:

- **Answer ALL THRE (3) questions, showing clearly all the calculations.**
 - **Use a PENCIL to draw clear diagrams where necessary**
 - **Calculators are allowed**
 - **Answers must be rounded off to two (2) decimal places**

Question 1 (40 Marks)

- a) A firm's short run production function is given by $Q = 6L^2 - 0.2L^3$
- (i) Find the size of the workforce that maximises output (use second derivative to prove). [7]
 - (ii) Find the value of output at the point in (i) [3]
 - (iii) Find the size of the workforce that maximises the average product of labour (use second derivative to prove). [7]
 - (iv) Calculate MP_L and AP_L at the value of L in (iii). What do you observe? [7]
- b) A firm's production function is $Q = 8K^{1/4}L^{1/2}$, where K and L are respectively, capital and labour costs. Unit capital and labour costs are 2 and 1, respectively. What is the minimum total of input costs (that is, costs due to capital and labour) if output is to be 240 units? **(Use Substitution Method)** [8]
- c) A consumer spends all her income of R120 on the two goods A and B. Good A costs R10 a unit and good B costs R15. What combination of A and B will she purchase if her utility function is $U = 4A^{0.5}B^{0.5}$? **(Use the Lagrangian Multiplier method)** [8]

Question 2 (30 Marks)

- a) What total revenue a firm will earn if it charges a price of R715 and its $MR = 969 - 0.15q^2$ [8]
- b) If a firm faces the marginal cost schedule $MC = 180 + 0.3q^2$ and marginal revenue schedule $MR = 540 + 0.6q^2$
- (i) And total fixed costs are R65, what is the maximum profit it can make?
(Assume that the second-order condition for a maximum is met. [10]
- c) Given the demand function $P = 70 - 4Q_d$ and the supply function $P = 5 + Q_s$, evaluate the consumer's surplus and the producer's surplus, assuming equilibrium. [12]

Question 3 (30 Marks)

a) Given the following 3x3 matrix:

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 3 \\ 4 & 0 & 0 \end{bmatrix}$$

(i) Find the determinant of matrix A using the expansion by co-factor [4]

(ii) Find the inverse matrix (A^{-1}) [6]

b) A company's input requirements over the next four weeks for the three inputs X, Y and Z are given (in numbers of units of each input) by the matrix

$$R = \begin{bmatrix} 2 & 0.5 & 1 & 7 \\ 6 & 3 & 8 & 2.5 \\ 4 & 5 & 2 & 0 \end{bmatrix}$$

The company can buy these inputs from two suppliers, whose prices for the three inputs X, Y and Z are given (in R) by the matrix

$$P = \begin{bmatrix} 4 & 6 & 2 \\ 5 & 8 & 1 \end{bmatrix}$$

Where the two rows represent the suppliers and the three columns represent the input prices. Use matrix multiplication to derive a matrix that will give the total input bill for the next four weeks for both suppliers. [6]

c) The demand functions for apricots (subscripted a) and blackberries (subscripted b) are inter-connected by the following demand functions:

$$Q_a^D = 96 - 4P_a + 2P_b \text{ and } Q_b^D = 40 + 8P_a - 5P_b$$

The supply functions for apricots and blackberries (similarly subscripted) are:

$$Q_a^S = -4 + 28P_a \text{ and } Q_b^S = -4 + 7P_b$$

(i) Find equilibrium prices and quantities for apricots and blackberries by using the Cramer's rule.

[14]