UNIVERSITY OF FORT HARE

Integral Calculus: A Theoretical Approach
Mat 121

Degree Examinations

November

2018

Time: 3 Hours

Subject: MAT 121

Marks: 100

This question paper consists of 3

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Instructions

Answer all questions.

Symbols used have the usual meaning

Question 1

1.1Express the complex number $z = \dfrac{3}{4-3i}$, in the form $a+ib$. (3)

1.2 Prove that $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$ for complex numbers z_1 and z_2. (3)

1.3 State without proof De Moivre's Theorem.
Hence use this theorem to find $(1-\sqrt{3}i)^5$. (5)

1.4 Find the cube roots of i and sketch the roots on the z -plane. (4)

1.5 For $z = x\sin y$ find $\dfrac{\partial^3 z}{\partial y^2 \partial x}$ (3)

Question 2

2.1 First use substitution and then integration by parts to evaluate the integral $\int x^5 e^{x^2} dx$. (4)

2.2 Evaluate the integrals (a) $\int \cot^2 x dx$ (b) $\int \cos 3x \sin 6x dx$ (6)

2.3 Chose a suitable trigonometric substitution to evaluate the following integrals:

$$\int_0^3 x^2\sqrt{9-x^2}\,dx$$ (5)

2.4 Write down the partial decomposition of $\dfrac{1}{x(x+1)(2x+3)}$. After evaluating for the constants use this partial decomposition to evaluate $\int \dfrac{1}{x(x+1)(2x+3)} dx$. (4)

2.5 Use an appropriate rationalizing substitution to evaluate the integral

$$\int \frac{1}{3\sin x + 4\cos x} dx$$ (4)

2.6 An integrating factor $I(x)$ is used to solve the linear differential equation $\dfrac{dy}{dx} + P(x)y = Q(x)$ .Use this procedure to find the integrating factor for the initial value problem $x^2 \dfrac{dy}{dx} + 2xy = \cos x$, $y(\pi) = 0$. (3)

2.7 Determine whether the equation $y' = \dfrac{xy + y^2}{x^2}$, is homogeneous, and solve it. (4)

2.8 Show that the equation $\sin y + (1 + x\cos y)\frac{dy}{dx} = 0$ is exact, and hence solve it. (4)

Question 3

3.1 If A is a $m \times n$ matrix and $B = A^T$, find the size of

(a) B

(b) BB^T

(c) $B^T B$ (3)

3.2 Given the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \\ 2 & 0 & 1 \end{bmatrix}$

(a) Find det(A) by

(i) Cofactor expansion method.

(ii) "Basket-Weave" method. (4)

(b) Find A^{-1} using the Adjoint method. (5)

3.3 (a)Write the system of linear equations $\begin{aligned} x - z &= 3 \\ 2y - 2z &= 2 \\ 2x + z &= 3 \end{aligned}$ in matrix form. (1)

(b)Solve the system using **Cramer's Rule.** (4)

3.4 Find all the solutions of the given linear system using the Gauss method with back substitution, $\begin{aligned} 2x_1 + x_2 + -x_3 &= 2 \\ x_1 - x_2 + x_3 &= 7 \\ 2x_1 + 2x_2 + x_3 &= 4 \end{aligned}$. (4)

Question 4

4.1 Find the equation of the conic, hyperbola that satisfies the conditions; the foci $(\pm 6,0)$ vertices $(\pm 4,0)$ and sketch its graph. (7)

4.2 Given that $P_1(1,-1,3)$ and $P_2(-1,4,5)$ find

(i) The coordinates of the midpoint of the line segment joining P_1 to P_2.

(ii) The unit vector in the direction of $\overrightarrow{P_1P_2}$.

(iii) Find the angle between the vectors $\bar{u} = 2i + 6j - 4k$ and $\bar{v} = -3i - 9j + 6k$. Are they orthogonal, parallel or neither? (6)

Question 5

5.1 Show that $\sinh^{-1} x = \frac{1}{\sqrt{1+x^2}}$. (3)

5.2 Find $\frac{dy}{dx}$ for the following $y = e^{\tanh x} \cosh(\cosh x)$ (5)

5.3 Let $a_n = \frac{4n-3}{3n+4}$, determine whether $\{a_n\}$ is convergent or diverges. (4)