UNIVERSITY OF FORT HARE

STA 111

DEGREE EXAMINATIONS

June 2017

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Time: 3 hours

Subject: Descriptive Statistics and Differentiation

Marks: 100

This paper consists of 8 pages including cover page

Internal Examiners

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Instructions

Answer all Questions

List of key formulae and Statistical tables are attached

Programmable calculators may not be used

Section A

Question one [10 marks]

1.1 Ask yourself a question and provide an answer for it. **[1]**

1.2 Two equally likely events **[1]**

a. Have the same probability of occurrence **b.** Cannot occur together
c. Have no effect on the occurrence of each other **d.** None of these

1.3 Two independent events are **[1]**

a. Always mutually exclusive **b.** Never mutually exclusive
c. Always complementary **d.** None of these

1.4 Two mutually exclusive events **[1]**

a. Have the same probability **b.** Cannot occur together
c. Have no effect on the occurrence of each other **d.** None of these

1.5 The collection of all outcomes for an experiment is called **[1]**

a. A sample space. **b.** The intersection of events
c. Joint probability **d.** None of this

1.6 Two mutually exclusive events **[1]**

a. Have the same probability **b.** Cannot occur together
c. Have no effect on the occurrence of each other **d.** None of these

1.7 The joint probability of two mutually exclusive events is always **[1]**

a. 1.0 **b.** Between 0 and 1 **c.** 0 **d.** None of these

1.8 Two independent events are **[1]**

a. Always mutually exclusive **b.** Never mutually exclusive
c. Always complementary **d.** None of these

1.9 A compound event includes **[1]**

a. All final outcomes **b.** Exactly two outcomes
c. More than one outcome for an experiment **d.** None of these

1.10 Chebyshev's theorem can be applied to **[1]**

a. any distribution **b.** bell-shaped distributions only
c. skewed distributions only **d.** none of these

Question two [45 marks]

2.1 Consider the distribution blood glucose levels for sample of 60 patients from a government clinic in Nkandla.

55	115	118	114	59	109	63	97	90	59	105
81	84	81	82	61	103	77	82	76	68	86
97	80	77	85	69	62	101	83	58	83	101
86	84	78	59	92	88	97	87	92	70	86
72	84	82	84	101	88	71	86	85	73	62
79	93	75	57	62						

Construct a frequency distribution table for this data, hence calculate: **[5]**

a. The Inter-Quartile-Range **[5]**

b. The Coefficient of Variation **[7]**

2.2 A team of four children is to be selected from a class of twenty children, to compete in a quiz game. In how many ways can the team be chosen if:

a. Any four can be chosen; **[2]**
b. The four chosen must include the oldest in the class; **[3]**
c. The four chosen must exclude the oldest in the class. **[3]**

2.3 A certain population of male adults possesses the three characteristics of being married, having college degree, and being a citizen of a specified state. It was found that 5% of the males possess all three characteristics, 15% have a college degree and are citizen of a specified state, 8% have a college degree and are married, and 10% are married and are citizen of a specified state. Furthermore 45% of the male adults have a college a college degree, 40% are citizen of a specified state whereas 50% are married.

a. Represent the situation using the Venn diagram, hence **[5]**
b. If one male is chosen at random from this population, determine the probability that he:
- ***i.*** Is married or has a college degree or is from a specified state. **[4]**
- ***ii.*** Has a college degree and he is married. **[2]**
- ***iii.*** Is not from the specified state but is married and has a college degree. **[3]**
- ***iv.*** Is not married or does not have a college degree, given that he is from the specified state. **[3]**
- ***v.*** Name two events that are mutually exclusive and justify your answer. **[1; 2]**

Question three [35 marks]

3.1 At Hopewell electronics all employees were asked about their political affiliations. The employees were grouped by type of work as executive or production workers. The results are given in the following table.

	Political Affiliation		
Employee Type	ANC	DA	COPE
Executive	5	34	9
Production Worker	63	21	8

Suppose an employee is selected at random from the Hopewell employees.

a. Calculate the probability of selecting:

i. an ANC supporter. **[2]**

ii. an Executive. **[2]**

iii. a DA supporter given that the supporter is a production worker. **[3]**

b. Are the events Cope and production Worker independent? Give a reason for your answer. **[2]**

c. Name any two mutually exclusive events. **[1]**

3.2 According to the USA Snapshot ® "Knowing drug addicts," 45% of Americans know somebody who became addicted to a drug other than alcohol. Assuming this to be true, what is the probability that out of a group of 30 randomly selected Americans:

a. at most 29 know somebody who became addicted to a drug? **[2]**

b. more than 1 know somebody who became addicted to a drug? **[2]**

c. between 10 and 15 know somebody who became addicted to a drug? **[3]**

3.3 The weights of metal components produced by a machine are distributed Normally with mean 14 kg and standard deviation of 0.12 kg.

a. What is the probability that a component sample randomly from production will have a weight

i. Greater than 14.3 kg? **[2]**

ii. Less than 13.7 kg? **[2]**

b. Components are rejected if their weights are outside the limits 13.7 to 14.3 kg. If the machine produces 500 components per day, how many would be expected to be rejected? **[9]**

3.4 Suppose a distribution has mean $\mu = 111$ and standard deviation $\sigma = 7.6$. If Chebyshev's Theorem tells us that 81.1% of the values are between 'a' and 'b' (symmetrical about the mean), then what are these limits? **[5]**

Section B

Question four [20 marks]

4.1 Find the following limits:

a. $\lim_{x\to\infty} \frac{x^3+1}{2x^2-3x+1}$ **[3]**

b. $\lim_{t\to 0} \frac{2e^{2t}-e^t-1}{t^2}$ **[4]**

4.2 Consider the following problems:

a. Using first principles differentiate the function $y = 2x^{-\frac{1}{2}}$ **[4]**

b. Using the quotient rule, differentiate the function $f(x) = \left(3x^2+2\right)\sqrt{(5x+4)}$ **[4]**

c. Find the equation of the tangent to the curve of $y = -3x^2 + 4x + 5$ at the point (3; 10). **[5]**

KEY FORMULAE

1. $\dfrac{\sum mf}{N}$

2. $\dfrac{\sum mf}{n}$

3. $\dfrac{\sum m^2 f - \dfrac{\left(\sum mf\right)^2}{n}}{n-1}$

4. $\dfrac{\sum m^2 f - \dfrac{\left(\sum mf\right)^2}{N}}{N}$

5. $L + \dfrac{\left[\left(\dfrac{n}{2}\right) - pcf\right] * i}{f}$

6. $L + \dfrac{(f_1 - f_0) * i}{2f_1 - f_0 - f_2}$

7. $L + \dfrac{\left[\left(\dfrac{jn}{4}\right) - pcf\right] * i}{f}, \quad j = 1,2,3$

8. $L + \dfrac{\left[\left(\dfrac{jn}{100}\right) - pcf\right] * i}{f}, \quad j = 1,2,3.....,99$

9. $\dfrac{SS_{xy}}{SS_{xx}}$

10. $\dfrac{bSS_{xy}}{SS_{yy}}$

11. $\dfrac{s}{\overline{x}} * 100\%$

12. $Q_3 - Q_1$

13. $n(n-1)(n-2)....1$

14. $\dfrac{n!}{(n-r)!}$

15. $\dfrac{n!}{(n-r)!r!}$

16. $\sum x^2 - \dfrac{(\sum x)^2}{n}$

17. $\sum y^2 - \dfrac{(\sum y)^2}{n}$

18. $\sum xy - \dfrac{(\sum xy)^2}{n}$

19. $\overline{y} - b\overline{x}$

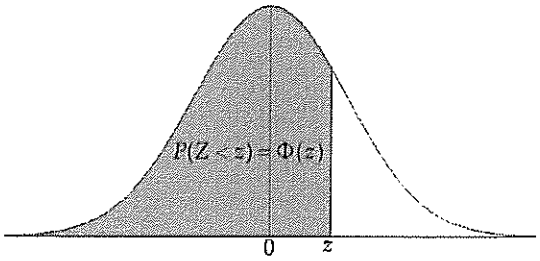
Table 1: Area under the standard normal curve

Normal probabilities to the left of Z.

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}x^2} dx$$

Entries in the table are values of $z \geq 0$

$\Phi(-z) = 1 - \Phi(z)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998

Table 2: Values of the inverse normal distribution

Entries in the table of the inverse function $z = \Phi^{-1}(u)$ for $u \geq 0.5$ where $u = \Phi(z)$ denotes the standard normal distribution function Note that $\Phi(-z) = 1 - \Phi(z) > 0.5$ when $\Phi(z) < 0.5$

$\Phi(z)$	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.50	0.0000	0.0025	0.0050	0.0075	0.0100	0.0125	0.0150	0.0175	0.0201	0.0226
0.51	0.0251	0.0276	0.0301	0.0326	0.0351	0.0376	0.0401	0.0426	0.0451	0.0476
0.52	0.0502	0.0527	0.0552	0.0577	0.0602	0.0627	0.0652	0.0677	0.0702	0.0728
0.53	0.0753	0.0778	0.0803	0.0828	0.0853	0.0878	0.0904	0.0929	0.0954	0.0979
0.54	0.1004	0.1030	0.1055	0.1080	0.1105	0.1130	0.1156	0.1181	0.1206	0.1231
0.55	0.1257	0.1282	0.1307	0.1332	0.1358	0.1383	0.1408	0.1434	0.1459	0.1484
0.56	0.1510	0.1535	0.1560	0.1586	0.1611	0.1637	0.1662	0.1687	0.1713	0.1738
0.57	0.1764	0.1789	0.1815	0.1840	0.1866	0.1891	0.1917	0.1942	0.1968	0.1993
0.58	0.2019	0.2045	0.2070	0.2096	0.2121	0.2147	0.2173	0.2198	0.2224	0.2250
0.59	0.2275	0.2301	0.2327	0.2353	0.2378	0.2404	0.2430	0.2456	0.2482	0.2508
0.60	0.2533	0.2559	0.2585	0.2611	0.2637	0.2663	0.2689	0.2715	0.2741	0.2767
0.61	0.2793	0.2819	0.2845	0.2871	0.2898	0.2924	0.2950	0.2976	0.3002	0.3029
0.62	0.3055	0.3081	0.3107	0.3134	0.3160	0.3186	0.3213	0.3239	0.3266	0.3292
0.63	0.3319	0.3345	0.3372	0.3398	0.3425	0.3451	0.3478	0.3505	0.3531	0.3558
0.64	0.3585	0.3611	0.3638	0.3665	0.3692	0.3719	0.3745	0.3772	0.3799	0.3826
0.65	0.3853	0.3880	0.3907	0.3934	0.3961	0.3989	0.4016	0.4043	0.4070	0.4097
0.66	0.4125	0.4152	0.4179	0.4207	0.4234	0.4261	0.4289	0.4316	0.4344	0.4372
0.67	0.4399	0.4427	0.4454	0.4482	0.4510	0.4538	0.4565	0.4593	0.4621	0.4649
0.68	0.4677	0.4705	0.4733	0.4761	0.4789	0.4817	0.4845	0.4874	0.4902	0.4930
0.69	0.4959	0.4987	0.5015	0.5044	0.5072	0.5101	0.5129	0.5158	0.5187	0.5215
0.70	0.5244	0.5273	0.5302	0.5330	0.5359	0.5388	0.5417	0.5446	0.5476	0.5505
0.71	0.5534	0.5563	0.5592	0.5622	0.5651	0.5681	0.5710	0.5740	0.5769	0.5799
0.72	0.5828	0.5858	0.5888	0.5918	0.5948	0.5978	0.6008	0.6038	0.6068	0.6098
0.73	0.6128	0.6158	0.6189	0.6219	0.6250	0.6280	0.6311	0.6341	0.6372	0.6403
0.74	0.6433	0.6464	0.6495	0.6526	0.6557	0.6588	0.6620	0.6651	0.6682	0.6713
0.75	0.6745	0.6776	0.6808	0.6840	0.6871	0.6903	0.6935	0.6967	0.6999	0.7031
0.76	0.7063	0.7095	0.7128	0.7160	0.7192	0.7225	0.7257	0.7290	0.7323	0.7356
0.77	0.7388	0.7421	0.7454	0.7488	0.7521	0.7554	0.7588	0.7621	0.7655	0.7688
0.78	0.7722	0.7756	0.7790	0.7824	0.7858	0.7892	0.7926	0.7961	0.7995	0.8030
0.79	0.8064	0.8099	0.8134	0.8169	0.8204	0.8239	0.8274	0.8310	0.8345	0.8381
0.80	0.8416	0.8452	0.8488	0.8524	0.8560	0.8596	0.8633	0.8669	0.8705	0.8742
0.81	0.8779	0.8816	0.8853	0.8890	0.8927	0.8965	0.9002	0.9040	0.9078	0.9116
0.82	0.9154	0.9192	0.9230	0.9269	0.9307	0.9346	0.9385	0.9424	0.9463	0.9502
0.83	0.9542	0.9581	0.9621	0.9661	0.9701	0.9741	0.9782	0.9822	0.9863	0.9904
0.84	0.9945	0.9986	1.0027	1.0069	1.0110	1.0152	1.0194	1.0237	1.0279	1.0322
0.85	1.0364	1.0407	1.0450	1.0494	1.0537	1.0581	1.0625	1.0669	1.0714	1.0758
0.86	1.0803	1.0848	1.0893	1.0939	1.0985	1.1031	1.1077	1.1123	1.1170	1.1217
0.87	1.1264	1.1311	1.1359	1.1407	1.1455	1.1503	1.1552	1.1601	1.1650	1.1700
0.88	1.1750	1.1800	1.1850	1.1901	1.1952	1.2004	1.2055	1.2107	1.2160	1.2212
0.89	1.2265	1.2319	1.2372	1.2426	1.2481	1.2536	1.2591	1.2646	1.2702	1.2759
0.90	1.2816	1.2873	1.2930	1.2988	1.3047	1.3106	1.3165	1.3225	1.3285	1.3346
0.91	1.3408	1.3469	1.3532	1.3595	1.3658	1.3722	1.3787	1.3852	1.3917	1.3984
0.92	1.4051	1.4118	1.4187	1.4255	1.4325	1.4395	1.4466	1.4538	1.4611	1.4684
0.93	1.4758	1.4833	1.4909	1.4985	1.5063	1.5141	1.5220	1.5301	1.5382	1.5464
0.94	1.5548	1.5632	1.5718	1.5805	1.5893	1.5982	1.6072	1.6164	1.6258	1.6352
0.95	1.6449	1.6546	1.6646	1.6747	1.6849	1.6954	1.7060	1.7169	1.7279	1.7392
0.96	1.7507	1.7624	1.7744	1.7866	1.7991	1.8119	1.8250	1.8384	1.8522	1.8663
0.97	1.8808	1.8957	1.9110	1.9268	1.9431	1.9600	1.9774	1.9954	2.0141	2.0335
0.98	2.0537	2.0749	2.0969	2.1201	2.1444	2.1701	2.1973	2.2262	2.2571	2.2904
0.99	2.3263	2.3656	2.4089	2.4573	2.5121	2.5758	2.6521	2.7478	2.8782	3.0902