UNIVERSITY OF FORT HARE
Department of Pure and Applied Mathematics

MAT 122

DEGREE EXAMINATIONS
NOVEMBER 2018

Time: 3 Hours

Subject: A PRACTICAL APPROACH TO DIFFERENTIAL CALCULUS

Marks: 100

This question paper consists of 2 pages

Internal Examiner(s)

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Instructions

Answer ALL Questions
Symbols have the usual meanings.

Question One

1.1 Prove by Mathematical Induction that $\forall n \in \mathbb{Z}^+$, $2^n > n$

(5)

1.2 Solve the following inequality and present your solution on a real number line.

$$\frac{2x}{x-2} > 1$$

(3)

1.3 State, without proof, the Binomial Theorem. If the first 3 terms of the expansion of $(1+ax)^n$ in ascending powers of x, are $1 + 12x + 64x^2$. Determine a and n.

(9)

1.4 Find the domain and range of g if $g(x) = \frac{x}{|x|}$

(4)

1.5 If $f(x) = \frac{1}{x}$, $g(x) = x^2$ and $h(x) = 4x - 5$ find

(a) $h \circ g$

(b) $g \circ h \circ f$

(5)

1.6 On the same xy-plane sketch the graph of the function (a) $f(x) = x^2 + 3x + 2$. From (a) sketch the graphs of

(b) $f(x) + 2$

(c) $f(x + 2)$

(5)

Question Two

2.1 Evaluate $\lim_{h\to 0} \frac{(3+h)^{-1}-3^{-1}}{h}$

(3)

2.2 Prove that $\lim_{x\to 0} \frac{sin(x)}{x} = 1$ and hence evaluate $\lim_{x\to 0} cot(8x)tan(5x)$

(6)

2.3 State, without proof, L'Hospital's Rule and hence evaluate $\lim_{x\to 0} \frac{sin(x)}{6x}$.

(5)

2.4 State the three conditions to be satisfied for a function to be continuous at a point $x = a$, and hence determine the value of b for which the following function is continuous, for all values of x.

$$f(x) = \begin{cases} x & if \quad x < -2 \\ bx^2 & if \quad x \geq -2 \end{cases}$$

(7)

Question Three

3.1 From definition, find $f'(x)$ for the function $f(x) = \frac{1}{\sqrt{3x}}$

(5)

3.2 Prove that $\frac{d}{dx}(tan(x)) = sec^2(x)$

(7)

3.3 Find $\frac{dy}{dx}$ if

(a) $y = sin(x) + 10x^4$

(b) $y = \frac{\sqrt{x}-1}{\sqrt{x}+1}$

(c) $y = 3^x - sec(7x)$

(15)

3.4 Differentiate implicitly $y^4 + x^2y^2 + yx^4 = y + 1$

(4)

Question Four

4.1 Find the equation of the tangent to the curve $y = x^3 - x^2 - 2$ at the point $x = 2$. Also, find the co-ordinates of the point where the tangent meets the line again.

(7)

4.2 Find the minimum and maximal values of the function $y = 4x^3 + 15x^2 - 18x + 7$ distinguishing between them. Sketch the graph of the function.

(10)