UNIVERSITY OF FORT HARE

PHY111F

DEGREE EXAMINATIIONS

May / June 2023

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Time: 3 HOURS

Subject: Mechanics of a Particle

Marks: 100

This paper consists of 5 pages including cover page

Internal Examiner

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Moderator

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Instructions

- Answer all five (5) questions
- Read the question more than once and clearly write down your data.
- On problem solving, specify useful definitions and draw clearly labelled diagrams.

Question 1 **(Dimensional Analysis and Conversion) [20]**

1.1. Gasoline is heavily taxed in Europe, with a recent cost of 1.20 euro per litre. What was the price in dollars per gallon if the currency conversion is 1.04 euro = $1 US? One gallon (gal) = 3.8 litres (L). [4]

1.2. The gravitational constant G is $6.67\times10^{-11}\ m^3/s^2.kg$. What is G in units of $cm^3/s^2.g$? [3]

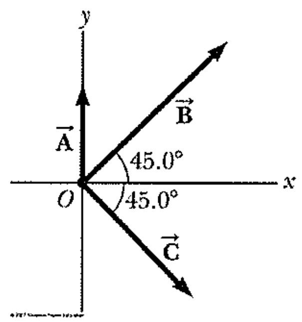
1.3. One of Einstein's most famous results is contained in the formula $E = mc^2$, where E is the energy content of the mass m and c is the speed of light. What are the dimensions of E.? [3]

(Position, Velocity and Speed, Instantaneous speed, acceleration, Motion in two dimensions)

1.4. Three displacement vectors of a cricket ball are shown in the diagram where $|\vec{A}| = 20.0$ units, $|\vec{B}| = 40.0$ units and $|\vec{C}| = 20.0$ units. Find

1.4.1. The resultant in unit vector notation. [5]

1.4.2. The magnitude and direction of the resultant displacement. [5]



Question 2 **(Motion in One Dimension and Free Fall) [20]**

2.1. A particle moves along the x axes. Its position is given by the equation $x(t) = (3.00t^2 - 2.00t + 3.00)$ m, where x is in meters and t in seconds.

Determine

2.1.1. The average speed between $t = 2.00s$ and $t = 3.00s$. [6]

2.1.2. The instantaneous speed at $t = 2.00s$ and $t = 3.00s$ [6]

2.2. A Moon rock is thrown upwards with a *velocity 7 m/s*. After *7 s*, it has a downward velocity of *4 m/s*.

2.2.1. What is the acceleration due to gravity on the Moon? [4]

2.2.2. How high above the starting point did the rock go before it began to fall? [4]

Question 3 **(Motion in Two Dimension) [20]**

3.1. In a local bar, a customer slides an empty beer mug down the counter for a refill. The bartender is momentarily distracted and does not see the mug, which slides off the counter and strikes the floor 1.40 m from the base of the counter. If the height of the counter is 0.86 m,

3.1.1. With what velocity did the mug leave the counter? [4]

3.1.2. What was the direction of the mug's velocity just before it hit the floor? [3]

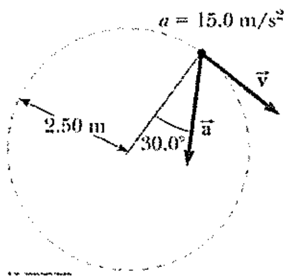
3.2. A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection? [5]

3.3. The diagram represents the total acceleration of a particle moving clockwise in a circle of radius 2.50 m at a certain instant of time. At this instant, find

3.3.1. The radial acceleration, [2]

3.3.2 The speed of the particle and [3]

3.3.3 Its tangential acceleration. [3]



Question 4 **(Applications of Newton's Laws) [20]**

4.1 Three forces acting on an object are given by $\vec{F}_1 = \left(-2.00\hat{i} + 2.00\hat{j}\right)N$, $\vec{F}_2 = \left(5.00\hat{i} - 3.00\hat{j}\right)N$, $\vec{F}_3 = \left(-45.0\hat{i}\right)N$. The object experiences an acceleration of magnitude 3.75 m/s^2.

4.1.1 What is the direction of the acceleration? (Hint: find the angle relative to x-axis) [5]
4.1.2 What is the mass of the object? [5]
4.1.3 If the object was initially at rest, what is the speed after 10.0 s? [5]
4.1.4 What are the velocity components of the object after 10.0 s? [5]

Question 5 **(Work Done and Work Energy Theorem) [20]**

5.1 For what values of the angle θ between two vectors is the scalar product positive and negative? [2]

5.2 If the speed of a particle is doubled, what happens to its kinetic energy? Does it double, or becomes four times larger, or becomes $\sqrt{2}$ times larger, or it becomes halve as large or one quarter as large. Show why you think so! [2]

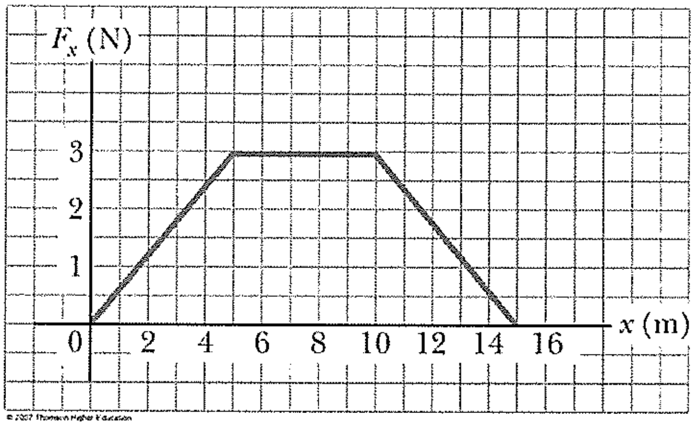
5.3. A particle is subject to a force F_x that varies with position as shown in the diagram. Find the work done by the force on the particle as it moves

5.3.1 From x = 0 to x = 5.00 m, [2]

5.3.2 From x = 5.00 m to x = 10.0 m and [2]

5.3.3 From x = 10.0 m to x = 15.0 m [2]

5.3.4 Find the net work done this force on the particle [2]



5.4 A force $\vec{F} = \left(6\hat{i} - 2\hat{j}\right)N$ acts on a particle that undergoes a displacement $\Delta\vec{r} = \left(3\hat{i} + \hat{j}\right)m$. Find

5.4.1. The work done by the force on the particle and [4]

5.4.2. The angle between $\vec{F}$ and $\Delta\vec{r}$. [4]

Formula Sheet: (PHY 111F)

$v_{xf} = v_{xi} + a_x t, \qquad v_{xf}^{\,2} = v_{xi}^{\,2} + 2a_x(x_f - x_i), \quad x_f = x_i + v_x t + \frac{1}{2}a_x t^2, \qquad x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$

$\Delta x = x_f - x_i, \qquad v_{x,avg} = \frac{\Delta x}{\Delta t}, \quad v_{avg} = \frac{d}{t}, \qquad v_x = \frac{dx}{dt}, \qquad a_{x,avg} = \frac{\Delta v}{\Delta t}, \qquad a_x = \frac{dv_x}{dt} \quad \Delta\vec{r} = \vec{r}_f - \vec{r}_i,$

$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}, \qquad \vec{v}_{ins} = \frac{d\vec{r}}{dt}, \qquad \vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}, \qquad \vec{a}_{ins} = \frac{d\vec{v}}{dt}, \quad \vec{r} = x\hat{i} + y\hat{j}, \qquad \vec{v} = v_x\hat{i} + v_y\hat{j},$

$h = \frac{v_i^{\,2}\sin^2\theta_i}{2g}, \qquad R = \frac{v_i^{\,2}\sin 2\theta_i}{g}, \qquad a_t = \left|\frac{d\vec{v}}{dt}\right|, \quad a_c = \frac{v^2}{r}, \quad a_r = -a_c = -\frac{v^2}{r}, \ a = \sqrt{a_t^{\,2} + a_r^{\,2}},$

$T = \frac{2\pi r}{v}, \ \sum\vec{F} = m\vec{a}, \qquad \vec{F}_g = m\vec{g}, \qquad \vec{F}_{12} = \vec{F}_{21}, \qquad f_s \le \mu_s n, \qquad f_k = \mu_k n, \ W = F\Delta r\cos\theta,$

$\vec{A}.\vec{B} = AB\cos\theta, \quad W = \vec{F}\cdot\Delta\vec{r}, \quad W = \int_{x_i}^{x_f} F_x dx = -\Delta U, \quad W_{net} = \int_{r_i}^{r_f}\sum\vec{F}\,d\vec{r}, \ F_s = -kx, \quad F_{app} = kx,$

$W_s = \frac{1}{2}kx_i^{\,2} - \frac{1}{2}kx_f^{\,2}, \qquad W_{app} = \frac{1}{2}kx_f^{\,2} - \frac{1}{2}kx_i^{\,2}, \qquad KE = \frac{1}{2}mv^2, \ W_{net} = \Delta KE = \frac{1}{2}m(v_f^{\,2} - v_i^{\,2}),$

$\Delta KE = KE_i - KE_f, \ F_x = -\frac{dU}{dx}, \ F_y = -\frac{dU}{dy}, \qquad F_z = -\frac{dU}{dz}, \qquad U_s = \frac{1}{2}kx^2, \quad Wg = mg((y_f - y_i)$

$\Delta E_m = \Delta KE + \Delta U = 0, \quad KE_i + U_i = KE_f + U_f, \qquad P_{ins} = \frac{dE}{dt}, \qquad P_{avg} = \frac{W}{t}, \qquad P = \vec{F}.\frac{d\vec{r}}{dt} = \vec{F}.\vec{v}$

$\vec{p} = m\vec{v}, \quad p_x = mv_x, \quad p_y = mv_y, \quad p_z = mv_z, \ \sum\vec{F} = \frac{d\vec{p}}{dt}, \quad \Delta\vec{p} = \int_{t_i}^{t_f}\vec{F}\,dt = \vec{I}$