UNIVERSITY OF FORT HARE

Advanced Classical Mechanics MAQ 521

Honours Degree Examinations

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Time: 3 Hrs

Subject: Applied Mathematics Honours

Marks: 100

This question paper consists of 4 pages

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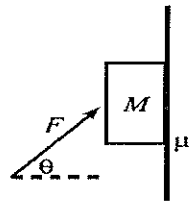
Instructions

Answer all questions.
Symbols have the usual meanings

Question 1

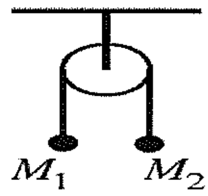
1.1 A book of mass M is positioned against a vertical wall. The coefficient of friction between the book and the wall is μ. You wish to keep the book from falling by pushing on it with a force F applied at an angle θ with respect to the horizontal $\left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$ as shown in the following diagram.



1.1.1 Redraw the above diagram and show all forces acting on the book. (4)
1.1.2 If μ is the coefficient of static friction, what is the frictional force acting on the book? (5)
1.1.3 For this given μ, what is the minimum F required to keep the book up? (6)
1.1.4 What is the limiting value for μ for which there exists an F which will keep the book up? (3)

1.2 A massless pulley hangs from a fixed support. A string connecting two masses, M_1 and M_2, hangs over the pulley (see the following figure). Show that the accelerations of the masses is

$$a = g\frac{M_2 - M_1}{M_2 + M_1}$$



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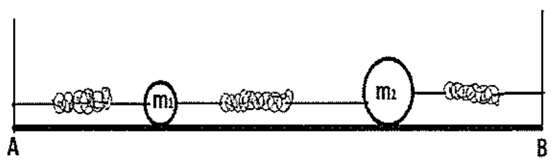
Question 2

2.1 Two unequal objects of masses m_1 and m_2 are connected by three different springs so that they are free to slide on a frictionless table AB, as shown below. The walls at A and B to which the ends of the springs are attached are fixed.

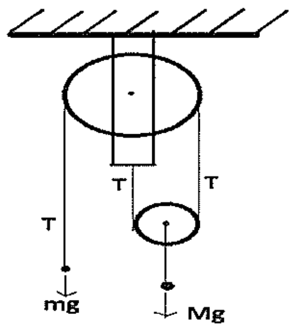
2.1.1 Set up the differential equations of motion of these two masses. (6)

2.1.2 Find the normal frequencies of vibration for the system. (8)



2.2 Two pulleys of masses m and M are connected as shown in following diagram; (10)

2.2.1 What mass m is needed to balance the mass M?
2.2.2 Find the acceleration of mass M if $m = \frac{3M}{4}$.



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Question 3

3.1 Let F_v be the net external force acting on the v^{th} particle of a system.

3.1.1 Prove that $\frac{d}{dt}\left\{\sum_v m_v \dot{\boldsymbol{r}}_v . \frac{\partial r_v}{\partial q_\alpha}\right\} - \sum_v m_v \dot{\boldsymbol{r}}_v . \frac{\partial \dot{r}_v}{\partial q_\alpha} = \sum_v \boldsymbol{F}_v . \frac{\partial r_v}{\partial q_\alpha}$. (6)

3.1.2 If $\boldsymbol{T}$ is kinetic energy of the system, show that (a) $\frac{\partial \dot{T}}{\partial q_\alpha} = \sum_v m_v \dot{\boldsymbol{r}}_v . \frac{\partial \dot{r}_v}{\partial q_\alpha}$, (3)

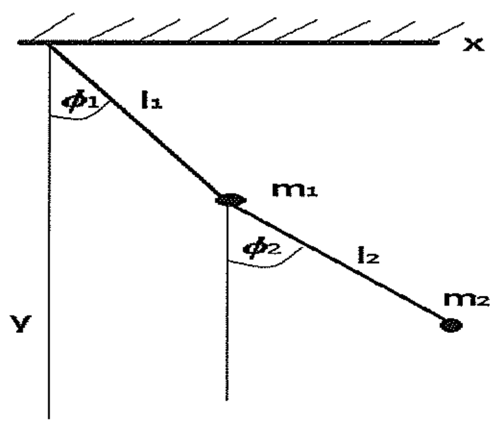
(b) $\frac{\partial \dot{T}}{\partial \dot{q}_\alpha} = \sum_v m_v \dot{\boldsymbol{r}}_v . \frac{\partial r_v}{\partial q_\alpha}$. (3)

3.1.3 Hence prove that $\frac{d}{dt}\left(\frac{\partial \dot{T}}{\partial \dot{q}_\alpha}\right) - \frac{\partial T}{\partial q_\alpha} = \Phi_\alpha,\ \alpha = 1, \ldots, n$ where $\Phi_\alpha = \sum_v \boldsymbol{F}_v . \frac{\partial r_v}{\partial q_\alpha}$. (5)

3.1.4 Suppose that the forces acting on a system of particles are derived from a potential function V, which is conservative. Prove that if $\mathrm{L} = \mathrm{T} - \mathrm{V}$ is the Lagrangian function, then (7)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_\alpha}\right) - \frac{\partial L}{\partial q_\alpha} = 0.$$

3.2 Consider the following coplanar double pendulum placed in a uniform gravitational field (acceleration $\boldsymbol{g}$)



3.2.1 Find the Lagrangian of this system. (8)

3.2.2 From the Lagrangian found above, find the equations of motion. (5)

3.2.3 If $l_1 = l_2 = l$ and $m_1 = m_2 = m$, show that the frequency is $W_{\pm} = \frac{g\sqrt{2 \pm \sqrt{2}}}{l}$. (5)

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