

UNIVERSITY OF FORT HARE

MAT 123

SUPPLEMENTARY EXAMINATIONS

January 2019

Time: 3 HOURS

Subject: A PRACTICAL APPROACH TO INTEGRAL CALCULUS

Marks:100

This question paper consists of 3 pages

Internal examiner(s)

Mr .I K Appiah
Mr Z Mahlasela

External examiner(s)

Instructions

Answer **any 4** questions **ONLY**.
Symbols have the usual meanings

Question One [25 marks]

1.1 Discuss and possibly evaluate the convergence of $I = \int_{-\infty}^{\infty} \frac{x^2}{9+x^6} dx$ (5)

1.2 Evaluate the integral $I = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{dx}{1+\sin x - \cos x}$ (7)

1.3 Integrate $I = \int \frac{(3x^2-4x+5) dx}{(x-1)(x^2+1)}$ (5)

1.4 Find $I = \int_1^4 x^{1/2} \ln x dx$ (4)

1.5 Integrate $\int \sqrt{1+2x^4} x^3 dx$ (4)

Question Two [25 marks]

2.1 (a) Use the reduction formula: $\int \sin^n x dx = -\frac{1}{n} \cos x \sin^{n-1} x + \frac{n-1}{n} \int \sin^{n-2} x dx$ to

show that $\int_0^{\pi/2} \sin^n x dx = \frac{n-1}{n} \int_0^{\pi/2} \sin^{n-2} x dx$ where $n \geq 2$ is an integer (4)

(b) Use part (a) to evaluate $\int_0^{\pi/2} \sin^3 x dx$ and $\int_0^{\pi/2} \sin^5 x dx$ (2,2)

2.2 Find $I = \int \frac{dx}{x^2 \sqrt{16x^2-9}}$ (5)

2.3 Use partial fraction to find $\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)} dy$ (5)

2.4 Find $I = \int \cos 3x \cos 4x dx$ (3)

2.5 Use Chain Rule to find $\frac{\partial w}{\partial t}$, if $w = \frac{x}{y} + \frac{y}{z}$, $x = \sqrt{t}$, $y = \cos(2t)$, $z = e^{-3t}$ (4)

Question Three [25 marks]

3.1 A tank contains 100L of brine with 15kg of dissolved salt. Pure water enters the tank at a rate of 10L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank

(a) after t minutes? (6)

(b) after 20 minutes? (2)

3.2 Solve the differential equation $xy' = y + x^2 \sin x$, $y(\pi) = 0$ (5)

3.3 Find the cosine of the angle between the plane $x + y + z = 0$ and $x + 2y + 3z = 1$ (5)

3.4 Determine whether the given vectors are orthogonal, parallel or neither:

$\vec{a} = \langle -2, 1 \rangle$, $\vec{b} = \langle 3, 5 \rangle$ (4)

3.5 Find a unit vector in the direction of the vector $\langle -2, 4, 3 \rangle$ (3)

Question Four [25 marks]

4.1 Solve the following system of linear equation by Gauss-Jordan elimination:

$$\begin{aligned} 2x - 2y + z &= 3 \\ 3x + y - z &= 7 \\ x - 3y + 2z &= 0 \end{aligned} \tag{8}$$

4.2 Use the Cofactor Method to find A^{-1} if $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 5 \\ 6 & 0 & 2 \end{bmatrix}$. (7)

4.3 Find the polar form of zw by first putting z and w into polar form:

$$z = \sqrt{3} + i \text{ and } w = 1 + \sqrt{3}i \tag{6}$$

4.4 Calculate f_{xxyz} if $f(x, y, z) = \sin(3x + yz)$. (4)

Question Five [25 marks]

5.1 Find the Maclaurin series of $h(x) = \sqrt{1+x}$ and state its region of convergence. (7)

5.2 (a) Use differentiation to find a power series representation for $f(x) = \frac{1}{(1+x)^2}$.

What is the radius of convergence? (3)

(b) Use part (a) to find a power series for $f(x) = \frac{1}{(1+x)^3}$ (4)

5.3 Find an equation for the ellipse that shares a vertex and a focus with the parabola $x^2 + y = 100$ and that has its other focus at the origin. (5)

5.4 A telephone line between two poles $14m$ apart in the shape of the catenary

$$y = 20 \cosh\left(\frac{x}{20}\right) - 15, \text{ where } x \text{ and } y \text{ are measured in meters.}$$

(a) Find the slope of this curve where it meets the right pole. (3)

(b) Find the angle θ between the line and the pole. (3)

END