REFORM-BASED APPROACHES IN THE LEARNING AND TEACHING FOR CONCEPTUAL UNDERSTANDING OF CALCULUS FOR DIPLOMA STUDIES AT A SOUTH AFRICAN UNIVERSITY

by

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A THESIS SUBMITTED IN FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY (EDUCATION)

at the

UNIVERSITY OF FORT HARE

SUPERVISOR: PROFESSOR K. J. MAMMEN

2017

Give thanks to the Lord,
for he is good;
his mercy endures forever
(Ps 106:1).

The hardest arithmetic to master is that which enables us to count our blessings

(Eric Hoffer).

(http://quotes-for-teachers.blogspot.co.za/2011/06/math-and-science.html)

PLAGIARISM DECLARATION

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ABSTRACT

This research tested whether Reform-Based Approaches (RBAs) in the learning and teaching of calculus could lead to improved conceptual understanding. The study adopted positivistic paradigm, quantitative approach and pre- and post-test in a quasi-experimental design. The theoretical framework was Constructivism. The interventions were grounded on learner-centred RBAs including Interactive Engagement (IE), Peer Discussion (PD) and Good Questions (GQ). The experimental group comprised 119 volunteering students from a population of 461 registered for Mathematics as a service subject for the National Diploma (ND) in science or engineering at a South African university. Those not in the experimental group were taught through teacher-centred traditional approaches which have been the norm. However, only 71 out of those in the traditionally taught cohort volunteered to write both Pre- and Post-tests. As such, the total number of subjects in the study was 190, i.e., 119 from the Reform-Based cohort and 71 from the Traditional cohort. The instrument, the Calculus Concept Inventory for Technicians (CCIT), consisted of 19 questions on functions, differentiation and integration. Based on a pilot test, the instrument was improved. The Reform-Based cohort did not receive any participation reward and test scores did not contribute to promotion scores. The students wrote Pre-tests in the second week after commencement of lectures and Post-tests during the last week of lectures. The data were analysed using various statistical tools, tests and measures such as Chi-squares, Student t-tests, Pearson's Product Moment correlation, Cronbach alpha, KR-20, the Difficulty Index, and Item Discrimination Point Biserial Index (PBI). The raw gain and normalised gains were also employed in data analyses. The main finding of this study was that RBA made a significant impact on the conceptual understanding of calculus of the experimental group. The gain achieved by the experimental group was in a low range and corresponded to the low use of IE (25% of contact time). A combination of RBA with Traditional teaching is recommended. Also, RBA will be most successfully introduced if supplemented and complemented through supportive environments.

ACKNOWLEDGEMENTS

This research and its culmination in this thesis would not have been possible without the cooperation and encouragement of many people whose support is gratefully appreciated.

I am indebted to my supervisor, Professor K.J. Mammen, for his encouragement, assistance, admirable work ethic and professional approach. He helped me to narrow my focus on a specific topic and provided me with an initial framework to get started. He was always available for consultation at short notice. He was particular about quality and detail. He was prepared to sacrifice at a personal level to advance his students. I am grateful to have had somebody with his exceptional qualities to guide me along the way.

I would also like to thank Dr M. Rabaza for his sacrificial commitment and meticulous attention to detail. He was gracious in his reviews, generous with his time and always quick to respond.

I would not have had comparison data without the gracious and timeous intervention of my colleague Dr Simfumene Stofile. Two field workers, Mr E. Ramothwala and Mr M. H. Komani, aided with the collection and capturing of data. Dr S. Mishi assisted with statistical analyses. Mr. J. Biggs kindly did the language editing. Thanks are also due to all my other colleagues, especially Mrs E. Oberholster and Mrs L. Bester, who assisted and supported me in various ways, and to all the students who participated in the research. I am also grateful to my employer, the Walter Sisulu University, for the generous financial assistance and for granting me sabbatical leave to focus on my studies.

I would like to thank my husband Kobus for his unwavering support and encouragement. My children, Giselle and Shaun, Abraham and Ilana, Jacobus and Kyle, siblings and in-laws acted as cheerleaders throughout this endeavour. Jacobus deserves special mention for his assistance with technical issues and statistical analysis. Abraham applied his skills to enhance the quality of drawings.

Special mention goes to my siblings and their spouses, Dr S.J. and P.A.J. van Rensburg, Dr P.W.A. and Elizabeth Möller, and Elizabeth and Johannes Prinsloo, for their profound devotion and lifelong support.

I would lastly like to pay tribute to my mother, Elizabeth Möller, who encouraged us all academically and resolutely believed in us. We remember her with fondness, gratitude and pride.

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LIST OF ACRONYMS

AL: Active Learning

ACL: Academic Literacy

AIDS: Acquired Immune Deficiency Syndrome

APOS: The theory defining a function in terms of action, process, object and scheme

APS: Admission Point Score

ARS: Audience Response Systems, or Clickers

BSDT: Basic Skills Diagnostic Test

C2005: Curriculum 2005

CAPS: Curriculum and Assessment Policy Statement

CAS: Computer Algebra Systems

CCH: Calculus Consortium based at Harvard

CCI: Calculus Concept Inventory

CCIT: Calculus Concept Inventory for Technicians

CCR: The Calculus Concept Readiness Instrument

CERM: Calculus Education Reform Movement

CHE: Council on Higher Education

CJ: Comparative Judgement

CK: Content Knowledge

DBE: Department of Basic Education

DE: Department of Education

DHET: Department of Higher Education and Training

DTBFS: Diagnostic Test of Basic Fractions Skills

ECSA: Engineering Council of South Africa

ESUM: Engineering Students' Understanding Mathematics

FCI: Force Concept Inventory

FET: Further Education and Training (Grades 10 – 12)

GET: General Education and Training (Grades 0 (or Grade R) to Grade 9)

GQ: Good Questions

HESA: Universities South Africa (formerly: Higher Education South Africa: HESA)

HIV: Human Immunodeficiency Virus

ICM: International Congress of Mathematicians

ICT: Information and Communications Technology (or technologies)

IE: Interactive Engagement

IEM: Interactive Engagement Methods

IMU: International Mathematical Union

JET: Joint Education Trust

KCS: Teachers' Knowledge of Content and Students

M1: First Semester course in Mathematics

M2: Second Semester course in Mathematics

M3: Third Semester course in Mathematics

M4: Fourth Semester course in Mathematics

MAT: Mathematics, section three of the NBT National Assessment Test

MWG: Mathematics Working Group

MKO: More Knowledgeable Other

NBT: National Benchmark Test

NCS: National Curriculum Statement

NRC: National Research Council

RNCS: Revised National Curriculum Statement

NCTM: National Council of Teachers of Mathematics (USA)

ND: National Diploma

NMAP: National Mathematics Advisory Panel

NSC: National Senior Certificate

OBE: Outcomes Based Education

PAL: Peer Assisted Learning

PCK: Pedagogical Content Knowledge

PCKg: Pedagogical Content Knowing

PI: Peer Instruction, used interchangeably in this study with Peer Discussion (PD)

PD: Peer Discussion, used interchangeably in this study with Peer Instruction

PBI: Point Biserial Index

QL: Quantitative Literacy

RBA: Reform-Based Approach

RM: Reform Movement

RNCS: Revised National Curriculum Statement

SAAIR: South African Association for Institutional Research

SD: Standard Deviation

SEFI: European Society for Engineering Education

SMICT: Science, Mathematics and ICT

SMK: Subject Matter Knowledge

STEM: Science, Technology, Engineering and Mathematics

T: Traditional courses that make little or no use of IE methods

TIMMSR: Third International Mathematics and Science Study-Repeat

TK: Technology Knowledge

TPACK: Technological, Pedagogical and Content Knowledge

UFH: University of Fort Hare

UoT: University of Technology

ZPD: Zone of Proximal Development (Vygotsky)

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CHAPTER ONE

INTRODUCTION AND MOTIVATION FOR THE STUDY

1.1 Background to the Study

Research on the understanding of, and connections between complex mathematical concepts has been ongoing (Schoenfeld, 1995; Wilson, 1997; Hake, 1998; Froyd, 2008; Méndez and Delgado, 2013; Freeman, Eddy, McDonough, Smith, Okoroafor, Jordt and Wenderoth, 2014; Tang and Titus, 2015; CBMS, 2016). Globally, learners and teachers struggle with calculus and its inherent complexities (Bressoud, 2009, 2015a, 2015b; Sonnert, Sadler, Sadler and Bressoud, 2015). Dropout and failure rates in calculus courses are high (Spaul, 2013; Spaull and Kotze, 2015). Students struggle with conceptual understanding of calculus, and have difficulties applying their knowledge in the various related fields. Calculus education reformers advocate active, hands-on and student-centred approaches to learning and teaching (Thomas, 2014a), combined with the utilisation of computers and graphical methods in order to facilitate understanding and inter-connection between these concepts. This approach to learning and teaching is currently referred to as Reform-Based Approaches (RBAs). A few important features associated with Reform Mathematics have been identified from literature on calculus learning and teaching, namely: addressing misconceptions (Stage and Kinzie, 2009); keeping students active in class (Hake, 1998); monitoring that learning takes place (Lee, Sbeglia, Ha, Finch and Nehm, 2015); developing students' conceptual understanding (Kueffer and Latterell 2001); acknowledging gaps in prior knowledge (Thomas, 2014a; Gaze, 2015) and consequently attempting to bridge the gap between secondary and tertiary education (Gallimore and Stewart, 2014); employment of cooperative learning styles (Johnson and Johnson, 2002); offering multiple perspectives on subject material (Huang and Cai, 2011; Huang, 2015), and using appropriate technology effectively (Caldwell, 2007; Chan, Tam and Li, 2011; Jurukovski, Callender and Schoberle, 2015; Chien, Chang and Chang, 2016; Hunsu, Adesope and Bayly, 2016). Furthermore, one of the problems that led to the establishment of the Reform Movement (RM), was the insistence that traditional teaching methods often lead to the detriment of conceptual knowledge (Engelbrecht, Bergsten and Kågesten, 2009). RBA offer a solution to the limitations and challenges experienced worldwide with the traditional teaching and learning of calculus (Steen, 1986; Stedman, 1997; Joiner, 1998; Nelson, 2005; Murphy, 2006).

Impressive results have been reported by studies utilising RBAS such as interactive-engagement methods in physics education (Hake, 1998; Ezrailson, 2004). Research has also been conducted on Interactive Engagement (IE) in other fields of study. Furthermore, a meta-analysis of eight studies of undergraduate Calculus found that RBAS in the teaching of calculus had a positive impact on undergraduate students' conceptual understanding of calculus (Kueffer and Latterell 2001). According to the data in the above study, reform teaching techniques increased undergraduate Calculus I students' conceptual understanding. Yet another study measured the effect of clickers on examination performance in calculus when the course was taught with clickers (Ross, Scott and Bruce, 2012). The low-performing group who used clickers in class showed a significant improvement. Literature has thus indicated the value of RBAS. No studies could however be located which used GQ, IEM, PD, ARS and ConcepTests for teaching calculus as a service subject for diploma studies in South Africa.

This chapter introduces the study and provides the background. An overview of calculus, its history, and the need for reform in the international context, African context and South African contexts are delineated. RBAs are presented as a way to overcome the problems experienced with the learning and teaching of calculus. RBA as used in this study, and the research question and objectives, the purpose of the study, the rationale and research problem, and the perceived significance of the study are all explicated. A brief overview of the research methodology and a concise summary of the delimitations of the study follow. One section provides the definitions of operational terms used in the study. An overview of the relationship between RBAs and teaching and learning of calculus is discussed broadly in the background.

In this study, RBAs were applied to the teaching of calculus with the aim of improving students' conceptual understanding. Various RBAs have been developed over the last few decades. For the purposes of this study, the focus was on Active Learning (AL) while using Good Questions (GQ) and Peer Instruction/Discussion (PI/PD) in combination with Audience Response Systems (ARS), also called clickers. Students' conceptual understanding of calculus was tested using a ConcepTest. In the following section, these approaches and tools will be explained and discussed.

1.1.1 Reform-Based Approaches (RBAs)

RBAs in the teaching of calculus have been developed to counteract the problems that are generally seen as the result of traditional teaching. Calculus education reform pedagogies align calculus instruction more closely with theories of how students learn. These student-centred Reform-Based pedagogies have been developed and are still being developed to enhance and improve the student experiences of learning calculus. The reformers advocate an active, hands-on, student-centred approach to learning and teaching, utilising computers and graphical methods in order to facilitate understanding and connection of complex mathematical concepts (Schoenfeld, 1995; Wilson, 1997; Hake, 1998; Froyd, 2008; Méndez and Delgado, 2013; Freeman et al., 2014; Tang and Titus, 2015; CBMS, 2016). Whilst also emphasising the relevance and usefulness of calculus, supporters of RBAs further encourage pedagogies such as cooperative learning, problem based learning and the utilisation of social media, blogs, web-based teaching, multimedia, tablets and interactive teaching tools, (Murphy, 2002; Stanley, 2002; Veldman, De Wet, Ike Mokhele and Bouwer, 2008; Tapare, 2013). In this study, the RBAs selected for the intervention were IE, GQ, PD and ARS.

1.1.2 Active Learning

AL is generally defined as any instructional method that engages students in the learning process in the classroom (Prince, 2004). AL is often contrasted with traditional lectures where students are expected to passively receive information from the instructor in the broadcast model of teaching.

Research has revealed a number of positive outcomes of AL, such as improved recall of information, increased conceptual understanding, improved attendance of lectures, higher retention in academic programs, enhanced critical thinking skills, interpersonal relationships and self-esteem, fewer misconceptions and improved teamwork skills (Drake and Battaglia, 2014).

Studies on AL have been conducted in various countries such as the USA (Hoekstra and Mollborn, 2012; Lumpkin, Achen and Dodd, 2015; Tang and Titus, 2015), India (Rajcoomar, 2013), the UK (Purchase, Mitchell and Ounis, 2004), Spain and Mexico (Méndez and Delgado, 2013) and also South Africa (Steyn and Steyn, 2002; Sekwena, 2014). Freeman et al. (2014) conducted a meta-analysis of 225 studies on AL in undergraduate science, technology, engineering, and mathematics (STEM) courses. They stated that on average, student performance on examinations and concept inventories increased by 0.47 standard deviations (SDs) under AL, and the average examination scores improved by about 6%. They further reported that students in classes with traditional lecturing were 1.5 times more likely to fail than those in classes where AL was used. Their analyses showed that failure rates in mathematics dropped by 15% on average when AL was used. AL was found to be effective across all class sizes and across the STEM disciplines and had a strong positive impact on female students. The researchers quite rightly raised questions about the continued use of traditional lecturing and expressed support for AL as the preferred, empirically validated teaching practice.

In the United States, North Carolina State University established a highly collaborative, hands-on, computer-rich, inter-active learning environment for large, introductory college courses called SCALE-UP. In the calculus-based introductory physics part of their study, they found that students' attitudes improved, their conceptual understanding and their problem solving abilities improved significantly, the success rates were higher than those achieved when compared to traditional instruction, particularly for females and minorities (Beichner, Saul, Abbott, Morse, Deardorff, Allain, Bonham, Dancy and Risley, 2007).

1.1.3 Good Questions (GQ) and Peer Discussion (PD)

The form of AL that is used in this study, is IE, which is AL with the emphasis on Peer Instruction (PI). PI, also called "pair/share" or "Think-Pair-Share" (Braun, Bremser, Duval, Lockwood and White, 2017) is an activity during which small groups discuss questions and have to take a stand on a challenge posed to them. PI provides students with the opportunity to collaborate with their classmates, explain their solution methods, and construct knowledge. Researchers have reported on various positive results obtained when employing PI in classrooms (Crouch and Mazur, 2001; Pilzer, 2001; Lucas, 2009; Cronhjort, Filipsson and Weurlander, 2013).

GQ are usually combined with PI. Miller, Santana-Vega and Terrell (2006, p. 199) claim that the combination:

- stimulates students' interest and curiosity in mathematics.
- helps students monitor their understanding.
- offers students frequent opportunities to make conjectures and argue about their validity.
- draws on students' prior knowledge, understanding, and/or misunderstanding.
- provides instructors a tool for frequent formative assessments of what their students are learning.

The students participating in Cronhjort's study in Sweden on using PI in calculus classes were more engaged than in traditional lectures and displayed higher motivational levels (Cronhjort et al., 2013), whilst Jones, Antonenko and Greenwood (2012) reported that the PI group in their study in the USA scored significantly higher on a test of transfer of knowledge than the control group. In a study based on AL in an engineering faculty in the USA, Prince (2004) reported improved interpersonal relationships, increased levels of social support and higher levels of self-esteem amongst participants.

In a study involving PI in Taiwan, Chien et al. (2016) posit that the effectiveness of PI may be attributed to the theory of scaffolding (Bruner, 1984). According to them, a student is enabled by the interaction with a More Knowledgeable Other (MKO). However, a peer is not necessarily a MKO other. It is more likely the verbalising and ordering of thought processes that bring about a better understanding of the concepts.

1.1.4. Audience Response Systems (ARS) or clickers

Various names such as audience response system, classroom response system, student response system, personal response system, classroom communication system, group response system and electronic voting system are used in the literature to refer to electronic voting systems that enable wireless mass communication between an audience and an instructor or facilitator (Caldwell, 2007). The hand-held devices that form part of these systems enable input into the system and are commonly called keypads, handsets, zappers, i-clickers or clickers. The instructor usually presents multiple choice questions on a PowerPoint slide. Following anonymous voting, the results are displayed graphically on a screen (Bode, Drane, Kolikant and Schuller, 2009).

Researchers have reported positive outcomes after using clickers in the classroom in various countries such as Canada (White, Syncox and Alters, 2011), the US (Kolikant, Drane and Calkins, 2010; Gibson, 2011; Brady, Seli and Rosenthal, 2013), Nigeria (Agbatogun, 2013) and South Africa (Simelane and Skhosana, 2012). Early reviews mainly focused on students' perceptions of the efficacy of clickers although little evidence had been reported about the actual efficacy of clickers. Nonetheless, recent studies such as the meta-analysis by Chien et al. (2016) have finally revealed evidence of the value of clickers. They report that the general outcomes accompanied with clicker-integrated instruction were greater than those of conventional lectures, regardless of what specific instructional strategies were used or whether the outcomes were assessed by immediate or delayed Post-tests. The mean effect sizes were statistically and practically significant. Clicker-integrated instruction was furthermore found to facilitate knowledge application. In previous studies the effectiveness of clicker-integrated instruction was attributed to other factors. The results from the study done by Chien et al. however suggest that this is not the case and that the use of clickers indeed is advantageous. The results of the meta-analysis therefore settle the debate.

Other studies have yielded similar results (Lucas, 2009). MacArthur and Jones (2008) cite a study by Poulis, Massen, Robens and Gilbert (1998b), who collected data over a 13 year period on clicker use by more than 2000 students in chemistry, physics, and various engineering courses. They found that when clickers were used, the pass rate was 85%, and when clickers were not used, the pass rate was 60%.

A study conducted by Pengfei (2007) in the USA used clickers for teaching electromagnetism. Data produced in his study revealed that students using clickers achieved a small but significant gain in conceptual learning. On average, students in the clicker section scored 10% better on common exam multiple choice questions. In a meta-analysis involving 53 articles with an overall sample of 26,095 participants, Hunsu et al. (2016) found significant effects of using clicker-based technologies on a number of cognitive and non-cognitive learning outcomes. Kay and LeSage (2009) analysed 67 peer-reviewed papers from 2000 to 2007 in order to encapsulate benefits and challenges associated with the use of an ARS. Main benefits listed include improvements in attendance, attention levels, participation and engagement, interaction, discussion, quality of learning, learning performance and assessment. Schmidt (2011) reported high satisfaction levels amongst the participants in the study conducted with clicker pedagogy in Denmark, even though he asserted that the use of clickers provided students with a more critical and possibly more realistic self-assessment of their conceptual understanding of the material.

As the use of technology such as clickers becomes increasingly common, teaching has to adapt to include new categories of knowledge, one being Technological Knowledge (TK). Content Knowledge (CK) and Pedagogical Knowledge (PK) are two categories of knowledge that are deemed essential for teaching (Shulman, 1987). Technological Pedagogical and Content Knowledge (TPACK) comprises of these three components, together with the relations between and among the components. When using technology such as clickers, TPACK is an essential element for effective teaching.

1.2 The rationale for the reform of calculus teaching

Studies show that students at both secondary and tertiary level find calculus difficult (Ahuja, 1998; Hourigan and O'Donoghue, 2007; Sonnert et al., 2015) and many students are underprepared for studies in calculus or have difficulties recalling pre-calculus materials (Hourigan and O'Donoghue, 2007). Service disciplines complain that students struggle to transfer skills learnt in mathematics (Roberts, 2004; Akinsola and Awofala, 2008) and are not able to apply these skills to solve problems (Roberts, 2004; Akinsola and Awofala, 2008). These difficulties lead to poor attitudes towards calculus (Machin and Rivero, 2002; Awang, Ilias, Che, Wan and Mokhtar, 2013; Bressoud, Carlson, Mesa and Rasmussen, 2013). The problems are compounded by students who are deemed to be underprepared for tertiary studies in calculus (Hourigan and O'Donoghue, 2007; Stroumbakis, 2010). Some experts blame an emphasis of procedural knowledge in pre-calculus to the detriment of the development of conceptual knowledge (Mahir, 2009; Zerr, 2010; Awang-Salleh and Zakaria, 2012; Zulnaidi and Zakaria, 2012; Muzangwa, 2013; Thompson, Byerley and Hatfield, 2013). All these problems contribute to poor pass rates in calculus (Hourigan and O'Donoghue, 2007; Maggelakis and Carl, 2007; Stroumbakis, 2010; Nelson, 2011; Ubuz, 2011).

1.2.1 Calculus

Calculus forms the most important part of the mathematics curriculum for engineers and scientists (Davis, Harrison, Palipana and Ward, 2005), and therefore deserves special attention. The word "calculus" is the Latin word for a small stone used for counting (Hake, 2013). Calculus is a branch of higher mathematics and originated in the sixteenth and seventeenth centuries. Kleiner (2001) claims that it is difficult to define calculus and that any definition is unlikely to capture the rich and multi-faceted nature of calculus. Kleiner further states that although calculus is difficult to define, modern day calculus can be viewed as the mathematics of motion and change and encompasses two main branches, differential calculus and integral calculus.

1.2.2 Importance and applications of calculus

According to Kleiner (2012), the invention of calculus "is one of the great intellectual achievements of civilization." Von Neumann claimed that "The calculus was the first achievement of modern mathematics and it is difficult to overestimate its importance" (von Neumann, 1960, p. 2). Almost every specialised field in science and technology applies calculus in some way. Calculus is the gateway to higher level mathematics courses, because it is used in all branches of science, and plays a major role in problem solving and inventions of all kinds. It is applied wherever variable forces are at work. Kleiner (2001, p. 138) describes its importance as follows:

> Calculus has served for three centuries as the principal quantitative tool for the investigation of scientific problems. It has given precise (mathematical) expression to such fundamental concepts as motion, continuity, variability and the infinite (in some of its aspects) – notions that have formed the basis for much scientific and philosophical speculation since ancient times.

Calculus is critical not only for mathematics but also for science and technology, its designs and applications. Calculus plays a major role in problem solving and inventions of all kinds. In the foreword of their book entitled *Calculus*, Finney, Thomas and Weir (1994, no pagination) describe some of the many and varied fields of applications of calculus such as economy, biology, oceanography, physiology, physics, psychology and engineering. Real analysis, complex analysis, differential geometry, and calculus of variations are branches of advanced mathematics that developed from calculus. “Calculus is also fundamental in probability, topology, Lie group theory, and aspects of algebra, geometry and number theory. In fact, mathematics as we understand it today, would be inconceivable without the ideas of calculus” (Kleiner, 2001, p. 138).

1.2.3 Pre-calculus

“The term *pre-calculus* is, at times, used to refer to any course which precedes calculus, but it is most frequently used for courses intended to prepare students for transition into calculus” (Stroumbakis, 2010, p. 9). In this study, pre-calculus refers not so much to a course, but more to the core mathematical skills required as a prerequisite for studying calculus.

1.2.4 First-year students are under-prepared for mathematics studies

In South Africa, “University teachers often complain that first-year students have little understanding of basic concepts of pre-calculus and even the high achieving students are only better in a procedural way of thinking” (Engelbrecht, Harding and Potgieter, 2005). First-year students in South Africa are undeniably under-prepared for studying mathematics courses (Padayachee, Boshoff, Olivier and Harding, 2011; Engelbrecht and Harding, 2015). It is regrettable, since deficiencies in knowledge, skills and academic proficiencies may mask students’ innate ability. Because of deficiencies in prior knowledge and the proliference of misconceptions, numerous students may be performing below their potential or even fail when they actually have the ability to pass (Malan, Marnewick and Lourens, 2010). The identification of at risk students, together with the development of programmes to prevent their failure, are necessary components of educational reform in universities (Donnelly, 1987). Furthermore, a current theme in the mathematics literature deliberates a progression problem related to numeracy, namely that students' difficulties with fractions have not necessarily been resolved by the time they enrol at university. Spaull and Kotze (2015) cite Taylor, Muller and Vinjevold (2003), who argue that any attempts to raise students' mathematical proficiency must first address deficits from prior knowledge if they are to be successful. One of the aims of RBAs is to acknowledge (lack of) prior knowledge and to attempt to bridge the gap between secondary and tertiary education. Fractions is a key threshold concept in mathematics and are prevalent in numerous topics in mathematics such as geometry, probability and trigonometry (Pienaar, 2014, p. 2). Proportional reasoning is deemed especially important (Bone, Carr, Daniele, Fisher, Fones, Innes, Maher, Osborn and Rockwell, 1984) and effects students understanding of key threshold concepts in pre-calculus and calculus such as trigonometry, slope, rate of change and derivative quantities (Cheng, 2010; Livy and Herbert, 2013; Ronda, 2015). Since these concepts are so basic and crucial in calculus, a lack of proportional reasoning skills may detrimentally effect students' conceptual understanding of introductory calculus. A study by the researcher revealed that entry-level students enrolled for Mathematics (M1) for science and engineering diplomas performed poorly in a test of numeracy skills and struggled with proportional reasoning. The average score (47.8%) in a test based on fraction skills was regarded as a cause for concern, especially considering that the test was pitched at a Grade eight level. The average score was far below lecturers' expectations. Furthermore, students displayed a lack of conceptual understanding of fractions and their applications (Coetzee and Mammen, 2016). Some fields of application require even higher levels of numeracy. Ramful and Narod (2014) reported that the proportionality tasks arising in chemistry were more complex and difficult than those covered in the mathematics curriculum for engineering.

Fisher and Scott (2011) argue that the under-preparedness of the student body as a whole, implies that traditional teaching approaches will not be sufficient, and that reform of teaching and learning is required. Outdated pedagogy, one aspect of so-called "staff under-preparedness", was identified by CHE as one of the issues negatively impacting access and throughput (CHE, 2010).

Not only are first-year students under-prepared for calculus studies, but fewer applications are received for calculus-based courses. The substantial decline in the number of applicants to calculus courses over the last 30 years seems to be a global phenomenon (Tall, Smith and Piez, 2008) and is a source of concern.

1.3 Difficulties with the learning and teaching of calculus

Learners and teachers all over the world struggle with calculus and its inherent complexities. Sonnert and Sadler (2014, p. 1) assert that “for many students, introductory calculus is a daunting course, one in which the pace is often quick and the coverage wide”. Literature abounds with studies on difficulties that learners experience with calculus at both secondary and tertiary levels.

In South Africa, the Diagnostic Report on the 2013 National Senior Certificate (NSC) Mathematics examinations mentioned the application of calculus as the section on which candidates performed most poorly. The average learner percentage for the questions on applications of calculus on cubic graphs was 27.6%, and for applications in rates of change, 28.3%. Even procedural skills in calculus were found lacking, as most learners struggled to find the derivative from first principles, even though it was also highlighted in a previous report. The performance on this question was however slightly better, at 49.6%. The Report further laments that “The conceptual understanding of the application of differential calculus is still seriously problematic” (page 9).

1.3.1 High failure rates in various calculus-based courses and programmes

High failure rates (between 35% and 40%) in first-year university mathematics have been reported internationally (Wilson, 1997; Wade, 2011). The US is one of those countries, having reported failure rates of between 40% and 50% in first-year mathematics over a long period (Wieschenberg, 1994). This amounts to a substantial number of students, considering the numbers registered for calculus at any given time. Nelson (2011) states that in any given semester, approximately 40% of all college students in the USA took calculus I. According to Bressoud (2015b), 300 000 students registered annually for a calculus course at a college or university in the US, whereas more than 750 000 students were enrolled for a calculus course at secondary level at the time. Nelson (2011, p. 1) cites the following statistics in her doctoral thesis:

> … in any given semester about 12,000 calculus instructors teach more than 750,000 students in 7,500 universities, colleges and high schools, at a cost of two hundred-fifty million dollars in tuition and fees and millions in textbooks. Those numbers were double the numbers of 1955.

Ferrini-Munday and Graham (1991) report that the failure rates for engineering calculus in the US were even worse: only 140 000 of the 300 000 who enrolled annually for engineering calculus passed. Some researchers attribute the high failure rates in calculus to lack of reform in mathematics teaching, others blame the lack of a unified mathematics school curriculum.

South Africa, like most countries internationally, has been struggling for many years with low first-year pass rates and high attrition rates. Student success rates in mathematics in South African higher education institutions have been unacceptably low and have contributed to low overall first-year pass-rates generally. According to Nicolene Murdoch, the executive director for teaching and quality at Monash South Africa, and the president of the South African Association for Institutional Research (SAAIR) at the time, the highest failure rates were in the maths and science programmes which covered medicine, science, technology and business studies (Mtshali, 2013). Luneta and Makonye (2010) cite Porter and Masingila (2000), who claimed that many students in university calculus classes possess a superficial and incomplete understanding of basic calculus concepts that most educators may not be aware of. South African students' poor performance at secondary level in mathematics and science is well-documented, as well as the subsequent under-preparedness of entry-level students**.** Poor performance of university students have therefore been blamed on unsatisfactory secondary level conditions and changes in school curriculums. Yet, in a study on poor pass rates of first-year engineering students at the University of Cape Town, Wolmarans, Smit, Collier-Reed and Leather (2010) came to the conclusion that there was a general decline in performance for the period 2005 to 2009, and that, contrary to general conviction, this decline was not necessarily linked to the introduction of the new NCS curriculum during that period. A study done by Potgieter and Davidowitz (2010) concurs with these findings. Poor retention and high attrition remain a national and international concern. However, Wolmarans et al. (2010) advised against simply raising entrance criteria, which would exclude a substantial number of students, especially since participation rates in higher education amongst the Black population for the 20-24 age group is still unacceptably low – measured at 13% in 2009 (Case, Marshall and Grayson, 2013).

First year mathematics courses, containing mostly calculus topics, act as a gateway to various diploma programmes at university. Failure in calculus, and therefore in first year mathematics courses, influences throughputs, retention and graduation rates of these programmes (Engelbrecht and Harding, 2015). The Council for Higher Education reported that only 5% of engineering diploma students completed their qualification in three years (CHE, 2007). Although the reasons for the high failure rates are complex and varied, some researchers ascribe blame to two main factors, namely under-preparedness of students and outdated pedagogies used by lecturers (Judd and Crites, 2014). No wonder Gangolli said “The mathematics profession as a whole has seriously underestimated the difficulty of teaching mathematics” (MER Workshop, May 31, 1991), as cited by Bressoud (2015a, p. 67).

1.4 An Overview of calculus education reform globally

Calculus education reform originated in the United States, and it is therefore fitting to introduce this section with a short history of the RM in the States. Hence an overview of the status of calculus education reform is presented, with reference to projects, activities and research from other parts of the world such as Australia, New Zealand, the United Kingdom, the Far East and Europe. The section concludes with an overview of calculus education reform in Africa and in particular South Africa. Lastly, concerns around calculus education reform is raised.

1.4.1. Calculus education reform in the United States of America (USA)

As already mentioned, the problems that learners and teachers experienced with calculus, coupled with low throughput rates and general dissatisfaction amongst mathematics teachers with students’ perceived lack of conceptual knowledge, kindled a *Calculus Education Reform Movement* (CERM) in the United States (Kaput, 1997; Stage and Kinzie, 2009), starting from the late eighties. Followers of the RM claimed that students were uninvolved in lectures and disinterested in calculus. Failure rates were high, retention rates low and students were unable to apply calculus to solve non-routine problems in client disciplines (Selden, Mason and Selden, 1989).

Furthermore, ConcepTests in physics and calculus showed that traditional instruction had remarkably little effect on basic conceptual understanding (Epstein, 2013). Claims were made that students who had completed courses in calculus, could not apply their knowledge and were unable to solve the most basic mathematical problems (Wilson, 1997). Various proposals were made on how to overcome the problems and innovative ideas were expressed to improve and enhance the learning and teaching of calculus. An important event in the reform calendar was the text compiled by the Harvard Calculus group, *Calculus: Single and Multivariable* (Hughes-Hallett, Gleason, Flath, Gordon, Lomen, Lovelock, Mccallum and Osgood, 1993), which is based on *the rule of three*—the concept that each topic should be viewed not just symbolically, but also numerically (often with data tables, sometimes with emphasis on calculator and computer applications) and graphically. The reform calculus text focused on discovery learning and the solution of "real-world" problems and aimed to encourage conceptual understanding of the calculus. This text has since been revised a number of times.

The general aims of the RM can be summarised as follows (Joiner, 1998):

(i) To facilitate active and interactive involvement of students and to enable them to take ownership of their own learning,
(ii) To make the learning environment more learner-centred,
(iii) To facilitate a move on the scale from the procedural end towards the conceptual end,
(iv) To highlight and emphasise the relevance of calculus,
(v) To encourage the use of multiple representations to aid the formation of rich connections amongst concepts,
(vi) To use collaboration and encourage communication, confidence and independence and
(vii) To promote problem solving abilities.

Although the RM has since infiltrated mathematics education globally, some educators do not approve of all changes made to calculus teaching. Opponents to the RM assert that students do not work as hard as in the past, and that calculus has been watered down to appear easy (Cipra, 1996). Critics further assert that students are over-dependent on technology and lack basic skills. One such critic is George E. Andrews, professor and Head of the Department of Mathematics at the Pennsylvania State University. He uses a musical metaphor to clarify his point of view: "But to learn the piano, you must learn scales and chords before you move to the 'Moonlight Sonata'" (Wilson, 1997, p. A13).

Initiating calculus reform efforts have however become widespread despite the criticism. By 1994, almost 70% of the 1048 institutions which offered calculus courses in the USA, had made some Reform-Based changes to the teaching of calculus (Windham, 2008). There is also substantial support for reforming undergraduate instruction in the mathematical sciences from influential bodies in various countries, such as the White House Office of Science and Technology Policy, the National Academies, and the Association of American Universities in the United States (Saxe and Braddy, 2016). In several other countries, government-funded programs support educational reforms such as Learning Support Centres in the UK and bridging programmes in Germany (Biza, Hochmuth, Khakbaz and Rasmussen, 2016).

Controversy regarding reform however remains rife. Some universities have even returned to traditional instruction following failed reform efforts or negativity from teaching staff towards reform efforts (Windham, 2008). "Among others, UCLA, USC, and the University of Iowa have all at one time or another scrapped efforts at reform" (Windham, 2008, p. 111).

Although numerous institutions have initiated some form of reform such as using a reform text, the pace of calculus reform at universities in the USA is still regarded as slow. University professors have apparently been slower to transform their pedagogies compared to school teachers. In a study by Schumacher and Kennedy (2008) it was found that school teachers out-performed university professors in the area of calculus education reform. Tuma and Reif (1980) quote Richard M. Cyert, former president of Carnegie Mellon University, who claimed "The academic area is one of the most difficult areas to change in our society. We continue to use the same methods of instruction, particularly lectures that have been used for hundreds of years".

1.4.2 Calculus education reform in Australia and New Zealand

Much research has been done in these countries on the use of constructivism in calculus learning and teaching. From New Zealand comes a valuable contribution of using counter-examples in calculus, which was found to improve students' conceptual understanding (Klymchuk, 2005).

Yost (2008) and Herbert (2013) challenged the traditional sequence of using differentiation as introduction to calculus. The researchers advocated an area approach to introducing integration and consequently refer to differentiation as anti-integration. They claimed that such an introduction to calculus will lead to fewer conceptual errors in the learners' constructions of the concepts involved.

Various researchers (Jacobs, 2005; Lim, 2008; Jungic, Kent and Menz, 2012) have investigated the use of computer aided instruction in calculus. Jacobs (2005) utilised interactive graphs and animations to enhance learning in a course on differential equations at the University of South Australia. Jungic et al. (2012) found the use of online assignments beneficial to students from the University of Wollongong. Lim (2008) encouraged school teachers to use Excel to allow learners to manipulate gradients numerically.

In a joint project between Germany, the Ukraine and New Zealand, project based research was used to teach applications of mathematics to first-year students in engineering (Gruenwald, Sauerbier, Zverkova and Klymchuk, 2005). The researchers used ecological models, endeavouring to show how relevant mathematics is to real-world environmental issues.

1.4.3 Calculus education reform in the UK

Jaworski and Matthews (2011) report on a project titled "Engineering Students' Understanding Mathematics" (ESUM) that attempted to raise the level of conceptual understanding displayed by first year engineering students in mathematics. Changes were brought about by using inquiry learning and utilising Geogebra for calculus studies.

In a study done by Masouros and Alpay (2010) at the Imperial College London, spatial visualisation, algebraic manipulations, abstract notations, complex numbers and mechanics were identified as problematic in engineering mathematics. Furthermore, lecturers complained about poor transfer of skills from mathematics to other learning areas and expressed a need for interactive graphics and tools to demonstrate or help visualise applications of mathematics in novel and appealing ways.

1.4.4 Calculus education reform in Europe

An international flavour was added to the conceptual versus procedural debate in a study by Jukić and Dahl (2012). Danish students were found to outperform the Croats on conceptual questions in calculus. The reverse was true for the procedural questions. These differences were ascribed to the teaching methods used in the two countries. In Denmark, as in most Scandinavian countries, a learner-centred approach was common, whereas teacher-centred approaches were still popular in Croatia. Furthermore, the retention of the Danish students was found to be superior to that of the Croatian students (Jukić and Dahl, 2012, 2014). Mahir (2009) added yet another dimension to this debate, when it was found that Turkish students who displayed conceptual understanding in integration, also performed successfully when tested in procedural questions.

1.4.5 Calculus education reform in the Far East

Asian learners have distinguished themselves in international studies in mathematics, from an early age to later pre-calculus and calculus levels. Much research has been focused on international comparisons in order to isolate causal factors for the excellent performance of learners from these countries. Although the research does not necessarily focus on calculus, pre-calculus proficiency is included in most of these studies.

Although East Asian students outperform their Western counterparts in mathematics, research does not indicate that this performance can necessarily be ascribed to what the West would view as superior education in mathematics. In a study titled "In search of an East Asian Identity in Mathematics Education", Leung (2001) asserted that mathematics teaching in East Asia was still predominantly teacher centred, content driven and old fashioned by Western standards, in other words, in need of serious reform from the constructivist point of view. Leung contrasted the Western and East Asian approaches to mathematics in terms of six dichotomies, and came to the following conclusions:

Product versus process: the Asians focus more on mathematics as a product versus the modern Western viewpoint of emphasising the process. According to Leung, East Asians believe that their Western counterparts have exaggerated the importance of the process point to the detriment of the product.

Rote learning versus meaningful learning: In contrast to the modern Western point of view, East Asians value rote learning, even if what is learnt is not yet fully understood. According to this point of view, understanding is a gradual process of improvement and growth and repetition and practice is a route along which these goals are eventually realised. “Learning is an interactive process of repeated practice, memorization and understanding” (Leung, 2001, p. 41). Furthermore, mathematics teaching in East Asia is less concerned with mathematics in context, and the usefulness and reality, than teaching in the West. East Asian teachers still teach mathematics as abstract subject matter, whereas constructivist Western teachers prefer to teach mathematics in context (Leung, 2005, p. 211).

Studying hard versus pleasurable learning: Asian parents stress the value of hard work. For East Asians, pleasure is not as much derived from playing games or doing enjoyable activities in mathematics, but from the innate satisfaction derived from hard work and a contentedness derived from achieving goals. Generally, East Asian teachers expect more from their charges from an early age. In a meta-analyses, Wang and Lin (2009) detected a gap opening up in the early grades between the mathematics performance of learners from China and learners from the US. The gap gradually increased in the later grades. According to Zhou and Peverly (2005), skills of argument and proof are introduced as early as the first grade in East Asian schools. Also, in Soviet texts as well as in Japanese classes, problems used were procedurally and conceptually more complex than those used in the US. However, some studies have pointed to the fact that East Asian students often become overburdened by their studies and the high expectations associated with these.

Extrinsic versus intrinsic motivation: In the West, educators treasure intrinsic motivation and negatively view extrinsic motivation caused by factors such as examinations. Conversely, East Asian education is highly competitive and centred around examinations. The difference between the social and economic status derived from academic achievement in East Asia is far greater than the status gained in the

West from comparative qualifications. East Asian educators thus often endorse and encourage the use of extrinsic motivation (Leung, 2001).

Whole class teaching versus individualised teaching: In East Asia, more emphasis is placed on the society than on the wellbeing of an individual, and it is expected that the individual's needs should fit into the collective structure. Whole-class teaching is therefore the norm in East Asia, whereas Western teaching caters more for the needs of the individual. The teacher as a role model is also of greater importance in East Asia than it is in the West (Leung, 2005).

Competence of mathematics teachers versus pedagogy: In East Asia, expertise in the subject matter of mathematics is regarded as more important than a teacher's pedagogical expertise. In the Western approach, the emphasis is on the teacher as facilitator of learning, and the pedagogical expertise of the teacher is thus more valued than in East Asia. That said, research (Ma, 1999; Leung, 2005) assert that East Asian mathematics teachers are more competent than their counterparts in the West, not only in terms of their knowledge of subject matter, but also in their choice of appropriate pedagogies. "They are able to provide clearer explanations, use teaching time more efficiently, develop smoother pedagogical flow, and engage students in inquiry using whole class instruction" (Wang and Lin, 2009). According to Ma (1999), the choice of an appropriate pedagogy is closely related, and not possible without, a profound understanding of the mathematics.

It is clear from this discussion that classroom practice is embedded in culture and the accompanying values of the society in which the learning and teaching takes place. This outcome has implications for decision making and policies. If the East Asian success story is culture dependent, then it implies that their success cannot be emulated by another culture by merely adopting their practices. Care has to be taken when deciding how much can or should be borrowed from successful pedagogies employed elsewhere. It is also clear that, contrary to the constructivist viewpoint, teaching practices which are widely regarded as old-fashioned and outdated are able to produce learners who excel in international competitions.

1.4.6 Calculus education reform in Africa

Africa still faces serious developmental challenges. The continent has thirty-four of the world's forty-eight poorest countries and struggle to contain diseases such as HIV/AIDS and malaria (Ottevanger, Van den Akker and De Feiter, 2007). The World Bank has been cooperating with international agencies in an attempt to achieve universal primary education and to expand secondary school access. Another aim is to develop science, mathematics and ICT (SMICT) in secondary education. A study by Ottevanger et al. (2007) includes data on 10 Sub-Saharan African countries: Botswana, Burkina Faso, Ghana, Namibia, Nigeria, Senegal, South Africa, Uganda, Tanzania, and Zimbabwe, and reveals a number of serious challenges in SMICT: poorly-resourced schools; large classes; outdated curriculums; a lack of qualified teachers; and inadequate teacher education programs (Ottevanger et al., 2007). In most of these countries, policies emphasise learner-centred education, but many studies reveal that in practice, this remains an ideal (Ottevanger et al., 2007). Despite serious challenges experienced in all levels of education, concerted efforts have been made by African countries to modernise higher education, especially teacher education (Obuobi, Adrion and Watts, 2006; Bass, 2007; Agyei and Voogt, 2011; Getenet, 2013). According to Getenet (2013, p. 1), the use of technology in the African education system has increased, together with the quality of education. Obuobi et al. (2006) however lament the increasing population growth and enrolment, inadequate infrastructure, poor connectivity, inadequate funding, inadequate educational resources and staffing, and a persistent *brain drain* of qualified instructors in higher education. They furthermore also lament the impact of globalisation, information technology growth, and international markets on the ability of African universities to adjust to the rapidly changing world. A report published in 2014 by the International Congress of Mathematicians (ICM) identifies these problems as common to most African countries: “Overall, the story of mathematical development in Africa is one of potential unfulfilled” (IMU, 2014, p. 2). University classrooms are often overcrowded and lecture-styles and curricula outdated and out of sync with career realities (IMU, 2014).

1.4.7 Calculus education reform in South Africa

Conceptual understanding of calculus, or the lack thereof, has been studied by a number of researchers, both at school level and at tertiary level (Smith, 1995; Bezuidenhout, 1998). Some attention has also been given to the value of technology to enhance the learning environment (Junqueira and Hay, 2006; Jaffer, Ng'ambi and Czerniewicz, 2007; Kizito, Wessels and Cortina, 2007; Howie and Blignaut, 2009; Marais, 2009; Kizito, 2012; Simelane and Skhosana, 2012; Stols, 2012; Mavhungu, 2013). The question arises: How much progress has been made in practice in South Africa in terms of transformation from the traditional pedagogies to the more modern RBAs in calculus teaching?

Brodie, Lelliott and Davis (2002) asserted that although learner-centred teaching was seen as an important element of the South African Curriculum Framework and Curriculum 2005, much teaching practice remained teacher-centred. They also claimed that teacher-centred practices were remarkably resistant to change internationally and supported these claims with citations from literature. In a study done more than 16 years ago in the USA, Wilson (1997) compared calculus exercises taken from a traditional textbook to exercises taken from a *reform* textbook. It is disconcerting that the examples taken from the traditional textbook are typical of the questions used in many of the assessments and texts used for teaching calculus for diploma courses in South Africa (researcher's own observation). It is an indication that calculus teaching for diploma courses is in need of transformation (Wilson, 1997). "Teachers tend to teach math and science in the way they were taught - presently in the ineffective passive-student lecture mode" (Hake, 2013).

Further evidence of the lack of transformation is evident from a detailed study of thirteen secondary schools from all over South Africa. Bernstein, Clynick and Lee (2004) found that the teaching methods used were traditional, teacher-centred and textbook-based. Hardly any teaching aids were employed in the teaching of mathematics other than white-boards. *Reform* teaching in mathematics is seemingly a scarce commodity, in our schools and possibly also at some universities.

South Africa in general has high-quality university education in mathematics (IMU, 2014), but many universities suffer from disadvantages imposed upon them by governments prior to 1994, and are struggling with overcrowding, poor facilities and lack of resources. In such environments, research and implementing new pedagogies may be a luxury that many lecturers cannot afford.

To summarise: It seems as if, despite the popularity of the RM, much still remains the same. In 2006, (Schoenfeld, 2006) wrote

> The “problem solving” movement of the 1980s arose partly in response to the realization that student mastery of the basics had not significantly improved after a decade of emphasis on core skills. Thus there have been changes over time—but core aspects of the traditional curriculum remain in place. Variants of the traditional curriculum (represented by the textbook series of the major publishers) remain dominant to this day. The strongest challenge to the domination of the traditional curriculum has come, in the past decade, from what are generally called reform or standards-based mathematics curricula (Schoenfeld, 2006, p. 15).

1.4.8 Concerns around calculus education reform

Despite successes claimed by the RM, controversy remains. Opponents to the RM assert that students do not work as hard as in the past, and that calculus has been watered down to appear easy (Cipra, 1996). Critics assert that students are over-dependent on technology and lack basic skills. A recent study done by Sonnert et al. (2015) in the USA reveals that the use of technology and ambitious pedagogical practices do not have a reliable, positive impact on students’ attitudes towards mathematics. These controversies resulted in conflict between the RM and the traditionalists, with reformers advocating pedagogies in line with constructivist theory about learning, whereas traditionalists on the other hand embrace understanding that comes through consolidation of mechanical skills.

1.5 Challenges of progression in calculus from school to higher education

In this section, the progression issue is first discussed from a global point of view, with specific reference to the USA. Hence an overview is given of the difficulties of progression from school calculus to university calculus in South Africa. Reference is made to the advice given by various experts regarding this matter.

1.5.1 International challenges of progression

Internationally, there has been concern about the apparent gap between secondary and higher education in mathematics (Gallimore and Stewart, 2014). Student difficulties with mathematics have been attributed to various factors such as “differences in teaching styles, instructional approaches, studying and learning strategies as well as views about mathematics, specific mathematical concepts, mathematical knowledge, and goals of learning” (Biza et al., 2016, p. 16). Stroumbakis (2010) identified the following causal factors: inadequate grounding; a gap in expectations regarding knowledge required for success in first year university courses; dissimilarities between high school and university teaching environments and poor teaching on the part of lecturers. Haskell (2001) views the problem as a vertical transfer problem, from prior learning to a new learning environment that is higher in the knowledge hierarchy.

The problem is by no means new. Almost two decades ago, Reyes, Anderson-Rowland and McCartney (1998) had already asserted that first-year engineering students from the UK struggled with mathematics. Since then, the London Mathematical Society (LMS, 1995) has also expressed concerns about the following two key areas: students displayed a lack of procedural fluency in numerical manipulation and simplification and secondly, students’ analytical skills have declined. Students seemed to lack an understanding of the precise and analytical nature of mathematics.

Although the problem of progression from school to university is not a recent one, there have been signs that it has worsened over the last decades. A marked decline in mathematical proficiency was observed by Lawson (1997) in the United Kingdom. Treacy and Faulkner (2015) tracked entry-level students in Ireland between 2003 and 2013. They found that the proportion of students at risk of failing their service mathematics end-of-semester examinations has increased significantly between 2003 and 2013. Furthermore, the performance of the entry-level students in 2013 was significantly below that of the performance of the beginning undergraduates recorded 10 years previously. Ten years prior to this study, Mustoe and Lawson (2002) cited a number of international studies that came to the same conclusion. They note that In 1998 the decline in entry competencies was the main theme under discussion at the

Mathematics Working Group (MWG) of the European Society for Engineering Education (SEFI) in Finland. Also, Nishimori and Namikawa (1996) reported that 78% of university lecturers in Japan were of the opinion that entry-level students' mathematical abilities have declined. The Japanese educators ascribed the decline to the increase in student numbers and trends in society. Elsewhere educators attributed the progression difficulties to the expansion of higher education and the more modular approaches recently adopted by secondary schools in an attempt to popularise their mathematics courses. As far as 15 years ago, the trend with respect to curricula seemed to be towards a more utilitarian, career-oriented curriculum, associated with continuous assessment practices that favour calculations at the expense of proof (Hoyles, Newman and Noss, 2001). Regardless of the country of origin, most educators seem to agree that students nowadays prefer a more intuitive approach to mathematics, compared to the rigour and precision displayed by and expected of past students.

Duah, Croft and Inglis (2014) employed Peer Assisted Learning (PAL) to assist students in their transition to university mathematics studies and to counteract a potential reduction in motivation. In response to the progression problem, universities worldwide have introduced more prescriptive admissions criteria in addition to support programs. Remedial programs in mathematics for first-year students have become commonplace. Some of these are web-based and encompass features such as formative and summative testing, and features to encourage collaborative learning and student feedback (Masouros and Alpay, 2010). It is interesting to note that students, who took part in this particular study conducted in the UK, did not find mathematics more problematic than their other courses. Furthermore, students with an A-level in Further Mathematics courses struggled less with mathematics than students who held a lower entrance qualification. This study indicated that higher entrance criteria resolved some of the progression problems. Surprisingly though, even students with higher entrance qualifications were found lacking in core-level, basic mathematical skills when entering higher education (Masouros and Alpay, 2010, p. 63). An added complication is the lack of long-term retention of skills and knowledge. In a study at 15 universities in the UK, even students with mathematics majors retained less than 20% of the first-year concepts after the completion of their final, third year studies (Anderson, Austin, Barnard and Jagger, 1998), as cited by Tall et al. (2008).

1.5.2 Challenges of progression in calculus in the USA

No national policy existed in the past in the USA on the content of the mathematics curriculum to prepare learners for tertiary studies in calculus. Each state has the right to decide independently on their school mathematics curriculum (Wade, 2011). Despite the introduction of the Common Core State Standards (CCSS) in 2010, consensus has neither been reached on the number of credits required for tertiary studies in calculus, nor on the contents of these modules (Wade, 2011). Despite years of research, agreement on articulation is still out of reach (Stroumbakis, 2010).

1.5.3 Challenges of progression in calculus in South Africa

Low pass rates and progression issues are inextricably linked, especially in South Africa. Various problems with secondary education have been widely reported and these effect students' transition to university. Case et al. (2013) cite various studies done in South Africa on high failure rates amongst first-year students. Reports that go as far back as 1936 attribute the failure rates to progression difficulties. Seemingly, not much has changed, other than the massification of higher education. Various proposals have been made by researchers in an effort to address low pass rates and progression issues. Government initiatives have centred on extra study time, which initially was cast as pre-diploma or foundation courses at the Universities of Technology (UoTs), but lately as part of an extended curriculum, especially for science and engineering. In 2006 the Department of Education instructed universities to restructure their access activities into credit-bearing extended programmes (Grayson, 2010).

In mathematics education, some researchers advocate pre-calculus courses to prepare students for their first-year studies. Various tertiary institutions offer such a course at the beginning of the academic year (Case et al., 2013). The efficiency and value of these courses have however not been researched sufficiently. Other initiatives include pedagogical practices such as enquiry based or problem-based learning, active student engagement, student-centred teaching approaches, attempts to improve students' conceptual understanding and their academic literacy and the promotion of collaborative learning. Engelbrecht, Harding and Phiri (2010) argued for a slower teaching pace at the first year level. They advised that course material be reduced at the entrance level and have consequently made alterations to the first year mathematics courses at the University of Pretoria. In a study done by Grayson (2010), students asserted that they were mostly unprepared for the amount of work expected from them in their first year and were furthermore unequipped for the degree of difficulty of lectures and the quick pace of lecturing. Additionally, students complained about the lack of support they received from lecturers.

In response to these issues, Blackie (2010) concluded we should all examine the efficacy of our introductory courses in higher education. Both Grayson (2010) and Blackie (2010) advised that work covered in the early stages, should be more familiar and less voluminous and that the lecturing pace should initially be slower and should increase gradually. As early as twenty years ago, Tinto (1993, p. 45) argued that the following principles should be addressed when attempting to improve pass rates: improved curriculum design, increased classroom interactions, accommodating diversity, financial support, helping students to develop a sense of belonging, appropriate placement of students to match course content to their abilities and developing student preparedness. Levits, Noel and Richter (1999) in turn advised that the following approaches should be considered to cultivate success:

(i) The institution creates a student success structure.
(ii) Intensive contact with 'at-risk' students.
(iii) Academic advisors that understand the needs and motivational levels of students.
(iv) Lecturers that are actively engaged with the students and take initiative for the relationship.
(v) Attention being paid to the adaptation and individual needs of the students.
(vi) Celebration and recognition of successes by members of staff who are actively dedicated in their quest to motivate students to move to the next level.

In a report on the 2013 National Senior Certificate, the authors lament the lack of basic knowledge of concepts, a prerequisite for higher level thinking (DBE, 2013). Pinker claimed that Mathematics is "ruthlessly cumulative, all the way back to counting to ten" (Pinker, 1998, p. 342). When this reality is perceived in the context of the extensively published problems experienced in mathematics and science in secondary education in South Africa, a collaborative effort is called for in order to find optimum ways to teach mathematics in higher education. One way of improving the teaching and learning of mathematics in higher education is by using RBAs in the teaching of calculus. As mentioned elsewhere, calculus education reform pedagogies align calculus instruction more closely with theories of how students learn (Stage and Kinzie, 2009) and may be employed to address misconceptions and improve conceptual understanding.

1.5.3.1 Changes in the mathematics curriculum in South Africa since 1994

Major changes have been made to the school curricula in South Africa since 1994. These changes were initially triggered by the necessity to revamp the curriculum used under the former apartheid government and were therefore politically driven as part of a democratising drive (Jansen).

An interim core syllabus, which came into effect in 1995, was replaced by Curriculum 2005 (C2005) in 1998. Learner-centred teaching as advocated by RBAs, was an important element of the South African Curriculum Framework and Curriculum 2005 (Brodie et al., 2002). C2005 underwent a few reviews, and problems experienced with C2005 led to a final, major curriculum review in 2000, which consequently produced the Draft National Curriculum Statement (NCS) in 2001 and the subsequent Revised National Curriculum Statement (RNCS) of 2002. Learner-centred education, one of the features of RBAs, is one of the core concepts of the new curriculum, as stated in various official documents:

> Curriculum reform perspectives in mathematics education articulated in many research papers and policy documents aim at deepening and increasing each learner’s mathematical learning and achievement (National Curriculum Statement (NCS), 1998; National Mathematics Advisory Panel (NMAP), 2008). The perspectives suggest shifts from teacher-centred to learner-centred approaches (Luneta and Makonye, 2010).

In January 2012, the Curriculum and Assessment Policy Statement (CAPS) was launched in the FET Band and is currently still in effect.

1.5.3.2 Outcomes Based Education in mathematics and some parallels drawn to reform calculus in the USA

Some parallels can be drawn between the CERM and opposition against it in the USA and the introduction and demise of Outcomes Based Education (OBE) in SA. In a paper titled "Are OBE-Trained Learners Ready for University Mathematics?" (Engelbrecht et al., 2010), the researchers describe the 2009 intake of university students as more confident, but weaker than past students with respect to their mathematical skills. This deficiency was evident despite the exceptionally good Grade 12 results achieved by this group of learners in mathematics at the end of their final school year. In contrast, the first-year mathematics results of the 2009 matriculants, were exceptionally poor and pass rates were significantly lower than in previous years. The Department of Education (DE) introduced OBE after serious concerns were voiced in the mathematical community about learners' lack of problem solving ability and critical reasoning skills. OBE was supposed to address these concerns and turn out students who were able to compete internationally. The 2009 intake of students was the first group that was fully schooled in OBE. Some of the criticism levelled at OBE, was harsh. Engelbrecht et al. (2010) quote a couple of articles from the popular press: Roodt (2009) wrote an article titled *When A stands for abysmal*, lamenting the apparent low standards and Smith (2009) asserted in the Saturday Star that the OBE system produced *confident illiterates.* OBE has since been phased out and was replaced by the NCS.

1.5.3.3 The foundation for calculus in the school curriculum in South Africa

As mentioned elsewhere, Pinker (1998, p. 342) claims that "Mathematics is ruthlessly cumulative, all the way back to counting to ten." It is therefore necessary and informative to look at the foundation of calculus, from Grade R onwards. Any deficits in learning and teaching will be carried to the higher grades where it will exacerbate existing and new difficulties.

South African learners display a skills deficit from early on in their schooling career. The section on Numbers, Operations and Relationships is the main focus of Mathematics in Grades R - 3. Apart from basic concept development, it is difficult to identify specific skills in this section that are directly linked to calculus, which is only introduced in Grade 11. However, in a project launched in 2001, the Department of Education (DE) evaluated the numeracy and literacy skills of a 5% sample of grade 3 learners, from urban schools, farm schools and rural schools all over the country (Bernstein et al., 2004). The learners' average score on the numerical test was 30%. Already, after only 4 years of learning and teaching, serious problems are apparent in concept development. Furthermore, in Grade 4 tests conducted from 1998 to 2002 by the Joint Education Trust (JET), learners scored an average of 30% in numeracy, the lowest of 12 countries tested.

By grade 8-9, the gap between the South African learners and the others has widened considerably (Bernstein et al., 2004). In the intermediate phase, the number range is extended to 9-digit whole numbers, decimal fractions to at least two decimal places, common fractions and fractions written in percentage form. Again, the link between specific skills in this phase and calculus is not an apparent one, but it is fair to say that the development of the concept of fractions proves to be unsatisfactory. These difficulties may later effect students' conceptual understanding at university level (Norton and Hackenberg, 2010; Bressoud, 2016; Coetzee and Mammen, 2016).

Four cognitive levels are used in the FET curriculum to guide assessment. These are knowledge (20%), routine procedures (35%), complex procedures (30%) and problem solving (15%). Students struggle with complex procedures and problem solving. This observation is supported by research done in South Africa as early as 1987 (De Villiers and Njisane, 1987). In the JET tests conducted from 1998 to 2002, the grade 9 learners scored an average of 22% in mathematics and the Grade 11 learners 20% (Bernstein et al., 2004). By grade 8-9, the gap between the South African learners and learners from most other countries has widened considerably. Very poor results were also obtained on Grades 5, 6, 7, 9 and 11 (Bernstein et al., 2004). All the tests had to be simplified after results from pilot studies indicated that learners could not cope with the degree of difficulty of the original tests. Algebra typically represents the students' first encounter with abstract mathematical reasoning and it therefore causes significant difficulties for students who has not developed beyond Piaget's concrete reasoning stage (Susac, Bubic, Vrbanc and Planinic, 2014).

In the Third International Mathematics and Science Study-Repeat (TIMSS-R), conducted in 1998–9, South Africa scored the lowest of the 38 countries in maths and science, and also in unemployment. South Africa however had the highest score on educational expenditure (Bernstein et al., 2004). It is apparent that problems surrounding mathematics education start at a very early stage in the education system. It is therefore also clear that teaching in mathematics at all levels will have to address the accumulative shortfalls of the previous years. Teachers should spend time and effort reflecting on effective pedagogies to overcome difficulties and to eliminate misconceptions. This will probably be more effective if done collaboratively by involving calculus teachers from all levels, starting from school level. Case (2006, p. 15) urges all concerned with engineering education not to wait for schooling to improve, since it will not happen overnight, but instead to take responsibility for the students that do get admitted.

1.5.3.4 Time allocation and utilisation in South African schools

One aspect that causes problems in South African schools, is lack of time on task. The time allocation for mathematics is specified in CAPS document (DBE, 2011) and is depicted in Table 1.1.

TABLE 1.1: Time allocation for Mathematics in CAPS

CAPS	**Time allocated to Mathematics per week**
Foundation Phase (Grade R-3)	7 hours
Intermediate Phase (Grades 4–6)	6 hours
Senior Phase (Grades 7-9)	4.5 hours
FET Phase (Grades 10-12)	4.5 hours

In an article titled “The gap between the Implemented and Intended Grade 10 to 12 Mathematics Curriculum”, Stols (2011) asserts that, out of a total of 150 teaching days (this allows for 49 days for assessment throughout the year), Grades 10, 11 and 12 teachers spent on average only 48.5, 56,6 and 53.9 days actively teaching Mathematics (Table 1.2). Most topics were therefore not covered sufficiently, and the most neglected topics in Grade 12 were functions and graphs, applications of differentiation, linear programming and solving triangles in three dimensions. Stols (2011) quite rightly laments the effect that this oversight has on higher education, because of the importance of functions and calculus in most branches of mathematics in higher education. As already mentioned elsewhere, Mathematics is cumulative and therefore the topics identified in Table 1.2 serve as a foundation for calculus.

It has to be mentioned that the phenomenon of ineffective use of allocated teaching time is not peculiar to South Africa. According to (Abadzi, 2007), it is a common occurrence in many third world countries that syllabi are not adequately covered as planned. Samuelsson (2008) stressed that for effective teaching to take place, the number of learning opportunities has to be maximised. These include the number of school days in a year, the amount of hours dedicated to mathematics, and the time-on task. Learning will be effected if this aspect of time management is compromised.

In addition to the problem of teaching too few days, problems have been identified with the quality of the teaching in secondary schools in South Africa. Teaching activities focused on procedural fluency and repetition of tasks, and very little time was spent on activities that demanded higher order cognitive skills (Engelbrecht et al., 2009; Stols, 2011). This is in sharp contrast to the teaching of mathematics in East Asia, where more time is spent on procedurally complex problems (Leung, 2005, p. 204). Teachers also did not teach topics that were not assessed in the Grade 12 syllabus, for example volume and surface area and the completion of the square.

TABLE 1.2: Intended versus Actual number of teaching days in Grade 12

Grade 12 mathematics topics	Intended no of days	Implemented no of days
Transformations	4	2.00
Solving triangles in three dimensions	9	1.00
Compound and double angles	10	5.80
Analytical geometry	10	3.40
Linear programming	4	1.00
Tangents	2	0.50
Cubic graphs	4	1.50
Max and Min problems	3	0.50
Differentiation	10	4.25
Inverses	4	0.75
Transformation of graphs	5	0.00
Functions and graphs	5	2.25
Financial mathematics	6	3.75
Patterns and sequences	15	11.50
Cubic equations: factor and remainder	4	2.50
Logarithms	4	5.00

Source: Stols (2011)

Furthermore, teachers wasted time teaching topics that were in the old syllabus, but did not appear in the new syllabus. These oversights all directly impacted the conceptual understanding of calculus as taught in higher education. University lecturers of calculus should take cognisance of this fact, and attempt to address the gaps. According to Spaull and Kotze (2015), skills deficits will impair future performance and subsequent learning.

1.6 An introduction to STEM Higher Education in South Africa

Four types of engineering professionals are recognised in South Africa, namely engineers and certified engineers, engineering technologists and engineering technicians. Their respective qualifications are: B.Sc. (Eng) or B.Eng/Ing for engineers, B.Tech for technologists and National Diploma (ND) for technicians.

Engineers have to pass a Government Certificate of Competency examination to become certified engineers, which will enable them to do work reserved for professionals with this qualification. The Engineering Council of South Africa (ECSA) is the statuary body responsible for registering engineering professionals and for granting accreditation to programmes at universities.

In South Africa, there are three types of universities, namely traditional universities, universities of technology (formerly called technikons) and comprehensive universities. There are currently 23 universities in South Africa, of which 11 are traditional, six are Universities of Technology (UoT's) and six are comprehensive institutions. Engineering programmes are offered at six of the traditional universities, at eight of the UoT's and at one comprehensive university. Traditional universities offer engineering degrees and universities of technology offer national diplomas as their basic qualification, and the B.Tech at the next level. The engineering degrees comprise 4 years of academic study. For the national diploma, students have to complete four semesters of academic study and two semesters of experiential training in industry. For the B.Tech degree, an additional two academic semesters are required.

In the past, only one technician was trained for every engineer. This was an imbalance that the government has since attempted to address. Since 1994 the number of students enrolled for engineering diplomas at universities of technology, has increased substantially (Case, 2006). The low throughput rates have however continued to be a source of concern.

1.6.1 Mathematics curricula for Science and Engineering

The Mathematics curricula of the former technikons were formulated and adjusted at three conferences that took place in the late eighties and early nineties, the first at Scottburgh in 1987, the second at Witwatersrand Technikon in 1991 and the third at the same venue in 1993 (Smith, 1995). The curriculum is semester based and a maximum of sixteen weeks are allowed to cover the syllabus, although fewer weeks are usually available because of various interruptions (researcher's own observation). Although there are slight differences between the universities, the syllabus for the first M1 at the university where this study was conducted, included determinants, logarithms, exponents, trigonometry, complex numbers, statistics, differential calculus and an introduction to integral calculus. The differential calculus covered basic principles and the various rules used for differentiation, as well as applications of differentiation such as graphs, rates of change and optimisation. Calculus therefore comprised at least 50% of the M1 syllabus and the rest was pre-calculus topics. The syllabus for engineering mathematics for the second course in Mathematics (M2) included matrix algebra, partial differentiation and more advanced integral calculus and some of its applications. Calculus comprised at least 70% of the M2 syllabus, and this proportion increased for the higher levels of engineering mathematics. The syllabus for the third semester (M3) included La Place transforms, more advanced techniques for solving differential equations, Fourier analysis, numerical methods and Z-transforms. The fourth course (M4) syllabi differ increasingly amongst universities, as the course is compiled to accommodate the needs of the various departments serviced. Most engineering students are required to take at least one, but usually two, semester courses (M1 and M2) of mathematics, and some programmes require three semesters of mathematics for the national diploma, such as Electrical Engineering. Since the nineties, when Technikons were converted or incorporated into universities, syllabi have diverted somewhat, yet engineering mathematics syllabi at the former Technikons do not seem to have changed much (researcher's own observation). A study conducted by Gaisman, Martínez-Planell and McGee (2016) on engineering mathematics reported that students regarded mathematics teaching as too theoretical, and complained that it did not connect sufficiently with other sciences and the engineer's eventual employment needs. Therefore, modernised curricula were called for.

1.7 Rationale and problem statement

In South Africa, most science and engineering diplomas incorporate a minimum of two calculus based semester courses in mathematics. The questions arise: What is the most effective way to learn and teach calculus? Which features of teaching are both essential and sufficient for calculus as a service course for NDs? Considering both the importance and the complexity of calculus, Zhang (2003) laments that the way it should be taught and learnt remains problematic, despite numerous research studies conducted on the topic.

Conceptual knowledge has been identified as critical in science and engineering (Engelbrecht, Bergsten and Kågesten, 2012). As mentioned elsewhere, traditional, teacher-centred approaches to teaching calculus result in low levels of conceptual understanding of calculus topics (Zhang, 2003), a proliference of misconceptions (Luneta and Makonye, 2010), and high drop-out and failure rates in calculus courses (Petrillo, 2016). Reflection on the difficulties experienced with learning and teaching of calculus nationally and internationally has indicated a need for RBAs for the teaching of calculus (Kueffer and Latterell 2001; Stage and Kinzie, 2009). These approaches are learner-centred and constructivist based, reveal misconceptions, and result in levels of conceptual understanding of the material that have not been attained by other, more traditional pedagogical approaches (Figure 1.1). Furthermore, improvements are seen in student attendance, engagement, and learning (Duah et al., 2014) and improved pass rates (Hieb, Lyle, Ralston and Chariker, 2015). Higher levels of conceptual understanding also improves the retention of mathematical knowledge (Engelbrecht, Harding and Du Preez, 2007). RBAs furthermore emphasise real-world applications of calculus, affording students an opportunity to see that calculus is in fact useful and important (Murphy, 2006).

In this study, clickers and GQ were used to facilitate AL combined with PD. Caldwell (2007, p. 4) summarises observations made by other researchers and hence lists a few advantages of using clickers: utilisation of clickers improves the atmosphere of lectures; students become active and more attentive; students feel less isolated when they struggle with the content and attend lectures more regularly, especially when participation is graded.

No studies could be located which focused on IEM used for teaching calculus as a service subject for diploma studies. Only one South African study which utilised clickers to teach Mathematics as a service course in a South African university could be found (Simelane and Skhosana, 2012). The latter study however did not mention GQ, and was not focused on calculus. It also did not specifically focus on AL and PI. The study furthermore did not use ConcepTests as an assessment tool. The contribution of the current study will be to investigate the effect of GQ, clickers, PI and AL on the conceptual understanding of calculus for Science and Engineering students registered for NDs.

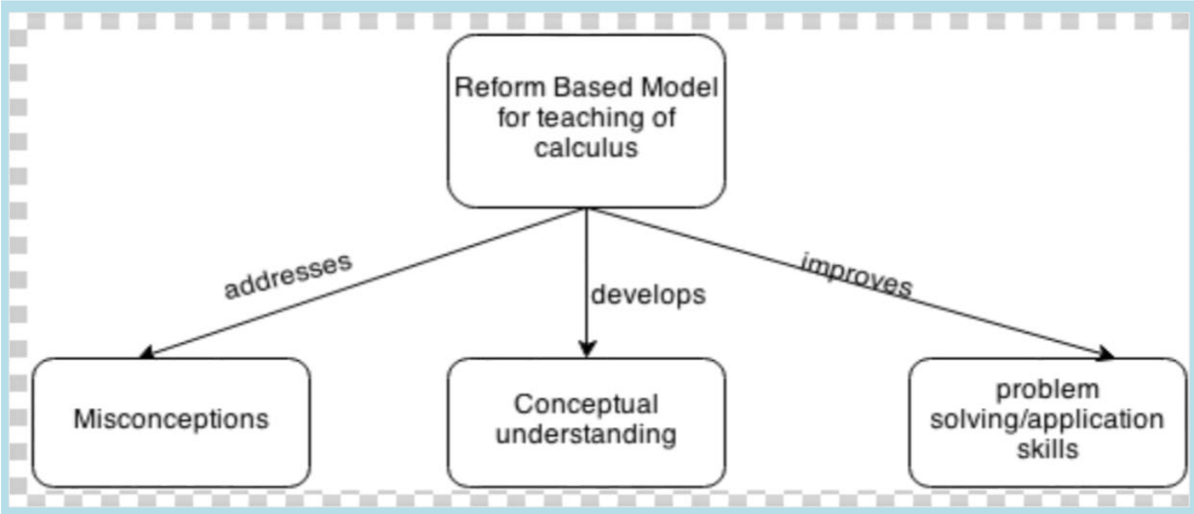

FIGURE 1.1: Reform-Based Approaches (RBAs)

Hence this study focuses on the effectiveness of RBAs for the teaching of calculus for NDs in science and engineering. Although similar studies have been conducted internationally, no study could be located which focused on IEM used for teaching calculus as a service subject for ND students in South Africa.

1.8 The research question:

The aim of this study was to determine whether a specific combination of RBAs applied to the teaching of calculus will affect positive change and bring about higher cognitive levels in the learning of calculus by students of the ND in Engineering and the ND in Analytical Chemistry at a state university in South Africa.

1.8.1 The main research question

Do RBAs in teaching improve conceptual understanding of calculus for ND studies in science and engineering?

1.8.2 Hypotheses associated with the research question

H_0: RBAs (as opposed to Traditional Approaches) used to teach calculus for science and engineering diploma students have no effect on students' conceptual understanding of calculus.

H_a: RBAs (as opposed to Traditional Approaches) used to teach calculus for science and engineering diploma students has a positive effect on students' conceptual understanding of calculus.

1.9 The purpose of the study

The purpose was to investigate whether a specific selection of RBAs utilised in the teaching of calculus, would improve students' conceptual understanding of calculus as a service subject for science and engineering diplomas. A minimum requirement for calculus lecturers is the need to have appropriate knowledge of, and training in, mathematics. However, calculus lecturers also need pedagogical skills. A literature search was done to identify the most important pedagogical approaches associated with RBAs. The approaches selected for this study were IEM and GQ. An intervention was designed, incorporating these features. A ConcepTest was employed to measure levels of conceptual understanding.

1.10 The objective of the study

The objective of this study was to test the effect of an intervention designed to improve students' conceptual understanding of calculus.

1.11 The significance of the study

The importance of calculus cannot be overemphasised. Calculus is a gateway for fields such as science, technology and engineering. Not only are current pass rates a source of concern, but also the low cognitive levels of understanding displayed by students of calculus. According to Epstein (2007, 2013), a surprising proportion of students displayed little or no understanding of the concepts that their lecturers assumed they knew at the end of an instructional module. A semester of instruction apparently made little or no difference to students' conceptual understanding of calculus concepts.

This study will not only create an increased awareness of RBAs in the teaching of Mathematics as a service course, but will also focus attention on ConcepTests and IE. As already stated, literature shows that these pedagogies have resulted in much improved learning and teaching internationally and it is hoped that these methods will also result in improved cognitive levels in mathematics as a service subject for diploma courses at universities in South Africa. It may be significant to point out that reformed calculus appears to be especially beneficial for students with a weak mathematics background.

Equipping students with critical thinking skills have the potential to lower high unemployment rates and alleviate related social problems. It has been shown that skills levels in mathematics can be used to predict future qualifications and levels of job satisfaction and income (Rivera-Batiz, 1992). Mathematical competencies can lead to higher paying jobs, and thus influence students' economic mobility. Furthermore, the shortage of engineers and scientists worldwide is a well-documented fact. South Africa needs these professionals to develop the country and improve its economy.

1.12 Research methodology

1.12.1 The design of the study

The study utilised a quasi-experimental research design. Pre-testing and Post-testing with experimental and control groups were conducted at the start and end of the lecturing period of the two semesters in 2015 when the intervention was done.

1.12.2 Study site

The university where the study was carried out, has four campuses. The research was conducted at two of the campuses where the NDs in Engineering and Analytical Chemistry were offered.

1.12.3 Sample/subject size

The experimental group comprised of 119 volunteering students from a population of 461 registered for Mathematics as a service subject for ND in science or engineering at a South African university. Those not in the experimental group were taught through teacher-centred traditional approaches which have been the norm. However, only 71 out of those in the traditionally-taught cohort volunteered to write both pre- and post-tests. As such, the total number of subjects in the study was 190, i.e., 119 from the RBAs cohort and 71 from the traditional cohort.

1.12.4 Instrument

The instrument was the Calculus Concept Inventory for Technicians (CCIT).

1.12.5 Data analyses

The data were statistically analysed using various statistical tools, tests and measures such as Chi-squares, Student t-tests, Pearson's Product Moment correlation, Cronbach alpha, KR-20, the Difficulty Index, and Item Discrimination Point Biserial Index (PBI). The raw gain and normalised gains were also employed in data analyses

1.13 Methodological Limitations

A non-equivalent control group design was used in the study. Almost by definition, these designs suffer from potential sample selection biases. The experimental groups were selected from one campus and the control groups from another, except for one control group, the class EC. This group was however located at a delivery site 30 km from the experimental group. The contamination effects were therefore kept at a minimum, since the control groups had minimal interaction with the experimental groups. This design may also have had other influences on the study, such as unique campus cultures that may have influenced the results of the research in non-obvious ways.

1.14 Outline of chapters

This thesis contains five chapters and are organised as follows:
Chapter One introduced the study and presented a background of the study, highlighting some of the factors that motivated the researcher in commencing the study, such as the need for calculus education reform. The RBAs used in this study were described. An overview of the challenges of progression from school mathematics to higher education calculus was presented. The objectives of the study, the research questions, the scope, a brief on population and sample, instrument, data analyses and delimitations of the study and the significance of the study were summarised.

Chapter Two provides a critical review of the theoretical framework and pertinent literature applicable to this study.

Chapter Three describes the research paradigm, approach and design. The population, sampling methods and data collection instruments are discussed. Issues such as validity and reliability; data collection procedures, analysis and ethics are addressed.

Chapter Four provides a detailed report of the results obtained during the collection of the data. Data are presented, interpreted and analysed.

Chapter Five presents a summary of the thesis and concludes with recommendations related to the research findings.

1.15 Definitions of operational terms

Active Learning: The process of keeping students mentally, and often physically, active in their learning through activities that involve them in gathering information, thinking, and problem solving (Thomas, 2014a).

Active Learning is the increase of student participation, or ‘interactivity’, for the purpose of positively affecting student learning and attitudes (Georgiou and Sharma, 2015).

In this study, Active Learning refers to engaging students in activity that supports reflection upon important concepts, in other words, learning by participating or contributing.

Reform-Based Approaches (RBAs) to teaching:

A hands-on, student-centred approach to learning and teaching, utilising computers and graphical methods in order to facilitate understanding and connection of complex mathematical concepts (Schoenfeld, 1995; Wilson, 1997).

Reform teaching has a change in instruction mode and/or implementation and integration of technology (Kueffer and Latterell 2001).

Calculus reform is defined as any attempt to move away from traditional modes of calculus instruction, including but not limited to the use of a ‘reform’ text (one whose purpose is to pursue the doctrines mentioned above), integration of technology into the calculus curriculum, and innovations beyond a lecture monologue in the delivery of instruction (Windham, 2008, p. 2).

For the purposes of this study, RBAs are active, learner-centred, constructivist approaches that incorporate technology in order to improve the learning and teaching of mathematics and emphasise conceptual levels of learning and teaching.

Calculus: The mathematics of motion and change (Sokolowski, 2014) and encompasses two main branches, differential calculus and integral calculus.

Calculus Education Reform Movement: movement in the USA (Kaput, 1997; Stage and Kinzie, 2009), starting from the late eighties, advocating RBAs.

Clickers: student input devices or electronic voting systems (Kenwrigth, 2009; Briggs and Keyek-Franssen, 2010; Kolikant et al., 2010; Jurukovski et al., 2015)

Concept Inventories: “Concept inventories are tests designed to measure the most basic knowledge in a field” (Thomas, 2014b, p. 1079).

ConcepTests: A test consisting of short, conceptual questions (Mazur, 1997), which are usually in multiple choice format.

Conceptual understanding: A network connection of knowledge structures. Conceptual understanding is thus a student's ability to apply and connect what he or she is currently learning to prior learning and other related fields (Hiebert and Carpenter, 1992).

Conceptions: meaningful features of knowledge specific to understanding a particular domain. Conceptions are in transition, and therefore often not fully developed, fragile and situation-bound (Leinhardt, Zaslavsky and Stein, 1990)

Conceptual Knowledge: knowledge that is rich in relationships (Hiebert and Carpenter, 1992, p. 3).

ConcepTests: Conceptual tests measure conceptual understanding of fundamental principles. Conceptual tests usually do not have a computational component (Hake, 2013).

Extended Stream: Students who do not qualify academically for the entrance requirements of their chosen course are allowed extra time to complete the course. Extended stream courses qualify for additional government subsidies.

Good Questions: multiple choice questions that stimulate discussion and help students and lecturers to probe and reveal misconceptions (Terrell, 2003; Miller et al., 2006)**.**

GoodQuestions: A project initiated in 2005 at Cornell University in the United States (Terrell, 2005). The project seeks to improve calculus instruction by adapting two methods developed in physics instruction, namely ConcepTests and Just-in-Time-Teaching. GoodQuestions is a pedagogical strategy that aims to raise the visibility of the key concepts and to promote a more AL environment.

Information and Communications Technology (or technologies): an umbrella term that includes any communication device or application, encompassing radio, television, cellular phones, computer and network hardware and software, satellite systems and so on, as well as the various services and applications.

Interactive Engagement: Heads-on (always) and hands-on (usually) activities which yield immediate feedback through discussion with peers and/or instructors (Hake, 1998). Interactive Engagement involves Active Learning with the emphasis on Peer Interaction.

Interactive Engagement Methods: methods that include activities designed to promote conceptual understanding through Interactive Engagement of students (Hake, 1998).

Mainstream: Students who qualify academically for the entrance requirements of their chosen course are placed in the mainstream.

Misconception: A feature of a student's knowledge which may develop as a result of overgeneralizing or may be due to interference from everyday knowledge. A misconception must have a reasonably well-formulated theory, it should be repeatable and/or explicit rather than random and tacit. They are usually robust and usually develop as a result of prior teaching (Leinhardt et al., 1990).

Peer Instruction: Peer Instruction/Discussion or “pair/share” is a learning opportunity during which small groups discuss questions and defend their point of view. Peer interaction provides students the opportunity to collaborate with their classmates, explain their solution methods, and construct knowledge. In this study, the term Peer Instruction is used interchangeably with Peer Discussion, but the term Peer Discussion in preferred, since the other person(s) in the “pair-share” group is/are not necessarily more knowledgeable.

The Rule of Three: Every topic should be presented geometrically, numerically, and algebraically (Hughes-Hallett, Gleason, McCallum, Flath, Lock, Tucker, Lomen, Lovelock, Mumford, Osgood, Quinney, Rhea and Tecosky-Feldman, 2005, p. vii).

The Rule of Four: Every topic should be presented geometrically, numerically, verbally, algebraically and in written form (Hughes-Hallett et al., 2005, p. vii).

Traditional courses: Courses that make little or no use of IE methods, relying primarily on passive-student lectures, recipe labs, and algorithmic-problem exams

Underpreparedness: For the purposes of this study, under-preparedness is understood as the situation where the knowledge and skills of the student entering an educational programme compares negatively with the assumed knowledge and competencies on which that programme is based (Hourigan and O’Donoghue, 2007).

1.16 Summary

In this chapter, the study was introduced and the background of the study was sketched. An overview was given of calculus, its history, and the need for reform in the international context, African context and South African contexts. RBAs were offered as a possible solution to the difficulties experiences with the learning and teaching of calculus worldwide and the RBAs used in this study were described. The research question was stated, together with the purpose of the study and the perceived significance of the study. The problem, together with the rationale and the objective of the study, were identified. A brief overview was hence given of the research methodology, after which the importance of the research was explicated. A concise summary of the delimitations of the study followed. This chapter was concluded with a section containing the definition of operational terms used in the study.

In the next chapter, a theoretical framework for the study will be presented, followed by a review of the pertinent literature.

CHAPTER TWO

THEORETICAL AND CONCEPTUAL FRAMEWORKS AND LITERATURE REVIEW

2.1 Introduction

The first chapter introduced the study. An overview of calculus, its history, and the need for reform with reference to the international context, the African context and the South African context was given. RBAs in the teaching of calculus were suggested as a solution to the difficulties experienced with the learning of calculus. A brief synopsis of the foundation for calculus in the school curriculum in South Africa was given. The research question, as well as the purpose of the study and the perceived significance of the study were formulated and discussed. A brief overview of the research methodology, including the delimitations, was thereafter presented.

This chapter covers a review of the pertinent literature, as well as the main theoretical framework of this study and how the theory is appropriate for the study. The main theoretical framework of this study is constructivism. As part of this theoretical framework, aspects of constructivism such as AL, Pedagogical Content Knowledge (PCK), the threshold concept, cognitive dissonance and formative assessment will also be addressed. The first part of the literature review will cover the technologies and methodologies used in RBAs in teaching calculus. The RBAs used in this study, namely IE methods and PD, ConcepTests, GQ, and ARS, will be described and critiqued. The second part of the literature review will contain a study of the topics covered by the ConcepTest used in the study, namely functions, differentiation and integration. Functions will be discussed along the framework of five *Big Ideas* compiled by the National Council of Teachers of Mathematics (NCTM). Common misconceptions and misunderstandings related to each of the three topics form an important part of PCK and will therefore be addressed in the discussions.

2.2 Theoretical framework

Theories are the conceptual basis for understanding, analysing, and designing ways to investigate relationships within social systems. The theoretical framework for this study was based on the constructivist theories of Piaget (Devries, 2000; Ojose, 2008), Von Glasersfeld (Von Glasersfeld and Steffe, 1991; Von Glasersfeld, 1995) and Vygotsky (Devries, 2000; Liu and Matthews, 2005). In the next section, the cognitive constructivism of Piaget and the Radical constructivism of Von Glasersfeld are explicated followed by the social constructivism of Vygotsky. These views are critiqued and compared. Hence constructivist applications to education are discussed. A decision regarding the use of direct instruction in the study is elucidated. The value of the constructivist pedagogic teaching approach used in this study, namely AL, is explained. The section concludes with a discussion of more peripheral pedagogical issues, some of which are directly linked to constructivism. These include knowledge for learning and teaching, cognitive dissonance, threshold concepts and formative assessment.

2.2.1 Constructivism

The roots of constructivism in Mathematics education can be traced back to three traditions, namely "Problem solving, Misconceptions, critical barriers and epistemological obstacles and Theories of cognitive development" (Confrey and Kazak, 2006, p. 3). These traditions gave a strong indication that students' learning problems were not only due to the complexity of the material, but could be ascribed to other factors (Confrey and Kazak, 2006). Mathematics educators sought solutions to problems identified as (1) students' limited conceptual understanding, (2) Curricula focused excessively on formal knowledge, isolated from experience and sense-making, (3) students' dependence on external sources for evaluation rather than self-regulation; and (4) mathematics instruction that was emotionally intimidating and alienating (Confrey and Kazak, 2006). Constructivism took hold in mathematics education because it addressed the major concerns of mathematics educators (Confrey and Kazak, 2006).

Constructivism has been described as a learning theory that understands learning as a recursive, non-linear constructive process (Fosnot and Perry, 2005). It has also been called an "epistemological stance regarding the nature of human knowledge" (Thompson, 2013, p. 1). Constructivism describes how participants, also referred to as cognising subjects (Von Glasersfeld, 1995), actively construe meaning (Thompson, 2013) via physical and social interactions with their environment (Abdulwahed, Jaworski and Crawford, 2012). Constructivists differ from the view of learners as receptacles of knowledge that can be transferred from the teacher to the learner. Unlike behaviourism, constructivists argue that "knowledge is not passively received but built up by the cognizing subject" (Von Glasersfeld, 1995, p. 18). Knowledge is created in the heads of persons, and “the thinking subject has no alternative but to construct what he or she knows on the basis of his or her own experience” (Von Glasersfeld, 1995, p. 1), a progression which differs from individual to individual. The process of getting to know consists of creating mental constructions. Over time, mental constructions adapt to new knowledge gained through new experiences. Knowledge is therefore self-adaptive and self-adjustable (von Glasersfeld, 1995).

Although Good (1993) managed to list some twenty different forms of constructivism in the literature, such as cognitive, contextual, critical, dialectical, empirical, humanistic, information-processing, methodological, moderate, Piagetian, post-epistemological, pragmatic, radical, rational, realist, social, sociocultural, socio-historical and trivial, the two principle schools of thought within constructivism remain cognitive constructivism (also referred to as individual or personal constructivism), and social constructivism (Thompson, 2013). In mathematics education the greatest influences are due to Piaget (cognitive constructivism), Vygotsky (social constructivism), and Von Glasersfeld (radical constructivism, derived from cognitive constructivism) (Amineh and Asl, 2015).

2.2.1.1 Piaget’s cognitive constructivism

Piaget (1962) described himself as a “genetic epistemologist”. Another term for *genetic* is *developmental*. (Hopkins, 2011). Piaget, a stage theorist, developed four universal and linear stages of development (Semmar and Al-Thani, 2015), namely sensorimotor, preoperational, concrete operational, and formal operational (Ojose, 2008). These are depicted in Table 2.1. A developmental stage can last months or years, depending on maturity, experience, culture, and the ability of the child (Papila and Olds, 1996). This view can create managerial difficulties for the curriculum at large, since students are usually grouped by chronological age, not by development levels (Weinert and Helmke, 1998). Personal experiences show that this type of grouping also creates challenges for teachers and learners.

TABLE 2.1: The four developmental stages of Piaget

Stage	Age Range	Description
Sensorimotor	0 – 2 years	Coordination of senses with motor response, sensory curiosity about the world. Language used for demands and cataloguing. Object permanence developed.
Pre-operational	2 – 7 years	Symbolic thinking, use of proper syntax and grammar to express full concepts. Imagination and intuition are strong, but complex abstract thought still difficult. Conservation developed.
Concrete operational	7 – 11 years	Concepts attached to concrete situations. Time, space and quantity are understood and can be applied, but not as independent concepts.
Formal Operations	11+ years	Theoretical, hypothetical, and counterfactual thinking. Abstract logic and reasoning. Strategy and planning become possible. Concepts learned in one context can be applied to another.

Source: The Psychology Notes Headquarter – http://www.PsychologyNotesHQ.com

Piaget's relational model of cognitive functioning encompasses two complementary processes, namely assimilation and accommodation. Piaget borrowed the word 'assimilation' from biology. Assimilation takes place when a cognising organism fits an experience into an existing conceptual structure or cognitive mental system, also called a schema. Assimilation is understood as the process of incorporating new sensory experience into an existing cognitive framework whereas accommodation occurs when a subject adjusts existing cognitive frameworks to incorporate experiences that cannot simply be assimilated. A disturbance in cognitive equilibrium occurs when new ideas do not fit existing schemas and hence the processes are out of balance. Such an imbalance is called a perturbation, cognitive conflict or cognitive dissonance (Campbell, 2010). If an existing schema has to be modified to assimilate new information, reflection may bring about a structural change - an accommodation that transforms the original cognitive structure. Accommodation thus comprises of reflective behaviour in order to reach a state of cognitive equilibrium. Hence "knowledge is a higher form of adaptation" (Von Glasersfeld, 1995, p. 58). Parallels exist between Piaget's "perturbation" and "cognitive dissonance" coined by Festinger (1957). In this study, the role of GQ is to kindle perturbations, which get resolved with the aid of PD.

Learning therefore takes place when a learner organises previous knowledge structures together with new experiences into a coherent collection of cognitive objects. Von Glasersfeld (1995, p. 57) cites Piaget (1937, p. 311) who asserts that "The mind organises the world by organising itself". The process is referred to as reflective abstraction, which plays a critical role in development of mathematical thought (Dubinsky, 1991).

2.2.1.2 Von Glasersfeld's Radical Constructivism

Radical Constructivism derives strongly from Piaget and is based on two propositions: a) knowledge is not passively received but actively built up by the cognising subject; b) the function of this cognitive process is adaptive and is compliant to the cognising subject's organisation of the experiential world (Von Glasersfeld and Kelley, 1981; Von Glasersfeld, 1982; Von Glasersfeld and Steffe, 1991; Von Glasersfeld, 1995)

Radical constructivists study learners in various social settings, attempting to understand learners' mathematical realities and how these develop (Thompson, 2013). Radical constructivist epistemology rejects objectivity of knowledge and contends that one's construction of the experiential world is subjective, although it acknowledges that there may be an objective reality beyond personal experience and reach. Knowing is seen as a personal or subjective experience, and one cannot gauge or measure the constructions that another person makes.

2.2.1.3 Vygotsky's Social Constructivism

Vygotsky argued that we learn best in a social environment, where we construct meaning through interaction with others (Vygotsky, 1978; Wertsch, 1985; Vygotsky, 1987b; Devries, 2000; Liu and Matthews, 2005). Constructing mathematical ideas is thus not only a cognitive, but also a social activity. Mathematicians operate within mathematical communities where proofs are accepted or rejected by peers.

Furthermore, learning takes place in a social community. Social constructivists focus on social and cultural mathematical and pedagogical practices and attend to an individual's internalisation of these. They deliberate on learners' participation in social settings. Social constructivism postulates that history and culture precede individual knowledge and that social practices predate individual participation.

Vygotsky's work focused mainly on the effect of social interaction, language, and culture on learning (Thompson, 2013). He was interested in the effect that language has on learning. He studied the role of the teacher and the learners' peers as they jointly negotiated meaning. He argued that higher order thinking developed first in social action and then in thought: "Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first between people …, then inside the child" (Vygotsky, 1978, p. 57).

Schoenfeld expounds mathematical problem solving as

> …an inherently social activity, in which a community of trained practitioners (mathematical scientists) engages in the science of patterns – systemic attempts, based on observation, study and experimentation, to determine the nature or principles of regularities in systems defined axiomatically or theoretically ("pure mathematics") or models of systems abstracted from real world objects ("applied mathematics") . The tools of mathematics are abstraction, symbolic representation, and symbolic manipulation. However, being trained in the use of these tools no more means that one thinks mathematically than knowing how to make shop tools makes one a craftsman. Learning to think mathematically means (a) developing a mathematical point of view – valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure - mathematical sense-making (Schoenfeld, 1992).

Vygotsky used the term Zone of Proximal Development (ZPD) to describe a developmental stage when a learner can solve a certain range of problems only when supported by a more knowledgeable person, such as a teacher, parent or more capable peers. The ZPD denotes the difference between what a learner can do without help and what a learner can do with help. Vygotsky (1978:86) defined the ZPD as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers" (Siyepu, 2013, p. 3). This zone varies from child to child and reflects the potential ability of the learner to grasp a specific concept (Thompson, 2013).

Vygotsky (1978) argued that if a learner needs assistance with problem solving or understanding, h/she is operating in his Zone of Proximal Development. Once a learner moves beyond the ZDP, he/she will be able to solve problems that initially could only be solved under guidance and in cooperation with the More Knowledgeable Other (MKO). The MKO refers to someone who has a higher level of ability or comprehension than the learner with respect to a particular task, process, or concept (Galloway, 2001). Chaiklin (2003, p. 2) cites Vygotsky (1987a, p. 211) who asserts: "… what the child is able to do in collaboration today he will be able to do independently tomorrow". As already mentioned, the ZPD can be designated by determining what a student is able to do with assistance. Wertsch (1985, p. 67) however warns that this is not a trivial task and emphasises that the ZPD "is to deal with two practical problems in the learning situation: the assessment of learners' intellectual abilities and the evaluation of instructional practices". In the context of this study, GQ and PD enable students to push the limits and progress continually beyond their current ZPD to a higher level.

> When consciousness shifts, or in other words, when the individual reorganises his or her conceptual system, he or she acquires potentials of perceiving new connections and of new possibilities of action. Consciousness, therefore, does not involve the complete knowledge of the absolute truth. It is a neutral concept referring to the general organisation of one's conceptual system, which orientates one's perception and sense-

> making. It emerges first on the social plane and then on the internal plane as generalised relationships are formed (Liu and Matthews, 2005, p. 10).

Wood, Bruner and Ross (1976) proposed the notion of "scaffolding" in the late seventies as a part of social constructivist theory (McLeod, 2012). They were influenced by the work of Vygotsky (1987a), whose original work on the ZPD was published in 1934. One of these researchers, Bruner (1978), continued to develop the theory of an expert assisting a novice or apprentice. He believed that children needed assistance from teachers and other adults when they learnt new concepts and called such assistance “scaffolding” (Wheeler, 2014). Scaffolding consists of adapting the level of support to suit the cognitive potential of the child. During the stage when the learner still observes the MKO, the learner is referred to as a peripheral participant (Dennen and Burner, 2008). As the learner gains expertise, scaffolding may be withdrawn gradually and hence the learner becomes a more central participant in the community of practice. The process is depicted in Figure 2.1.

Cambourne (1988) describes scaffolding as "raising the ante" and lists the most important phases of a scaffolding process: focusing on a learner's conception; extending or challenging the conception; encouraging reflection; and redirecting by offering new possibilities for consideration.

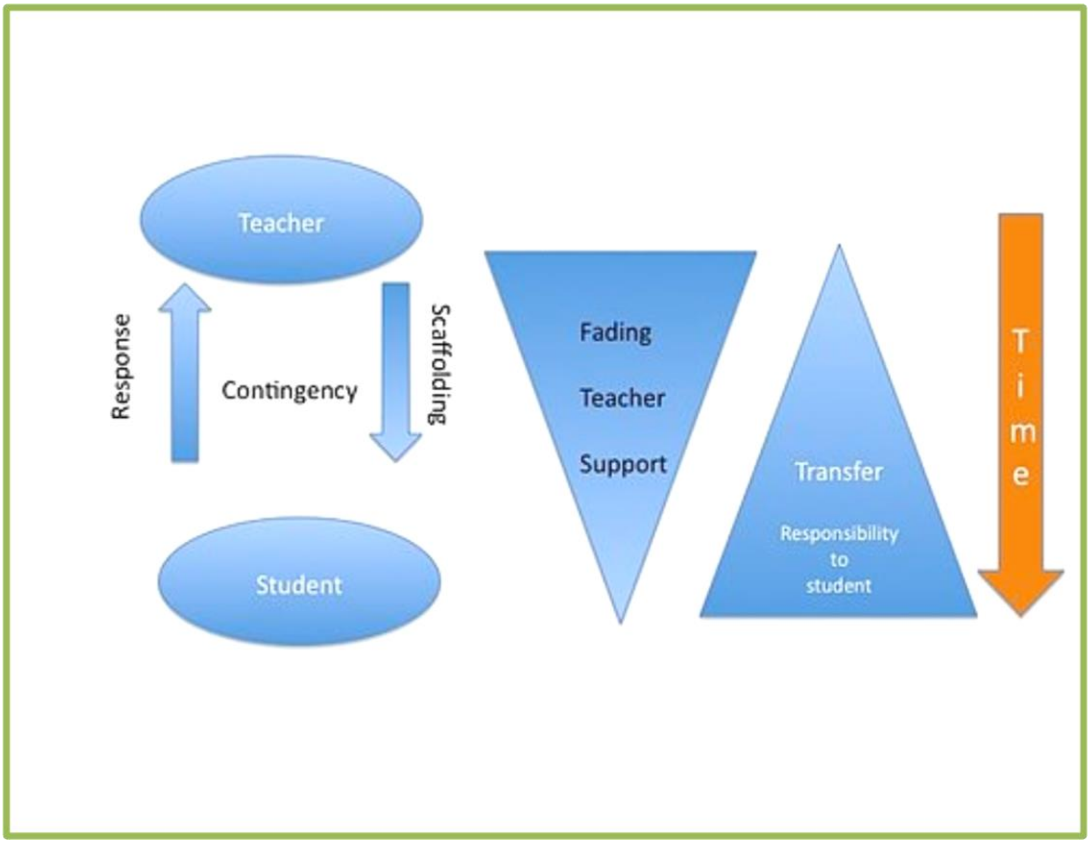

FIGURE 2.1: Scaffolding

Source: https://en.wikipedia.org/wiki/Instructional_scaffolding

An essential element to the ZPD and scaffolding is the acquisition of language. Vygotsky's theory regards language as fundamental to children's cognitive growth because language provides purpose and intention in order to direct behaviours. Speech enables children to communicate, which is vital for progressing in the ZPD. Dialogue offers opportunities for the child's unsystematic, disorganized, and spontaneous concepts to be challenged by the more systematic, logical and rational concepts of the skilled helper (Sajaniemi, 2013). The Vygotskian theory contends that there is a strong link between practical activities, speech, writing and the individual's intellectual activity (Singer, Ruddell and Ruddell, 1985). In support of this theory, Bruner (1960) posits the concept of the *spiral curriculum*, a structure that allows complex ideas to be taught at a simplified level first, and then re-visited at more complex levels later on.

Both Bruner and Vygotsky emphasised a child's social environment more than Piaget did. They also agreed that the MKO should play an active role in assisting the child's learning (McLeod, 2012).

2.2.1.4 Three forms of constructivism compared and critiqued

All forms of constructivism seem to agree on the need to establish the learner’s prior knowledge of a topic. The cognitive constructivists and the sociocultural constructivists further agree that cognitive re-organisation has to occur, but differ on the factors that activate these processes. The cognitive constructivists regard learning as an individual, cognitive self-organisation whereas the social constructivists see it as a social and cultural process (Table 2.2).

TABLE 2.2: Contrasts between the Individual Cognitive and the Sociocultural Constructivist Views

	Cognitive Constructivist	Sociocultural Constructivist
The mind is located:	in the head	in the individual-in-social interaction
Learning is a process of:	Active, cognitive reorganization	acculturation into an established community of practice
Goal is to account for:	the social and cultural basis of personal experience	constitution of social and cultural processes
Theoretical attention is on:	individual psychological processes	social and cultural processes
Analysis of learning sees learning as:	cognitive self-organization	acculturation, implicitly assuming an actively constructing child
Focus of analyses:	building models of individual students' conceptual reorganization and by analyses of their joint constitution of local social situation of development	individual's participation in culturally organized practices and face-to-face interactions
In looking at a classroom, we see:	an evolving micro-culture that is jointly constituted by the teacher and students	instantiation of the culturally organized practices of schooling

Extract from Duffy and Cunningham (1996, pp. 11-12)

Hopkins (2011) describes three major criticisms of Piaget’s work. Firstly, he asserts that Piaget’s theory does not offer a complete description of cognitive development. Piaget’s methodology is often unclear. He used very small samples, concentrating mostly on his own children as research subjects. Secondly, he criticises Piaget’s stage theory, which has become less prevalent in developmental research. Stages may seem apt for describing human behaviour, yet often miss the complexities of intra-individual and inter-individual variation in development. Thirdly, Piaget has been criticised for both underestimating the abilities of young children, as well as overestimating the abilities of older learners (Ojose, 2008). Critics claim that abstract instructions may cause young children to fail at tasks they otherwise may be able to execute (Gelman, Meck and Merkin, 1986). The theory implied that formal operations, such as hypothesis testing, abstract thinking, and deductive reasoning, should be attained by all children at about 12 years of age. Teachers may hence assume that their older students can always think logically in the abstract, yet this is not always the case (Shaffer and Kipp 2010).

Furthermore, Piaget's theory has also been criticised on the grounds that it is "empirically false, or philosophically and epistemologically untenable" (Machado and Lourenço, 1996, p. 143). They however claim that most of the criticisms of Piaget's work either derive from inaccurate translations or widespread misinterpretations, or ignore various modifications of Piagetian theory developed later in his life (Machado and Lourenço, 1996). One such misinterpretation is Piaget's view of the role that social factors play in the child's development. Despite the fact that Piaget is not generally viewed as a social constructivist, some researchers have softened this stance after reviewing Piaget's work. Piaget did in fact emphasize the central role of social factors in the construction of knowledge, as the following quotes demonstrate (Devries, 2000, p. 6):

> "… social life is a necessary condition for the development of logic" (Piaget, 1928/1995, p. 120); "… social life transforms the very nature of the individual" (Piaget, 1928/1995, p. 210); "… (the progress of) reason is due to social mechanisms" (Piaget, 1928/1995, p. 199); "… relations among individuals. . . modify the mental structures of individuals" (Piaget, 1950/1995, p. 40).

Devries (2000, p. 6) elaborates:

> Further, he unequivocally equated intellectual and social operations as identical, stating that: In the realm of knowledge, it seems obvious that individual operations of the intelligence and operations making for exchanges in cognitive cooperation are one and the same thing, the "general coordination of actions" to which we have continually referred being an interindividual as well as an intraindividual coordination because such "actions" can be collective as well as executed by individuals (Piaget, 1967/1971, p. 360)

Whereas Piaget supported a stage view of cognitive development as a universal process, Vygotsky regarded cognitive development as a catalyst of history, culture language and society (Newman and Newman, 2012). Vygotsky consequently argued that the environment impacted the cognition of the learner, stressing that a child could not develop based purely on exploration and experimentation, but required the help of more able adults or peers (Rajcoomar, 2013).

Bruner (1960) opposed Piaget's stage theory (Piaget, 1952). He argued that it is impractical and unfeasible to expect teachers to match the complexity of subject material to a child's cognitive stage of development. Because of this view, learners are sometimes underestimated and hence disadvantaged as teachers regard certain topics as too difficult for the learners. Such topics are then taught at a later stage when the teacher believes the child has reached the appropriate state of cognitive maturity. Bruner however believes that a child of any age is capable of understanding complex information: "We begin with the hypothesis that any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (Bruner, 1960, p. 33).

Martínez-Delgado (2002) asserts that general radical constructivist principles contradict their pedagogical implementation, and found parallelisms between constructivism and behaviourism. "The constructivist linking of activities with knowledge offers an obvious parallelism with the association of stimulus (activities)–response (new knowledge), characteristic of behaviourism. The two models differ in emphasis: in constructivism on the subject and in behaviourism on the environment" (Martínez-Delgado, 2002, p. 851).

Radical constructivist epistemology has also been criticised for rejecting objectivity of knowledge and the contention that one's construction of the experiential world is subjective. Radical constructivists readily acknowledge that there may be an objective reality beyond their experience, but posits that this objective reality is unknowable, since no tools exist to examine it. This stance has been interpreted as a statement of solipsism (Vergnaud, 1987) but refuted by others (Goldin, 1989; Von Glasersfeld, 1995; Milne and Taylor, 1998, p. 32). However, the pedagogical dilemma of the radical constructivist teacher remains – how to determine whether the pupil has constructed meaning in the way the teacher has intended (Jaworski, 1994). Radical constructivism, therefore, sees no escape from subjectivity. Some researchers see this as a problem: Martínez-Delgado (2002) laments the wide gap between the primary postulates of radical constructivism and their pedagogical development. For education to be effective, meaning has to be negotiated or constructed between participants. Unless some form of common understanding is constructed between teacher and pupil the educative process is doomed to failure.

Also, from a Radical constructivism perspective, each child may require an individual education plan, which is regarded as impractical:

> This is laudable, but the reality is that due to time constraints the teacher needs to feel reasonably confident that shared understandings are being assembled in the classroom. Without such common understandings the teacher will collapse under the theoretical pressure of radical constructivism in believing that he has to consult with each and every child about each and every aspect of knowledge acquisition. It seems to me that a social rather than a radical approach to constructivism has a better chance of informing actual classroom practice (McCarthy, 2016, p. 36).

According to the social constructivist theory, other people influence an individual's experiential environment and have considerable power in determining which behaviours, concepts, and theories are considered 'viable' in the individual's physical and linguistic interactions with them. Goldin (1990) laments the undue influence that people such as the MKO is seen to exercise on the learner. Also, not everybody agrees with Vygotsky that teaching, or peer mediation, is a necessary condition for learning (Clark, 2013). Although Bruner (1984) supported the concept of the ZPD and scaffolding, he questioned the concept that learners could only progress from the current ZDP through social interaction.

2.2.1.5 Constructivism and applications to education

It could be argued that constructivism is a theory of learning and that it is therefore pointless to study it from a teaching point of view (McCarthy, 2016). On the other hand, pedagogy is certainly influenced by the theory, and constructivism therefore has implications for instruction. Also, by acknowledging the role of MKOs such as peers, parents and teachers in children’s learning, the act of teaching becomes significant: “Otherwise, terms like ‘scaffolding’, ‘social interaction’ and ‘solving problems together’ will remain as slogans but with no real, practical meaning for teachers” (McCarthy, 2016, p. 14). According to the Vygotskian perspective, learning is very closely linked to teaching, and is often described as an apprenticeship or enculturation, a process of “integration into a community of practice” (Lave and Wenger, 1991). McCarthy (2016, p. 13) agrees and cites Fosnot and Dolk (2005, p. 175) who assert that learning and teaching are closely related and should be integrated in learning/teaching frameworks. They further state that “If learning doesn’t happen, there has been no teaching. The actions of teaching and learning are inseparable”.

Therefore, it stands to reason that improved teaching will enhance pupils’ learning experiences (McCarthy, 2016). Different approaches have been applied to facilitate conceptual understanding and constructivist learning such as collaborative learning, inquiry learning, problem-based learning and using real-world contexts. Others can be explored in Abdulwahed et al. (2012).

In the early nineties, discussions in the literature focused on the difficulties that teachers experienced when attempting to teach from a constructivist perspective Jaworski (1994). Brophy lamented constructivism’s vague prescriptions for teaching “If it is to be of much practical use, however, such input will have to become much more specific, prescriptive and empirically based…It will have to come to grips with the challenge facing the typical … teacher” (Steffe and Kieran, 1994, p. 76). That is also the main reason why, years later, diSessa and Cobb (2004, p. 80) called constructivism “a grand theory”, lamenting that grand theories are often “too high-level to inform the vast majority of consequential decisions”. To overcome these allegations, various researchers (Tall, 1977; Von Glasersfeld and Steffe, 1991) have proposed building models of the students’ mathematical knowledge in terms of schemes of actions and operations. Such models may include aspects of the learning environment. Knuth and Cunningham (1993) argue that constructivist learning environments should:

- provide experience with the knowledge construction process;
- embed learning in realistic and relevant contexts;
- encourage the use of multiple modes of presentation;
- provide experience in and appreciation for multiple perspectives;

- embed learning in social experience;
- encourage self-awareness of the knowledge-construction process and
- encourage ownership and voice in the learning process.

Constructivism therefore has implications for instruction:

> learning is a student-centred process, students' autonomy should be fostered; learning should be contextualised and associated with authentic real-world environments and examples; social interaction and discourse form an important part of learning; the taught elements should be made relevant to the learner; the taught elements should be linked with the learners' previous knowledge; it is important to facilitate continuous formative assessment mechanisms, … teachers should act as orchestra synchronisers rather than speech givers; and teachers should consider multiple representations of their teachings. (Abdulwahed et al., 2012, p. 13).

In addition, the teacher should realise the value of language as a tool in the process of guiding the students construction (Jaworski, 1994). Some general principles of learning derived from constructivism may also be helpful to keep in mind:

> • Learning is not the result of development; learning is development. It requires invention and self-organization on the part of the learner. Thus teachers need to allow learners to raise their own questions, generate their own hypotheses and models as possibilities, test them out for viability, and defend and discuss them in communities of discourse and practice.
>
> • Disequilibrium facilitates learning. Challenging, open-ended investigations in realistic, meaningful contexts need to be offered which allow learners to explore and generate many possibilities, both affirming and contradictory. Contradictions, in particular, need to be illuminated, explored, and discussed.
>
> • Reflective abstraction is the driving force of learning. As meaning makers, humans seek to organize and generalize across experiences in a representational form. Allowing reflection time through journal writing, representation in multi-symbolic form, and/ or discussing connections across experiences or strategies may facilitate reflective abstraction (Thompson, 2013, p. 20).

Furthermore, dialogue encourages reflection. Fosnot (2005) stressed that the classroom needs to be seen as "a minisociety - a community of learners engaged in mathematical activity, discourse, and reflection". Thompson (2013, p. 20) noted:

> The learners (rather than the teacher) are responsible for defending, proving, justifying, and communicating their ideas to the classroom community. Learning is the result of activity and self-organization and proceeds towards the development of structures. As learners struggle to make meaning, progressive structural shifts in perspective are constructed - in a sense "big ideas" (Schifter and Fosnot, 1993). These "big ideas" are learner constructed, central organizing principles that can be generalized across experiences, and that often require the undoing, or re-organizing of earlier conceptions. This process continues throughout development.

Constructivism is a theory of learning, not a theory about teaching. However, when one analyses the theory, one can begin to formulate a reformed practice that "supports rigor, empowerment, and the construction of genuine understanding" (Fosnot, 2005, p. 14). In a paper titled "Implementing Constructivism in Mathematics Classrooms", Cometto (2008, p. 88) argues in favour of facilitating calculus learning rather than using the lecturing approach and compares the two approaches (Table 2.3).

TABLE 2.3: Facilitation versus Lecturing

Lecturer	Facilitator
Dictates material	Guides Students to the material
Informs	Asks/Involved in dialogue with students
Teaches from the front	Supports by walking around
Gives answers	Provides guidelines, asks good questions
Learner is passive	Learner is active
Focus on the material	Focus on the student

Source: Cometto (2008, p. 88)

According to the constructivist approach, new material should be linked to something familiar in the learners' former experiences before formal methods are used (Makgato, 2012). New knowledge should furthermore be presented in the context of real-life applications (Rajcoomar, 2013). Hughes-Hallett et al. (2005) state that the following question is a good example of how questions can be used in a constructivist approach to calculus:

Let $N = f(t)$ be the total number of cans of carbonated drinks Sean has consumed by age *t* in years. Interpret the following in practical terms, paying close attention to units.

$$f(14) = 400$$

$$f^{-1}(50) = 6$$

$$f'(12) = 50$$

$$\left(f^{-1}\right)'(450) = \frac{1}{70}$$

This question adheres to the requirements mentioned above. The question is set in familiar contexts and based on prior knowledge. Also, no calculations are required when answering this question, but the learner needs prior knowledge and conceptual understanding of the derivative and the inverse of a function. The units for the answers can be regarded as an indication of the level of conceptual understanding displayed.

Some critics are concerned that the constructivist approach to learning dismisses the active role of the teacher or the value of expert (Rajcoomar, 2013). Others stress that teachers have a responsibility to “continually ascertain where the student currently is with respect to understanding and either set up situations that promote cognitive conflict for the student (Piagetian), or promote understanding through the notion of scaffolding (Vygotskian)” (Manson, 2009, p. 47).

Student-centred pedagogy therefore requires “teachers to make links between students’ current meanings and new knowledge” (Brodie et al., 2002, p. 98). Tall (1977) regards the role of the teacher in the resolution of cognitive conflict as of paramount importance. The essential role of the teacher is assisting the learner with schematic restructuring. Simple signs of cognitive conflict in the learner are confusion, annoyance or frustration and these will be apparent to the sensitive teacher. According to Tall it is the responsibility of the teacher to resolve the conflict in a suitable manner.

McCarthy (2016, p. 43) however poses a relevant question: “Does constructivism exclude direct instruction?” He proceeds to explain the predicament of teachers: direct instruction seems to be in conflict with the guidelines of social constructivism, which state that cognitive development is a product of social interaction between partners who solve problems together. Yet, direct instruction has always been popular in mathematics teaching. Another question therefore needs a response: “are there any circumstances in which simply telling pupils information could be compatible with social constructivist pedagogy?” (McCarthy, 2016, p. 44). Brophy answers in the affirmative and contends that direct instruction works best when it is used for teaching canonical knowledge (initial instruction establishing a knowledge base). On the other hand, social constructivist techniques are best used for “constructing knowledge networks and developing processes and skills (synthesis and application)” (Brophy, 2006). Although Love and Mason (1995) seem to agree that there are many circumstances in which it is not only proper and effective, but essential to tell people things, they go on to qualify this statement by saying that “as long as what is said or explained is at the edges of what the pupils can do for themselves, rather than in the core” (Love and Mason 1995, p. 58). This stance is quite the opposite of Brophy’s.

Nevertheless, teachers are under curricular and time constraints, and these limitations often result in direct instruction. Love and Mason (1995) acknowledge the need for direct instruction in order to maintain pupils' motivation and helping them avoid frustration. It may also promote clarity of thought. Hyslop-Margison and Strobel (2007) suggest that direct instruction is especially effective in classrooms where students already possess considerable subject knowledge.

Mathews (2000) views theories which advise against direct instruction as problematic. His concern is that knowledge of some complex conceptual schemes have been constructed over decades by exceptional minds and questions children's access and ability to master these without any direct instruction. Kirschner, Sweller and Clark (2006) agree and lament constructivist views that have become ideologically and often epistemologically opposed to the presentation and explanation of knowledge. They warn that empirical studies over the past half-century indicate that minimally guided instruction is less effective and less efficient than instructional approaches that place a strong emphasis on guidance of the student learning process. They also note that students subjected to pure-discovery methods and minimal feedback, often become lost and frustrated, and their confusion can lead to misconceptions. They reference studies that demonstrate unguided discovery as inefficient and concludes that students learn more deeply from strongly guided learning than from discovery.

Initially, constructivism envisioned the goals of mathematics instruction along the lines of Piaget‘s epistemology. At the end of the 1980s, Cobb argued that the goal of instruction is or should be to help students build [mental] structures that are more complex, powerful, and abstract than those that they possess when direct instruction was used (Cobb, 1988). The pedagogical problem was to create the classroom conditions for the development of complex and powerful mental structures. This challenge is still the focus of research today.

Also, it is necessary to explore the meaning of the word 'understanding' in the teaching and learning of mathematics. The word is controversial, and mathematics educators, researchers and mathematicians have different interpretations for the word (Siyepu, 2013). Kirschner et al. (2006) define learning as a change in long-term memory. Some constructivists however regard memory as less important and focus on understanding of concepts as the main issue. Fox (2001) argues that neither view holds true. I differ from him, and believe that both views in fact present a perspective of reality. For example, learning the names of fractions requires memorisation rather than understanding. Also, it remains important to keep in mind that mathematics pedagogy approaches mathematics as a conceptual system rather than a collection of discrete procedures.

> A mathematical idea or procedure or fact is understood if it is part of an internal network. More specifically, the mathematics is understood if its mental representation is part of a network of representations. The degree of understanding is determined by the number and the strength of the connections. A mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections (Hiebert and Carpenter, 1992, p. 67).

The advent of constructivism had a profound impact on the field of mathematics education. Confrey and Kazak (2006, p. 335) elaborate on the contribution that constructivism has made to learning and teaching. Constructivism has revealed:

> … how to make effective use of the resources, language, inscriptions, and ideas they bring to the enterprise of learning. It has produced many practical accomplishments from curricula to new technological tools, as well as documented a number of substantial considerations of student thinking about which all teachers need to know. Because of the theory, we have realized that careful attention must be paid to how students become increasing aware of what they believe and know, and how this is refined and developed in the company of others. Our views of the role of teachers has been transformed to recognize their critical contributions as stimulators, guides, facilitators and critics – assisting students in developing the fundamental reasoning abilities that are the hallmark of mathematics as students complete a tour of the rich variety of topics in the fields Confrey and Kazak (2006, p. 335).

2.2.2 Active learning

Constructivism as a description of human cognition is often associated with pedagogic approaches that promote learning by doing (Dalgarno, 1996) or AL. Constructivist theorists highlight the crucial role that activity plays in mathematical learning and development (Cobb, 1994). Piaget asserted that “knowledge arises from the active subject's activity, either physical or mental, and that it is goal-directed activity that gives knowledge its organization” (Von Glasersfeld, 1995, p. 56). According to Noddings (2004, p. 10), “An acceptance of constructivist premises about knowledge and knowers implies a way of teaching that acknowledges learners as active knowers” (p. 10). AL refers to classroom practices that promote higher-order thinking by providing opportunities to engage in mathematical investigation, communication, and group problem-solving, while students also receive feedback on their work from peers and experts (CBMS, 2016). As already mentioned earlier, teachers need to “continually ascertain where the student currently is with respect to understanding and either set up situations that promote cognitive conflict for the student (Piagetian), or promote understanding through the notion of scaffolding (Vygotskian)” (Manson, 2009, p. 47).

Various sources from the literature confirm the value of using AL approaches at university (Prince, 2004; Méndez and Delgado, 2013; Freeman et al., 2014; Tang and Titus, 2015; Petrillo, 2016). Freeman et al. (2014) conducted a meta-analysis of 225 studies that compared student performance in undergraduate science, technology, engineering, and mathematics (STEM) courses in terms of traditional lecturing versus AL. The results were overwhelmingly in favour of AL.

> The effect sizes indicate that on average, student performance on examinations and concept inventories increased by 0.47 SDs under Active Learning (n = 158 studies), and that the odds ratio for failing was 1.95 under traditional lecturing (n = 67 studies). These results indicate that average examination scores improved by about 6% in Active Learning sections, and that students in classes with traditional lecturing were 1.5 times more likely to fail than were students in classes with Active Learning (Freeman et al., 2014, p. 8410).

In recognition of this evidence, the Conference Board of the Mathematical Sciences in the United States has requested all stakeholders to invest time and resources on the incorporation of AL into tertiary mathematics courses (CBMS, 2016).

AL has a positive impact on learning and students’ attitudes towards learning (Lumpkin et al., 2015) and increases student performance, outcomes and retention (Jurukovski et al., 2015). Advocates of the AL approach advise lecturers to present shorter lessons combined with activities designed to reach specific goals (Bonwell and Eison, 1991). Active learners are engaged in activities such as reading, discussing, writing, and reflection on these activities. In other words, AL has two components, namely doing, and thinking about doing. "By challenging learners to reflect on their own thinking, teachers and their peers help them to make unconscious processes overt and explicit and so making these more available for future use" (Black and Wiliam, 2009). Some researchers regard AL as one aspect of a more encompassing concept of student engagement, which includes behavioural, emotional and cognitive facets (Zepke, 2014). When students' attitudes improve, their motivation improves (Bonwell and Eison, 1991; Bonwell, 1996; Bonwell and Sutherland, 1996). Furthermore, the use of AL is promoted for its potential to support equity, diversity and access in mathematics education (CBMS, 2016). "For students from different socioeconomic, cultural, and educational backgrounds, and for students with different approaches to learning and social interaction, a supportive community of learners can be cultivated using AL techniques" (CBMS, 2016, p. 6). However, the implementation of AL techniques involves a substantial learning curve for students and teachers alike, and care has to be taken when introducing these. Concerns include increased preparation time, and difficulties with time management and coverage of course contents within the allocated time (CBMS, 2016).

2.2.3 Knowledge for teaching and learning

Shulman (1987), as cited by Depaepe, Verschaffel and Kelchtermans (2013), identified seven categories of knowledge that a teacher should have, namely PCK, CK, general pedagogical knowledge, curriculum knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge of educational ends, purposes, and values, and their philosophical and historical grounds. Teachers need knowledge from different domains such as knowledge of the subject matter, knowledge of students and lately also knowledge of technology. Shulman investigated various ways in which teachers transform their Subject Matter Knowledge (SMK) into a form that makes it more understandable and accessible to their learners (Depaepe et al., 2013). In this study, the same process is under investigation, but with the addition of technology. According to Niess (2005, p. 510), TPACK is "the integration of the development of knowledge of subject matter with the development of technology and of knowledge of teaching and learning". The focus of this study is therefore on SMK and its relation to the effective use of technology (TPACK).

2.2.3.1 Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK)

Shulman (1986) distinguished three categories of CK, namely SMK, PCK, and curricular knowledge. Teachers first and foremost need SMK. The most common view of SMK is that of academic knowledge obtained through formal training at universities and colleges, in other words, knowledge attained through training in a specific discipline. A more inclusive view encompasses teachers' beliefs about their specific discipline.

PCK was originally defined by Shulman (1987) as consisting of two parts, namely the knowledge that teachers have of common conceptions, misconceptions, and difficulties held by students regarding particular subject matter, and secondly, the knowledge that teachers have of a range of instructional strategies and representations that could be used to make particular subject matter accessible for students (Depaepe et al., 2013). PCK is the knowledge that instructors have about their subject, their students and the curriculum (Rollnick, 2010). PCK encompasses teachers' view of what constitutes good teaching. It also includes specialist knowledge such as which illustrations and examples work best, how to scaffold learning and which preconceptions exist in the mind of the learner on a specific topic. A preconception is a conception that students have as result of prior learning as learners do not appear as blank slates. If those preconceptions are misconceptions, teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners (Depaepe et al., 2013). PCK comprises the knowledge and explanation skills that make a particular topic more comprehensible to the learner. An experienced teacher with good PCK will therefore be able to prevent certain misconceptions forming in the mind of the learner. According to Shulman (1986), teachers need PCK for the effective teaching of a subject. Teachers lacking sufficient PCK may inadvertently and unknowingly make mistakes.

This view is supported by the notion of curricular saliency (Geddis and Wood, 1997), which refers to the teacher's understanding of the place of a topic in the curriculum and the value of the topic in the bigger scheme. When a teacher lacks sufficient curricular saliency, the teacher may decide to skip certain aspects of a topic or the topic itself, not knowing that the topic may be vital and essential at a later stage in the curriculum. Teachers who lack SMK, often lack curricular saliency. SMK is critical to the development of PCK.

Even experienced teachers may struggle with PCK when making changes to their pedagogy, such as moving from a more traditional approach to RBAs learner-centred approaches (Speer and Wagner, 2009). Additionally, experienced teachers who teach a topic for the first time, may be unaware of learners' errors and misconceptions in that specific subject area (Makonye, 2012).

Particular types of knowledge (Teachers' Knowledge of Content and Students (KCS)) used in teaching correlate with reform-oriented teaching practices and with student achievement (Hill, Ball and Schilling, 2008). Constructivist teachers must be able to compile and present tasks that bring about appropriate conceptual reorganizations in students.

> This approach requires knowledge of both the normal developmental sequence in which students learn specific mathematical ideas and the current individual structures of students in the class. Such teachers must also be skilled in structuring the intellectual and social climate of the classroom so that students discuss, reflect on, and make sense of these tasks (Clements and Battista, 1990).

One aspect of criticism of Shulman's conceptualizations of PCK relates to his static view on teachers' PCK. For instance, Cochran et al. (1993) used the term pedagogical content knowing (PCKg) instead of PCK to stress its dynamic nature (Depaepe et al., 2013). A more dynamic perspective of PCK views it as a knowing-to-act that is inherently linked to and situated in the act of teaching within a particular context. This aspect is especially pertinent when teaching with technology, since technology and its applications to teaching have developed so rapidly over the last few decades.

Another critical perspective of Shulman's PCK questions whether PCK can be theoretically and empirically distinguished from CK. It is claimed that one cannot meaningfully distinguish between CK and PCK since choices that teachers make in the act of teaching are based on multiple dimensions of knowledge as opposed to one specific knowledge type (Depaepe et al., 2013).

2.2.3.2 Technological Pedagogical and Content Knowledge (TPACK)

RBAs make use of technology. When teaching with technology, teachers need TPACK. TPACK (originally TPCK) is a framework that extends Shulman's construct of PCK to include Knowledge of Technology:

> … effective teaching depends on flexible access to rich, well-organized, and integrated knowledge from different domains, including knowledge of student thinking and learning; knowledge of subject matter; and increasingly, knowledge of technology (Koehler, Mishra and Cain, 2013, p. 13).

TK is difficult to define because technology changes so rapidly (Koehler and Mishra, 2009). TK includes perceptions about technology, information about technology, tools and resources and how to productively apply these in various situations to achieve specific goals (Koehler and Mishra, 2009, p. 65).

As the use of technology becomes more commonplace, teaching has to adapt. According to Koehler et al. (2013), teaching is a complicated, ill-structured discipline requiring the application of complex knowledge structures in dynamic contexts. Thus, effective teaching depends on access to integrated knowledge, namely CK, PK and TK. TPACK comprises of these three components, together with the relations between and among the components, which result in four more types of knowledge, namely Technological Pedagogical Knowledge (TPK), Technological Content Knowledge (TCK), PCK, and TPACK (Figure 2.2). These seven components together comprise TPACK (Mishra and Koehler, 2006). In a later paper, Koehler and Mishra (2008) added context to the framework, as context determines to a large extent the way technology can be used in educational practice (Kafyulilo, Fisser, Pieters and Voogt, 2015).

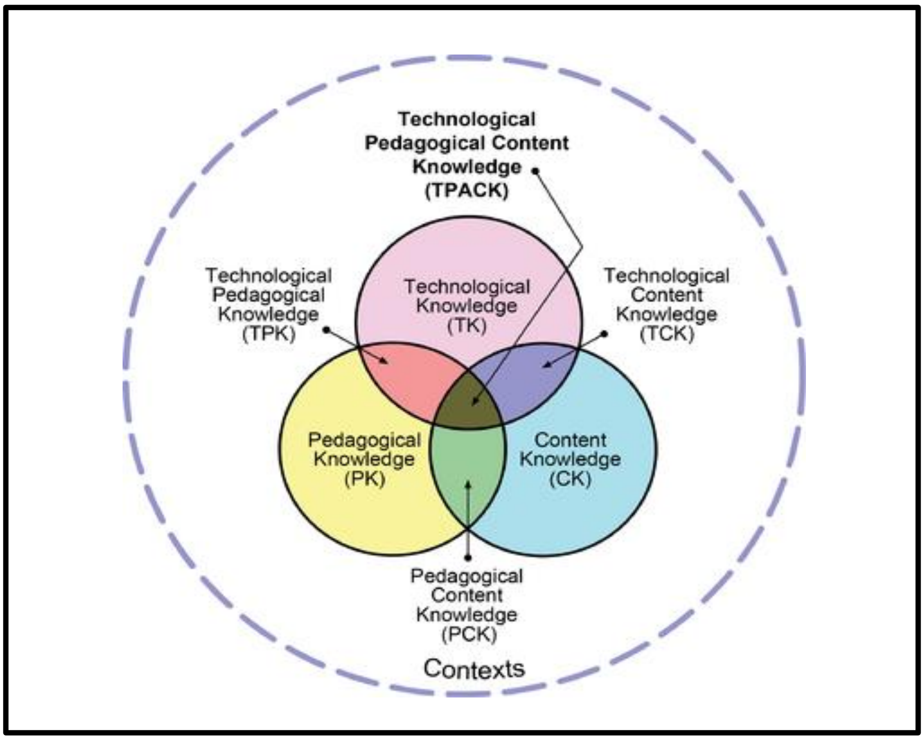

FIGURE 2.2: Technological Pedagogical Content Knowledge (TPACK)
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Koehler and Mishra (2009) provide the following concise definitions of TCK, TPK and TPACK:

Technological Content Knowledge (TCK) is

> … an understanding of the manner in which technology and content influence and constrain one another. Teachers need to master more than the subject matter they teach; they must also have a deep understanding of the manner in which the subject matter (or the kinds of representations that can be constructed) can be changed by the application of particular technologies. Teachers need to understand which specific technologies are best suited for addressing subject-matter learning in their domains and how the content dictates or perhaps even changes the technology—or vice versa (Koehler and Mishra, 2009, p. 65).

Technological Pedagogical Knowledge (TPK)

> An understanding of how teaching and learning can change when particular technologies are used in particular ways. This includes knowing the

> pedagogical affordances and constraints of a range of technological tools as they relate to disciplinarily and developmentally appropriate pedagogical designs and strategies (Koehler and Mishra, 2009, p. 65).

Technological Pedagogical Content Knowledge (TPACK)

> Underlying truly meaningful and deeply skilled teaching with technology, TPACK is different from knowledge of all three concepts individually. Instead, TPACK is the basis of effective teaching with technology, requiring an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge to develop new epistemologies or strengthen old ones (Koehler and Mishra, 2009, p. 66).

The importance of TPACK is evident from a study conducted by Leendertz, Blignaut, Nieuwoudt, Els and Ellis (2013), who reported that TPACK of mathematics teachers contributes towards more effective Grade 8 mathematics teaching in South African schools (Leendertz et al., 2013). The researchers analysed data from the South African Mathematics teachers' dataset of the Second Information Technology in Education Study (SITES 2006), which included 640 Grade 8 mathematics teachers and investigated their pedagogical use of Information Communication Technology (ICT). The purpose of the study was to investigate the level of TPACK of mathematics teachers, and how TPACK attributes contribute towards more effective Grade 8 mathematics teaching in South African schools. They reported that increases in TPACK of mathematics teachers, confidence in their ability to use ICT and levels of mathematical content knowledge contributed towards more effective mathematics teaching. They specifically mentioned IE. They reported that adequate TPACK enabled teachers to be more innovative with their teaching and learning activities and to facilitate interactive lessons.

2.2.4 Threshold Concepts

A threshold concept is a core concept, associated with a portal or gateway that leads to better conceptual understanding of a subject area. It plays an important role in dealing with what Perkins (1999) coined as 'troublesome knowledge'. When a student finally grasps a threshold concept (an 'aha' or 'Eureka' moment), a transformation takes place and the student gains new perspectives that will likely be retained. Pettersson (2011) asserts that using the framework of threshold concepts can be a powerful way of improving student learning, whilst Meyer and Land (2005, p. 374) use the term 'pedagogically fertile' when referring to threshold concepts. Threshold concepts are termed "transformative (occasioning a significant shift in the perception of a subject), irreversible (unlikely to be forgotten, or unlearned only through considerable effort), and integrative (exposing the previously hidden interrelatedness of something)" (Meyer and Land, 2005, p. 373). McCartney, Boustedt, Eckerdal, Mostrom, Sanders, Thomas and Zander (2009) claim that a student may get stuck in the process of mastering a concept, and that mastering a threshold concept may take years. Pettersson (2011) asserts that knowledge about these potential snares or "epistemological obstacles" (Brousseau, 1997, p. 87) are important for effective teaching.

Quinn (2011) in (Easdown and Wood, 2014), mentions fractions as one of the important threshold concepts in mathematics, while Pettersson (2011) lists a number of threshold concepts in calculus, such as function, limit, derivative and integral. She claims that all of these topics are problematic for students and laments the lack of research on threshold concepts in mathematics education. Pettersson (2011) cites Carstensen and Bernhard (2008) who assert that students scored higher when the teaching was focused on threshold concepts.

The study of thresholds prompted "… a 'big rethink' about the structure of subjects, the cognitive and affective difficulties of mastery and how best to learn and teach a subject". Furthermore, Meyer and Land (2005, p. 374) claim that a "shift in perspective is usually accompanied by (or occasioned through) an extension of the student's use of language". Thus, threshold concepts not only result in transformed thought, but also in an extended discourse. Hence, the researcher in this study also investigated the effects that language difficulties have in conceptualisation of fractions (Coetzee and

Mammen, 2016), a threshold concept which is regarded as part of the foundation of some calculus concepts such as rate of change.

2.2.5 Cognitive dissonance or conflict

Festinger's theory of cognitive dissonance stresses internal consistency as an ideal state (Festinger, 1957). According to this theory, individuals become distressed when they experience inconsistency (dissonance). Several educational interventions have been designed to promote cognitive dissonance in students by increasing their consciousness of conflicts between their prior viewpoints and new information (such as requiring students to defend prior beliefs) and then guiding students to new, correct explanations that will resolve the conflicts. Cognitive conflict thus stimulates conceptual change. When utilising GQ combined with Student Response Systems (SRS) and effective PD, students are challenged on their preconceptions and misconceptions. An important feature of the process is the PD that takes place after students are confronted with a Good Question. Knowledge is constructed socially, according to “Vygotsky’s principle that ideas appear first in the external ‘social’ plane, then become internalised by the individual” (Black and Wiliam, 2009, p. 20). The dissonance will be resolved when students abandon the misconceptions and adapt correct interpretations. Also, when learners reflect on their own learning, their metacognition is developed. Chien et al. (2016) posit that the effectiveness of PI may be attributed to the theory of scaffolding (Bruner, 1984). According to Chien and colleagues, a student is enabled by the interaction with a MKO. However, a peer is not necessarily a MKO. In my opinion, the verbalising of the problem and solution strategies, combined with the ordering of the thought processes enable the learner and bring about a better understanding of the concepts at play.

2.2.6 Formative assessment

One of the major advents of constructivism was the renewed focus on assessment. “Assessment was viewed as a means to support constructivist practices in a variety of ways” (Confrey and Kazak, 2006, p. 329). A need was identified to evaluate students’ knowledge more sufficiently. Students furthermore needed support to enable them to become aware of their own learning (metacognition), and to also reflect on it (Ezrailson, 2004). Teachers likewise required an increased understanding of student reasoning (Confrey and Kazak, 2006). Assessment was seen as an avenue to accomplish these goals.

Lutz and Huitt (2004) mention four guiding principles for teachers using the constructivist approach; the fourth of which addresses assessment: "Assessment, measurement, and evaluation should be a natural part of the learning process rather than an activity completed at the end of the learning process" (Lutz and Huitt, 2004). Black and Wiliam (2009, p. 7) offer the following definition of formative assessment:

> Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited.

According to Black and Wiliam (2009, p. 4), formative assessment can be conceptualised as consisting of five key strategies:

(i) Clarifying and sharing learning intentions and criteria for success
(ii) Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;
(iii) Providing feedback that moves learners forward;
(iv) Activating students as instructional resources for one another; and
(v) Activating students as the owners of their own learning.

Formative assessment provides evidence about learning, in order to adapt the teaching and to facilitate improved learning. It is a continuous process that informs both teacher and learners (Stull, Varnum, Ducette, Schiller and Bernacki, 2011). IE combined with GQ and clickers allow continuous, almost effortless formative assessment, coupled with PD and immediate feedback. PD or "pair/share" is a learning opportunity during which small groups discuss the questions and defend their point of view. PI provides students the opportunity to collaborate with their classmates, explain their solution methods, and construct knowledge.

IE and PI per definition satisfy the second, third and fourth of the strategies conceptualised by Black and Wiliam (2009). The lecturer will be the responsible agent to activate the first and fifth of the strategies. IE combined with PI are the ultimate formative assessment methodologies, as assessments are done continuously. Also, the immediacy and ease with which feedback is provided with ARS, makes clickers the ideal formative assessment tool.

Black and Wiliam (1998, p. 61) describe the potential value of formative assessment as follows:

> … significant learning gains lie within our grasp….The gains in achievement appear to be quite considerable, and as noted earlier, among the largest ever reported for educational interventions.

Formative assessment is an effective approach for achieving high-performance student outcomes for lifelong learning (OECD, 2008). Evidence point to the positive effect that formative assessment has on learning. Studies show that the effect size on groups where formative assessment is employed, is around 0.5 and that formative assessment may be especially effective for low-achieving mathematics students (Pengfei, 2007; Dibbs and Oehrtman, 2014).

However, not everybody agrees. A meta-analysis by Kingston and Nash (2011) disputes the validity of many of the studies making the claims of the apparent excessive efficacy of formative assessment.

> An effect size of about 0.70 (or 0.40 - 0.70) is often claimed for the efficacy of formative assessment, but is not supported by the existing research base. More than 300 studies….were reviewed. … Moderator analyses suggested that… two types of implementation of formative assessment, one based on professional development and the other on the use of computer-based formative systems, appeared to be more effective than other approaches, yielding mean effect size of 0.30 and 0.28, respectively (Kingston and Nash, 2011, p. 1).

Nevertheless, it is generally accepted that formative assessment is a powerful tool to improve learning and teaching, It is however important to note that formative assessment should be entrenched in the teaching and learning process, and not as a separate add-on. Furthermore, students' self-assessment should be promoted by creating opportunities for students to express their understanding. When formative assessment is applied effectively, confidence of teachers and students will increase (Black and William, 2006). Both the quality of the feedback and the length of the time lapse between the assessment and the feedback determine the potential value of the feedback and the quality of learning that will subsequently take place – the more immediate the feedback, the higher the potential value of the feedback. IE (IE) certainly falls into this category. IE methods are defined by Hake (1998, p. 3) as "those designed at least in part to promote conceptual understanding through active engagement of students in heads-on (always) and hands-on (usually) activities which yield immediate feedback through discussion with peers and/or instructors."

The relationships amongst the various concepts of the conceptual framework of this study are depicted in Figure 2.3. The relationship between the Theoretical Framework and the Conceptual Framework will be discussed after the next section.

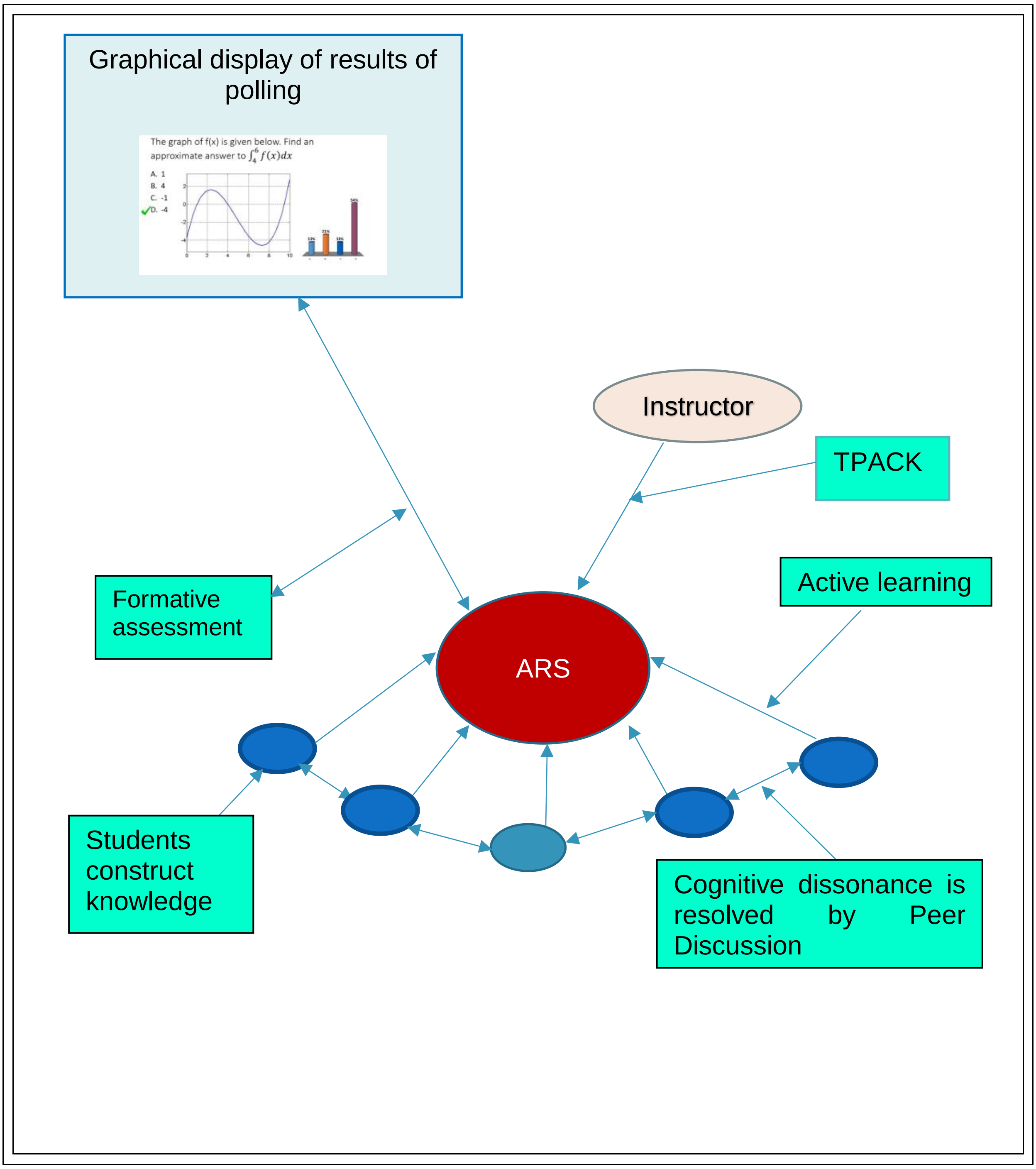

FIGURE 2.3: Conceptual Framework. Source: Researcher

2.3 Literature review

The first part of the literature review will cover the technologies and methodologies used in RBAs in teaching calculus. The second part of the literature review will cover the material included in the ConcepTest used in this study, namely functions, differentiation and integration.

2.3.1 The technologies, methodologies and issues linked to Reform-Based Approaches (RBAs)

In the next section, a short overview is given of the history of software developed for mathematics instruction and the effect it has had on methodologies of calculus instruction. The specific Reform-Based methodologies such as IEM and PI, together with accompanying tools used in this study such as clickers, GQ and ConcepTests, are hence discussed. After a summary of a few studies conducted internationally, all of which were based on these approaches and tools, the link between RBAs and procedural and conceptual knowledge is explained. The core Reform position taken by the Harvard Calculus Consortium (CCH) is explicated. Hence a meta-analysis of eight studies on calculus reform is summarised.

2.3.1.1 The role of new technologies in the development of Reform-Based Approaches (RBAs) to teaching

By the 1980’s, the quality of student learning in mathematics was increasingly being questioned internationally. At the same time, an explosion in the development of technology was experienced worldwide. Experts in the field of mathematics education proposed that the new technologies be used to address the poor quality of learning. This step was generally regarded as the birth of the Reform Movement in Mathematics Education. A few important features associated with Reform Mathematics have been identified from literature on calculus learning and teaching, namely: addressing misconceptions, keeping students active in class, monitoring that learning takes place, developing students’ conceptual understanding, acknowledging gaps in prior knowledge and consequently attempting to bridge the gap between secondary and tertiary education, employment of cooperative learning styles, offering multiple perspectives on subject material and using appropriate technology effectively. This approach to learning and teaching is currently referred to as RBAs. One of the problems that led to the establishment of the Reform Movement, was the emphasis that traditional teaching methods placed on procedural knowledge, often to the detriment of conceptual knowledge (Engelbrecht et al., 2009). Also, Cobb (1994) agrees with Confrey (1990) and Hiebert and Carpenter (1992) who observed that significant qualitative differences existed in the understandings that students developed in instructional situations. Furthermore, these understandings frequently differed from those that the teacher intended. RBAs attempt to address these problems.

2.3.1.2 The development of technology for mathematics instruction

The Calculus Education Reform movement proposed in the 1980s that technology be used to improve the learning and teaching of calculus. According to Garrett (2010), technology can be used as a window into student thinking, as an aid in the use of standard representations, as an aid to reasoning, as an aid to mathematical communication, to reveal and clear up misconceptions, and to empower students. It therefore stands to reason that the development of technology for the learning and teaching of calculus has received the most attention of all the topics in mathematics (Tall et al., 2008). The earliest developments included the introduction of computer languages such as BASIC to program solutions to numerical methods. In the early 1980s, a visual approach to graphs was made possible by the introduction of high-resolution graphics. The introduction of the mouse allowed active interaction with the graphics to translate, stretch and reflect graphs (Tall et al., 2008). Graphing applications in 2-D and 3-D, such as Geometer's Sketchpad (developed in the 1980s), Autograph (developed from 1990) and Geogebra (developed from 2001/2002), combine graphs with dynamic pictures and animations and include topics such as statistics. A current list of almost 100 such graphics software systems is available from Wikipedia. 'Technology in mathematics has evolved from four-function calculators to scientific calculators to graphing calculators and now to computers with computer algebra system software' (Kumar and Kumaresan, 2011, p. 374). Computer Algebra Systems (CAS) were developed in the 1970s, and evolved out of research into artificial intelligence. A current list of more than 45 systems, such as Maple, MathCad, MATLAB and Mathematica, is available from Wikipedia. Despite the proliference of technology and literature to support it, move towards technology use in the classroom has been slow – by 1991 only seven percent of tertiary calculus courses in the United

States employed technology, in spite of calls for their inclusion (Ferrini-Munday and Graham, 1991), as cited by Windham (2008). Some supporters became concerned that calculus reform would die an early death, since reform approaches required mind-set changes and efforts on the part of the instructors, which were more difficult to accomplish than simply retaining the status-quo of traditional lectures. One example is the early versions of Mathematica, which demanded a level of coding skill not normally expected from entry-level calculus students. Kilpatrick, Swafford and Findell (2001), as cited by Windham (2008, p. 18), lamented that "Change in education is notoriously complex, difficult, and unpredictable. Reform movements in mathematics education turn out neither as advocates hope, nor as detractors fear".

2.3.1.3 Reform-Based methodologies

Literature covers a variety of RBAs in calculus education, such employing clickers, mobile phones, social media, multimedia and interactive teaching tools to increase active participation of learners and students (Lucas, 2009; Strasser, 2010; Engel and Green, 2011; Roth, 2012). A number of studies focus on using technology to improve conceptual understanding of calculus (Dimiceli, Lang and Locke, 2010; Engel and Green, 2011; Jaworski and Matthews, 2011; Leng, 2011; Awang-Salleh and Zakaria, 2012; Awang and Zakaria, 2012; Zulnaidi and Zakaria, 2012; Berkeihiser and Ray, 2013; Haciomeroglu and Andreasen, 2013), adapting the sequence of the syllabus for optimal performance (Breetzke, 1988; Keynes and Olson, 2000; Herbert, 2013), using problem-based approaches (Judd and Crites, 2014), the use of cooperative education (Johnson and Johnson, 2002) and using counter-examples (Klymchuk, 2005).

When considering a change in pedagogy, one has to consider a number of factors, such as which approach will fit a particular audience, how to integrate the technology with the pedagogy in a meaningful way and how to balance time constraints with the demands that a change in pedagogy necessarily impose.

In this study, the Reform-Based methods employed were IEM and PD, ConcepTests, GQ and ARS or clickers.

Hake defines IEM as

> … those designed at least in part to promote conceptual understanding through Interactive Engagement of students in heads-on (always) and hands-on (usually) activities which yield immediate feedback through discussion with peers and/or instructors'. Traditional courses are viewed as those 'that make little or no use of IE methods, relying primarily on passive-student lectures, recipe labs, and algorithmic-problem exams (Hake, 1998, p. 65).

According to research done by Hake (1998) on large groups of physics learners and students, significant (two standard deviation) gains were made when lecturers used IEM. It is important to note that the *immediate feedback* part of IE is critical (Black and Wiliam, 1998; Hake, 1998; Epstein, 2013). Epstein (2013) states emphatically that the aspect of immediate feedback, more than any other aspect of RBAs, is the cause of the improvement measured by ConcepTests worldwide.

Wood (2004, p 797) had the following to say about IEM:

> One study after another over the past decade has shown that students who engage interactively with each other and the instructor in the classroom learn concepts better, retain them longer, and can apply them more effectively in other contexts.

PD or "pair/share" is an important phase of IE, during which small groups discuss the questions and defend their point of view. These discussions usually generate a lively atmosphere in the classroom and engage even the more passive students. A sense of community develops, yet another by-product of IE. Students also reported improved time management and test-taking skills (Lucas, 2009, p. 220). Research suggest that the main benefit of using IEM emanates from the PDs (Wood, 2004; Miller et al., 2006). PD provides students the opportunity to collaborate with their classmates, explain their solution methods, and construct knowledge while participating in a community of practice (Lave and Wenger, 1991). The environment is a safe space in which individual point of views can be defended and tested against others' ideas.

Learners share perceptions with each other and with the teacher, and their ideas become modified, selected or deselected, as common meanings develop. This enables learners to become clearer and more confident about what they know and understand (Fosnot, 2005, p. 10).

The PD phase of IE should not be underestimated. Rittle-Johnson, Siegler and Alibabi (2001) claim that students who attempt to explain the conceptual basis of the procedures that they use, gain knowledge from this reflection. It is thus beneficial to prompt learners to generate such explanations, since improved learning may result. Samuelsson (2008, p 247) references Granström (2006) who notes the value of discussion: Dialogue enables students to compare their own knowledge, reasoning and deductions to their peers' knowledge and thinking. Hence adjustments can be made such as correcting misconceptions. Most importantly, students need to convince themselves and their partners of the correctness of a particular method.

The value of PD may be in the scaffolding that ensues when students assist one another (see Figure 2.4 for two configurations of interaction) to 'advance in the Zone of Proximal Development (Vygotsky, 1934/1986)' (Samuelsson, 2008, p. 240). Furthermore, motivation improves when students discuss problems in small groups (Samuelsson, 2008).

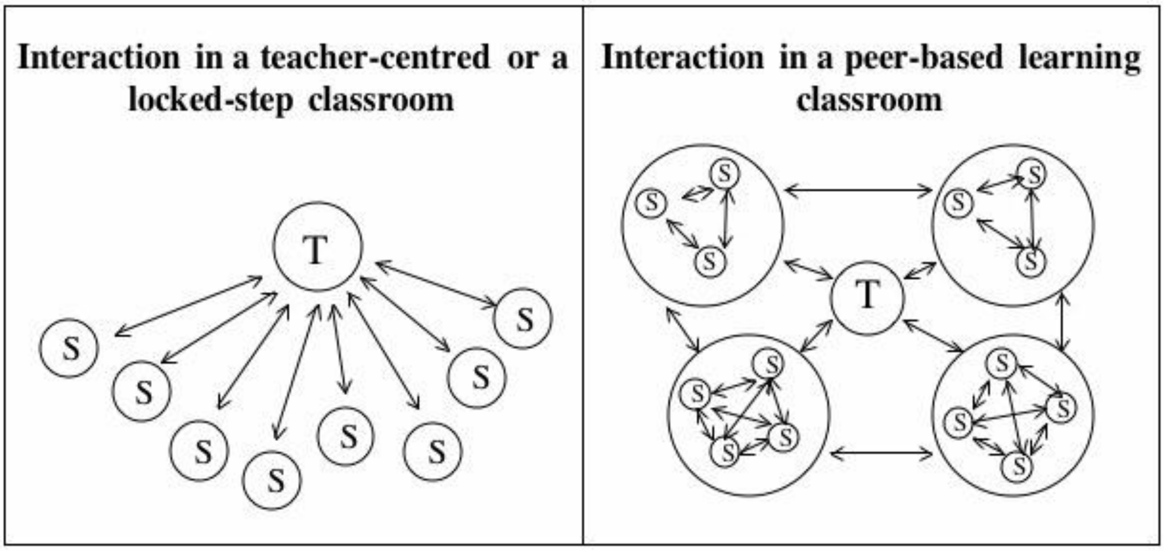

FIGURE 2.4: Interaction configurations. Source: arts.unimelb.edu.aus

Conceptual tests measure conceptual understanding of fundamental principles. Conceptual tests usually do not have a computational component. Conceptual tests have a twofold objective, namely to reveal misconceptions and/or weak conceptions, and to measure conceptual gains of groups of students. Concept assessments have been used widely to ascertain the prevalence of misconceptions, guide instructional decisions, and determine the effects of instructional interventions (Couch and Knight, 2015)

ConcepTests were developed by Harvard physics instructor Eric Mazur (Mazur, 1997) to assess patterns of student thinking and misconceptions. The instructor poses a "conceptual" question or problem presented in a multiple choice format. Students vote using a response system in class that summarises the students' choices and gives immediate feedback to the lecturer. Each student explains/defends his or her answer in small collaborative groups. This is also called PI or PD. Students vote again on the best answer. The instructor displays the results of the voting and explains the best choice.

"Concept inventories are tests designed to measure the most basic knowledge in a field" (Thomas, 2014b, p. 1079). Concept inventories are mostly used in the form of Pre-tests and Post-tests to measure gain in conceptual understanding after a course of instruction. Concept inventories are therefore usually used as measuring instruments to evaluate instruction which uses ConcepTests.

Results achieved by various researchers worldwide have been quite dramatic. Pilzer (2001) found the following results for students taught by ConcepTests and PD (Table 2.4).

TABLE 2.4: Results of teaching with ConcepTests and PD

	Conceptual Problems	**Standard Problems**
Taught with ConcepTests	73%	63%
Traditional Lecture	17%	54%

A major breakthrough for concept inventories in Mathematics was the Calculus Concept Inventory (CCI) developed by Epstein (2013). The CCI was modelled on the Force Concept Inventory (FCI), used to measure conceptual gains in an introductory course on mechanics (Hestenes, Wells and Swackhamer, 1992). The CCI contains items on functions, derivatives, and a third section on limits, ratios, and the continuum. The CCI measures conceptual understanding of the fundamental principles of differential calculus. The test consists of 22 multiple choice questions, and distracters are designed to elicit misconceptions.

Conceptual tests should however be used with caution. The overuse of conceptual tests could lead to despondency and lack of motivation amongst students, especially if the tests are pitched at a level too high for the average student. One has to take care with the selection of the material, especially in the case of students enrolled for STEM diplomas, whose prior knowledge in mathematics may not match those of students registered for STEM degrees.

The GoodQuestions project was initiated in 2005 at Cornell University in the United States (Terrell, 2005). The project is described on their webpage as follows:

> The GoodQuestions project seeks to improve calculus instruction by adapting two methods developed in physics instruction, namely ConcepTests and Just-in-Time-Teaching. GoodQuestions is a pedagogical strategy that aims to raise the visibility of the key concepts and to promote a more AL environment (www.math.cornell.edu/).

According to Terrell (2005), the essence of the approach is to develop questions that:

(i) stimulate students' interest and curiosity in mathematics;
(ii) help students monitor their understanding;
(iii) offer students frequent opportunities to make conjectures and argue about their validity;
(iv) reflect the role of student prior knowledge and misconceptions in building conceptual understanding;
(v) provide instructors with frequent formative assessments of what their student are learning and
(vi) support instructors' efforts to foster an AL environment.

GQ are therefore multiple choice questions that stimulate discussion and help students and lecturers to probe and reveal misconceptions. These questions do not necessarily have a unique correct solution, but could have more than one interpretation, or no solution. GQ elicit students' prior knowledge and connect it to the mathematics they are learning (Miller et al., 2006). GQ are questions that are mostly non-computational, that are related to students' experiences, are memorable and surprising, and help build on students' prior knowledge and partial understanding. GQ usually lead to lively discussions and debate (Terrell, 2003). As mentioned elsewhere, the role of GQ in this study was to kindle perturbations, which get resolved with the aid of PD.

Clickers are devices used for input into electronic voting systems or ARS which allow communication between an audience and a facilitator or instructor. The instructor usually presents multiple choice questions on a PowerPoint slide. After anonymous voting, the results are displayed graphically on a screen (Bode et al., 2009).

IEM require the use of technology for immediate feedback. The technology that is used worldwide is called ARS, clickers, keypads, handsets or zappers. The handheld devices used in an ARS are transmitters, about the size of a small cellular phone. Students use clickers to transmit answers by pressing a button on the clicker that corresponds to the correct answer to a multiple choice question (Figure 2.5 and Figure 2.6). Modern clickers usually have a 10-digit numeric keypad and use unique radio-frequency signals in order to receive and record every individual's response. These responses are immediately available to the lecturer. Answers are displayed on a screen, usually in the form of a histogram. Although polling is done in such a fashion that students are not able to identify each other's individual responses, the responses are recorded in the computer that it is linked to, and the software allows statistical analysis of each students' individual response, as well as the groups' responses. Students therefore feel safe to answer, since their answers are not publically linked to their identities. The software typically allows the instructor to specify which answer(s) is correct, and the answers can be graded. These tools also permit different point values to be given for correct versus incorrect answers (Caldwell, 2007). The systems can thus be used for formative assessment and can also be used to upload data to, and interact with, with classroom management systems such as Blackboard and WebCT (Caldwell, 2007).

FIGURE 2.5: Clickers used in this study. Source: http://www.clickers.psu.edu

A histogram is displayed on the data projector depicting the ratio of each multiple choice answer chosen by the group. Students are able to see the distribution of choices and rate themselves accordingly. The instructor uses this as a teaching opportunity to resolve conflicts and address misconceptions.

FIGURE 2.6: Turning Technologies' Graphical Interface. Source: http://turningpoint-uk.com

Caldwell (2007) make reference to a number of authors in a summary of the advantages of using clickers. Clickers help instructors to:

- increase or manage interaction, through questions that:
  - start or focus discussions (Jackson and Trees, 2003).
  - require interaction with peers (Knight and Wood, 2005).
  - collect votes after a debate (Draper, 1998)
- assess student preparation and ensure accountability, through:
  - questions about reading or homework (Knight and Wood, 2005).
  - prelab questions.
- find out more about students by:
  - surveying students' thoughts about the pace, effectiveness, style, or topic of a lecture.
  - polling student opinions or attitudes.
  - probing students' pre-existing level of understanding.
  - asking how students feel about clickers and/or AL.
  - assessing students' understanding of material in a lecture.
  - revealing student misunderstandings (e.g. Wood, 2004).
  - testing students' understanding of previous lectures or notes.
  - assessing students' ability to apply lecture material to a new situation.
  - determining whether students are ready to continue after working a problem (Poulis, Massen, Robens and Gilbert, 1998a).
  - allowing students to assess their own level of understanding at the end of a class (Halloran, 1995).

The information collected during a clicker session helps to determine the future direction of lectures, and also the optimal level of difficulty that students will be able to process with success. The immediacy of the feedback, and the opportunity that is created for cooperative learning and PD, make clickers a valuable teaching tool.

A number of empirical studies have been conducted to compare courses that have used clickers with those that have used other methods of instruction. According to these studies, improved student performance can be attributed to the high frequency of formative assessment when employing IEM and PD as the pedagogy. The value of clickers used for formative assessment has been reported in various fields, such as physics (Cruz, Dias and Kortemeyer, 2011; Deslauriers, Schelew and Wieman, 2011; Majerich, 2011; Guo and Shekoyan, 2014), dentistry (Briggs and Keyek-Franssen, 2010), mathematics for engineering (Bender and Thiele, 2014) and other disciplines (Hoekstra and Mollborn, 2012). All these studies reported positive results and attributed this improvement to regular formative assessment facilitated by the employment of clickers.

Many studies have been conducted to study IEM by using the quantitative paradigm. One of the biggest research projects involved more than six thousand introductory physics students over diverse student populations in high schools, colleges and universities (Hake, 1998). Data from pre/Post-tests, using the Halloun–Hestenes Mechanics Diagnostic test or FCI, were reported for 62 introductory physics courses. The effectiveness of a course in promoting conceptual understanding, the average normalised gain <*g*>, was calculated as the ratio of the actual average gain to the maximum possible average gain. The class performance was measured by the normalized gain, defined to be:

$$< g > = \frac{\text{gain}}{\text{possible gain}} = \frac{\text{class mean posttest \% score}}{100 - \text{class mean pretest \% score}} = \frac{\bar{X}_f - \bar{X}_0}{100 - \bar{X}_0}$$

> The normalized gain score measures the fraction of previously unknown material that is learned throughout the course. For example, if a class average on a pretest were 40% and 70% on the posttest, the normalised gain would be *G* = 0.5, meaning that class, on average, correctly answered half of the 60% of the material they answered incorrectly at the beginning of the semester (Thomas, 2014a).

The FCI was administered once at the start and once at the end of a first course in physics. Fourteen ‘‘traditional’’ courses, which made little or no use of interactive-engagement (IE) methods, achieved an average gain of 0.23 ± 0.04 of a standard deviation. In sharp contrast, 48 courses (N = 4458) which made substantial use of IE methods achieved an average gain of 0.48 ± 0.14 standard deviations, almost two standard deviations above that of the traditional courses (Hake, 1998). The average *G* for IE courses thus exceeded the average for traditional courses by about two standard deviations. Furthermore, results for 30 (N = 3259) of the above 62 courses on the problem-solving Mechanics Baseline test of Hestenes–Wells imply that IE strategies enhance problem-solving ability. The conceptual and problem-solving test results strongly suggested that IE methods can increase effectiveness of courses on mechanics well beyond that obtained in traditional practice (Hake, 1998). Epstein (2013) reported as follows:

> According to Thomas (2014a), results from studies using the Calculus Concept Inventory (CCI) have been less clear than those from the FCI. In a study involving 1342 Calculus I students at the University of Michigan (U-M) in 2008, the CCI was used to test the gain in conceptual understanding of students after being taught with IE methods.
> The results of the CCI at U-M include the following:
> - The average gain *G* over all fifty-one sections was 0.35.
> - Ten sections had a gain of 0.40 to 0.44.
> - The range of the gain scores was 0.21 to 0.44

Some studies using IE combined with the CCI reported smaller gains, such as the one by Mantini, Trigalet and Davis (2014). They reported gains between 10.2 percent and 17 percent in two consecutive semesters.

RBAs stress the importance of conceptual understanding. Since the aim of this study was to improve students' conceptual understanding of calculus, it becomes necessary to investigate the relationship between procedural and conceptual knowledge. Engelbrecht et al. (2009, p. 704) offered the following definitions of the two concepts, adding that the word *knowledge* can be replaced with understanding, thinking, or fluency:

> Procedural knowledge is the ability to physically solve a problem through the manipulation of mathematical skills, such as procedures, rules, formulae, algorithms and symbols used in mathematics.
>
> Conceptual knowledge is the ability to show understanding of mathematical concepts by being able to interpret and apply them correctly to a variety of situations as well as the ability to translate these concepts between verbal statements and their equivalent mathematical expressions. It is a connected

network in which the linking relationships are as prominent as the separate bits of information.

Procedural knowledge encompasses the language of mathematics as well as the rules, algorithms and procedures used in mathematics. Procedural fluency is the skilful execution of procedures and algorithms. Rittle-Johnson et al. (2001) defined procedural knowledge as the execution of sequences in order to solve specific types of routine problems. The knowledge is therefore linked to specific problem types and not generic. Conceptual knowledge is knowledge about connections and interrelations between concepts. As already mentioned, mathematics is accumulative in nature and new concepts are built on, and linked to, previously established ideas, knowledge and procedures. Conceptual knowledge thus includes procedural knowledge and can be demonstrated by "applying known principles or techniques in new situations" (Thomas, 2014b, p. 1078). Unlike procedural knowledge, conceptual knowledge is more flexible and generic and not linked to specific types of problems (Rittle-Johnson et al., 2001). Conceptual knowledge cannot be learned by rote.

Competing theories have been proposed about which type of knowledge develops first (Rittle-Johnson et al., 2001). Davis, Gray, Simpson, Tall and Thomas (2000) assert that learners first have to negotiate a procedural phase before they can attain conceptual knowledge. Chappell and Killpatrick (2003) propose that conceptual understanding aids the gaining of procedural skills. An iterative model of knowledge development put forward by (Rittle-Johnson et al., 2001), suggests that procedural and conceptual knowledge develop symbiotically with problem representation as the mediation factor between the two knowledge types. Authors agree that the two concepts are not mutually exclusive. Procedural and conceptual knowledge are tightly interwoven, with no strict demarcations (Mahir, 2009). It is not always easy to distinguish between the two knowledge types, since they lie on a continuum (Rittle-Johnson et al., 2001). The novelty of the task to be executed can be used as a factor to distinguish which of the two knowledge types are at play. Engelbrecht et al. (2012) however caution that conceptual problems can eventually change to procedural if encountered repeatedly by a student. A problem can only be classified as conceptual if the student has not encountered it before. It is clearly not easy to distinguish between the two types of knowledge. In fact, measuring the difference between procedural and conceptual knowledge with an acceptable degree of validity and reliability has always presented a major challenge to mathematics education researchers. Bisson, Gilmore, Inglis and Jones (2016) allude to two traditional approaches to measurement, the first being psychometrically validated instruments to measure knowledge of a particular concept such as the Calculus Concept Inventory (Epstein, 2007, 2013), and the second being one-to-one clinical interviews combined with a scoring rubric to rate the quality of each participant's understanding. Both these methods share the disadvantage of being long and resource-intensive processes that must be repeated for every concept of interest. Furthermore, their measures may be disputed by other experts. Also, with respect to interviews, the raters' skills may be disputed and the results may hence be unreliable. Bisson et al. (2016) propose a more reliable approach, Comparative Judgement (CJ), which allows student responses to short, open-ended questions to be judged in pairs by several experts. The judges have to decide which of the two answers presented to them shows superior conceptual understanding of the topic. Once all the answers have been judged several times, a ranked order is constructed representing the collective expertise of the judges. The ranking is hence used in statistical modelling to calculate a standardised parameter estimate (z-score) representing the quality of each student's answer. CJ uses no detailed assessment criteria or scoring rubrics and is based on a psychological principle that humans are better at comparing two objects against one another than they are at comparing one object against specified criteria (Thurstone, 1994), as cited by Bisson et al. (2016).

Bisson et al. (2016) argue that CJ offers a cheaper and more efficient approach than traditional test development, which can take years to design, test and refine, but mention instances where CJ may be less efficient. One such example is large scale studies that focus on a specific concept such as the CCI to assess undergraduate understanding of calculus. Using CJ for such studies may invoke prohibitive costs.

Emphasis on procedural fluency has typically been associated with traditional mathematics instruction whereas RBAs have put more emphasis on conceptual understanding (Thomas, 2014b). Advocates of the reform movement in calculus argue that students lack conceptual understanding and that teachers spend too much time promoting procedural fluency to the detriment of conceptual understanding. This is a hotly debated issue fundamental to the differences between those in the reform camp and those advocating traditionalist instruction. It is not a simple matter – even Jerome Bruner, the godfather of many of the constructivist ideas championed by the reform movement, noted that “adults typically require a certain amount of motoric skill and practice before they are able to develop an image representing their actions” (Driscoll, 1999, p. 226). Windham argues that although there may have been too much emphasis on procedural methods as the reformers point out, mechanical skill might still be a required necessary precursor to deeper understanding (Windham, 2008). Rittle-Johnson et al. (2001) likewise warn against downplaying the value of procedural knowledge in reform approaches. They argue that both knowledge types are essential for competency in mathematics. It is also worth noting that some researchers believe that constructivist approaches which focus on conceptual knowledge become feasible only after basic skills have been acquired. One such researcher asserts that a skills base is established through direct instruction combined with extensive drill and practice (Schollar, 2004). Engelbrecht and Harding (2015) assert that the transition from school mathematics to tertiary mathematics in South Africa is convoluted by the disproportionate emphasis on procedural approaches in mathematics in secondary education and the practice of examination coaching. Therefore, students have to undergo a change in thinking approach, from procedural to more conceptual, and a culture of independent learning needs to be fostered.

Stroumbakis (2010) alludes to a more profound type of procedural knowledge mentioned in the literature, sometimes termed utilisation competence. This type of knowledge involves “knowing when to use a procedure, understanding available shortcuts, seeing the relationships among and goals of parts of the procedure without necessarily understanding the concepts that are operated upon” (Stroumbakis, 2010, p. 35).

Kilpatrick et al. (2001) view procedural and conceptual knowledge as part of mathematical proficiency, while also listing three other parts, namely strategic competence (this includes the capacity to formulate and plan a solution of a mathematical problem); adaptive reasoning (this includes the capacity for mathematical-logical thought and expression) and lastly, a productive disposition. It is clear that relationship between procedural and conceptual knowledge is complex and despite a proliferation of literature on the topic, still not fully understood.

RBAs and calculus cannot be used in the same context without reference to the CCH. The CCH is ‘a consortium of eight diverse schools (including a high school and a community college), with Harvard University as the base. Its mission was to completely redesign the calculus curriculum. A Reform-Based book was written (Hughes-Hallett et al., 1993) and RBAs in calculus were designed by the Consortium. Both the book and the approach differ from the traditional versions that had become standard since the 1960’s. The approach places heavy emphasis on graphical analysis, numerical estimation and the concepts underlying calculus, with less time spent on algebraic manipulations, formalisms, and rote calculations. The Harvard Consortium Calculus advocates a multiple perspective approach to calculus, using technology to visualise concepts (Knill, 2009).

Although use of technology is incorporated into the course, it is not tied to a particular technology. The text and approach have been used at over 500 institutions in the last four years and have been judged by teachers and students alike to be a success and an exciting and refreshing experience’ (Lomen, Lovelock and McCallum, 1993). The multi-perspective approach was one of their basic guiding principles called The Rule of Three: ‘Every topic should be presented geometrically, numerically, and algebraically’ (Hughes-Hallett et al., 2005, p. vii). The Rule of Three has since been extended to The Rule of Four (Figure 2.7), which also includes verbal and written representations, such as lectures, discussions, PD, notes, textbooks and online resources (Harvard-Rule-of-4; Hughes Hallett, 2007).

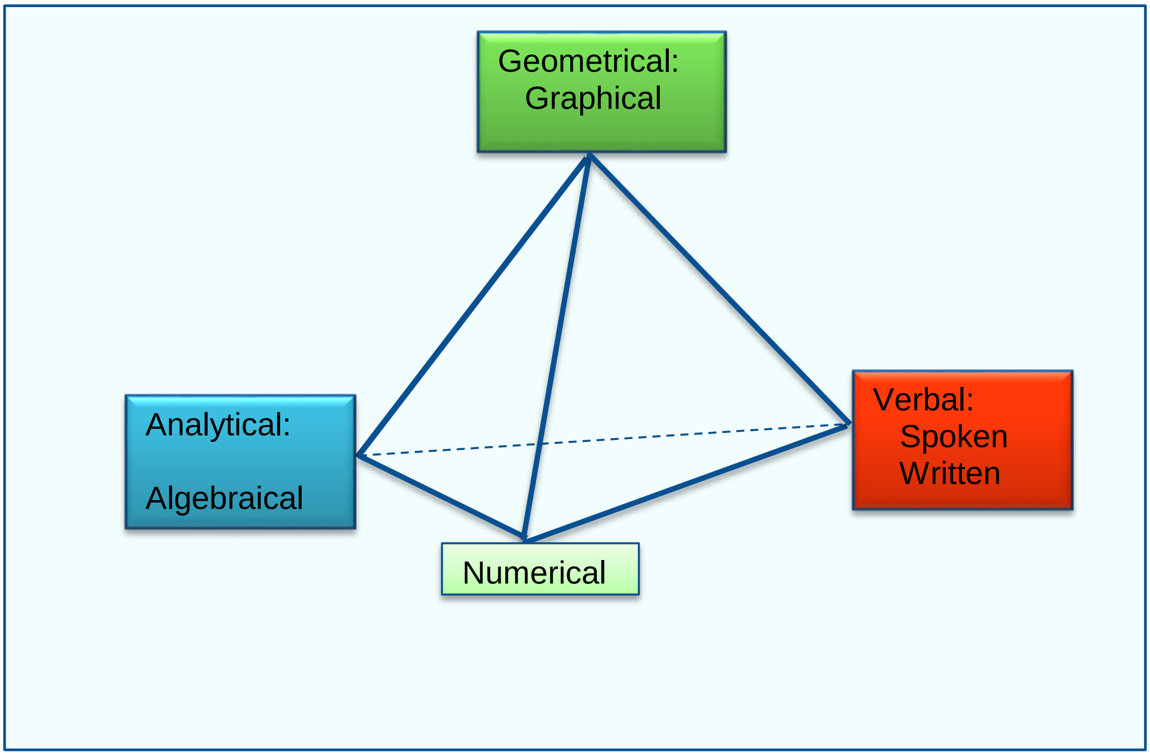

FIGURE 2.7: The Harvard Rule of Four.

Source: Harvard-Rule-of-4 (Knill, 2009, p. 3)

The Rule of Four may be extended to The Rule of Nine, which advises to "Look at a calculus problem algebraically, analytically, geometrically, historically, graphically, numerically, conceptually, psychologically, as well as experimentally" (Knill, 2009, p. 3). Although this advice makes sense, one has to take care. When taking too many approaches to a topic, students may feel overwhelmed with information and complexity (Knill, 2009, p. 3). The same care should be taken when conceptual problems become the main focus of the pedagogy. This should be done in moderation and the ratio of conceptual problems should be limited, especially for students with a weak background in mathematics.

Some studies conducted on RBAs used in the learning and teaching of calculus have shown positive results. Kueffer and Latterell (2001) conducted a meta-analysis of eight studies on Reform-Based calculus. Studies were selected based on definitions of reform calculus and conceptual understanding as well as various methodological criteria.

In the first study (Beckmann, 1988) examined two different courses, both dependent on the use of computer graphics. The students used computer graphics software to complete supplemental assignments. In the traditional course, skills development was emphasized and no graphical software was used. Findings indicated that the subjects from the reform section displayed higher levels of understanding and interest, yet retained compatible skills levels.

A second study by (Brunett, 1995) used a reform textbook and students participated in a weekly lab that comprised group work and real world problems. In this study, the experimental and control groups were given items from a placement test that was originally published by the Mathematical Association of America. The traditional section outperformed the reform section on these items.

The third study (Cooley, 1995) required students in the reform class to attend weekly sessions in a computer based on activities and assignments using *Mathematica* software. These were intended to enhance their conceptual understanding of calculus. Based on final exam results and the results of a researcher-written conceptual exam, it was found that the reform class did better on both conceptual and traditional calculus questions, including computational questions.

In the fourth study done by Cunningham (1991), the two groups involved in the study were similar in nature, except for the amount of technology used. The control group was only exposed to the computer during classroom demonstrations. The experimental group had classroom demonstrations and also had outside access to the software for homework and as a study aid. The experimental group frequently saw things done on the computer before they learned how to solve the problems manually. These students were allowed to use the computer during examinations. It was concluded that no significant differences were found in conceptual understanding between the two groups.

The experimental group in the fifth study by Garner (1998) used a reform text whereas the control group did not. The experimental group was exposed to activities and more student interactions, emphasizing conceptual skills and application problems. No significant difference between reform and traditional groups was found on the written test mean scores.

In the sixth study by Heid (1998), the reform group of students used various approaches to study calculus concepts and ideas and made use of personal computers for homework and assignments. The students in the experimental class outperformed the experimental group on conceptually oriented questions and they were able to discuss calculus concepts in more detail.

In a study by Hurley, Koehn and Ganter (1999), the reform class had a computer lab day instead of a one hour lecture and also participated in a group problem solving session, during which students worked on problems that were both conceptual and computational. The reform students did better than the traditional students when given the same final exam.

In the final study by Keller and Russell (1997) the reform section employed Mathematica projects. They participated in written labs and worked in groups of 3 to 4 students. The traditional students did not use technology. It was found that the reform students did better than the traditional students when tested in the departmental two-hour comprehensive final.

Although Kueffer and Latterell (2001) concluded that reform teaching techniques increased the conceptual understanding of undergraduate calculus students, it is worth noting that only four of the eight studies included in the meta-analysis had results in favour of reform methods, two showed no difference and two were in favour of traditional methods.

2.3.2 Literature review of the Mathematics topics included in this study

2.3.2.1 Misconceptions

Teachers should be informed about which concepts students struggle with and the ways to overcome the common difficulties and misconceptions (Zeytun, Çetinkaya and Erbas, 2010). I therefore included this topic in each of the discussions.

A misconception can be defined as a “feature of a student's knowledge which may develop as a result of overgeneralizing or may be due to interference from everyday knowledge. A misconception must have a reasonably well-formulated theory, it should be repeatable and/or explicit rather than random and tacit” (Leinhardt et al., 1990, p.

30). Smith, DiSessa and Roschelle (1993) define misconceptions as amateur and naïve preconceptions that learners believe to be valid. Olivier (1989) takes the opposite view, calling misconceptions intelligent constructions based on correct or incomplete (but not wrong) previous knowledge. Nesher (1987) in turn views the idea of a misconception as ‘a line of thinking that causes errors, all resulting from an incorrect underlying premise, rather than sporadic, unconnected and non-systematic errors’**.** According to Stroumbakis (2010), misconceptions arise when the concept image is distorted, possibly as a result of limited exposure to examples.

Seemingly, misconceptions cannot be avoided. They may however hamper the development of conceptual understanding of fundamental mathematical concepts (Olivier, 1989). Also, conceptual knowledge is an interrelated network and misconceptions may affect other parts of this network of knowledge.

Not all misconceptions are robust. Learners release some misconceptions fairly easily if this process is facilitated by a more knowledgeable other. These transitional misconceptions could be referred to as transitional or amateur concepts (Vygotsky, 1978) or “preceptions” (Chi, 2005). All levels of misconceptions are important in mathematics teaching and learning.

One aspect of misconceptions that most researchers agree on is their *stickiness*. Most misconceptions are deeply engrained and robust, defying all attempts to eradicate them, such as attempts to create cognitive conflict or dissonance. Such misconceptions are well rationalised by learners (Makonye, 2011). Confrey (1987) reported that misconceptions are resilient even in the face of instruction meant to overcome them and that the attraction of misconceptions is compelling to students. Misconceptions in tertiary mathematics may be more ‘sticky’ or resistant to remediation, because of years of reinforcement (Campbell, 2010).

However, even though they are tenacious, misconceptions are potentially viable and challengeable, that means that they could change. Errors and misconceptions are linked to both procedural and conceptual knowledge (Hiebert and Wearner, 1986), as cited by Makonye (2011). Learners' misconceptions in mathematics often arise out of a backdrop of misunderstanding of how new concepts link with the old. According to Leinhardt et al. (1990), misconceptions in science often originate in observations and interpretations of real-life events, whereas those in mathematics mostly originate in formal learning. Difficulties experienced by students do not always point to an underlying misconception, but could imply that something about the specific task makes it especially difficult.

2.3.2.2 Functions

In the NCTM's *Developing Essential Understanding of Functions (NCTM, 1989)*, functions are organized into five *Big Ideas*. This book mentioned five *Big Ideas* worthy of teachers' focus, the Function Concept; Covariation and Rate of Change; Families of Functions; Combining and Transforming Functions and lastly Multiple Representations of Functions. I used a similar framework to discuss functions, but altered the second category to also include other research perspectives of functions.

A function is a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output. A more formal definition follows:

A function $f: X \rightarrow Y$ is a subset of $X \times Y$ such that if $(x; y_{1)} \in f$ and $(x; y_{2)} \in f$ implies that $y_1 = y_2$, which is based on the Bourbaki definition:

> Let E and F be two sets, which may or may not be distinct. F is called a functional relation in y if, for all $x \in E$, there exists a unique $y \in F$ which is in the given relation with x (Bourbaki, 2003).

The function concept has two fundamental properties: the univalence and the arbitrariness conditions. The univalence states that every element of the domain must be assigned to a unique element in the co-domain. The arbitrariness suggests that a function could implement transformation in an arbitrary manner; it therefore rules out attributing a mechanical rule, algebraic or otherwise, to the concept (Malik, 1980).

Function concepts are crucial ideas in the study of calculus and in many other mathematical areas, such as limits, derivatives, and integration. The importance of functions is such that some researchers have labelled them "gatekeepers" to tertiary mathematics (Nilklad, 2004). Judd and Crites (2013) cite various studies that claim the importance of not only function, but aspects of functions such as covariation, function composition, function inverse, quantity, exponential growth, and trigonometry in pre-calculus and calculus. Several aspects of calculus are also interconnected to function concepts, such as the derivative as a rate of change, the various rules of differentiation, the derivative and anti-derivative as inverses of each other, multiple representations of the derivative and the derivative as a limit (Sofronos, DeFranco, Vinsonhaler, Gorgievski, Schroeder and Hamelin, 2011). It follows that students with a strong conceptual understanding of functions should succeed in their studies of calculus and conversely, students who experience difficulties in understanding key ideas of calculus, have weak understanding of the function concept (Tall, 1993; Nilklad, 2004; Othman, Asshaari, Tawil, Ismail, Nopiah and Zaharim, 2012; Thomas, de Freitas Druck, Huillet, Ju, Nardi, Rasmussen and Xie, 2012; Attorps, Björk, Radic and Virman, 2013; Vrabel, 2014). The importance of the function concept is reiterated by the urgent appeal of the NRC (1989) of the USA in Everybody Counts: "if undergraduate mathematics does nothing else, it should help students develop function sense" (NRC, 1989, p. 51). Carlson, Madison and West (2010a) mention other national bodies and policy makers which have made similar calls for pre-calculus level curricula to place greater emphasis on functions in the USA. They lament that students' inability to reason about and represent how two quantities change together causing problems for students in learning ideas of calculus such as limit, derivative, accumulation and the Fundamental Theorem of and ideas in differential equations. Limited understanding of the function concept has furthermore been shown to negatively affect the transition from secondary to tertiary education (Thomas et al., 2012).

Balyta (2007, p. x) asserts that the concepts of functions are not only useful and important in mathematics, but are also used outside the mathematics field and in real-life situations, such as accounting and the stock market:

> Debates over such international issues as those of global warming, population control, radioactive waste, inflation rates, and the national debt, often revolve around understanding the mathematical behaviour of functions, especially how one quantity changes in relation to another, or how one quantity changes over time. The importance of this for an educated citizenry is indisputable.

Although functions are central concepts in algebra, many research studies on secondary and tertiary students have shown that these concepts are some of the most difficult for students to understand (Nilklad, 2004). Various studies have shown that students do not have a deep understanding of the concept of function (Vinner, 1992), and are unable to solve problems as expected (Judd and Crites, 2014). In South Africa it has been no different. The Diagnostic Report on the 2015 NSC Mathematics examinations, mentions that the second lowest marks for a question were achieved for the section on functions and graphs - straight lines, inverses and their integration with calculus, for which the average was 26% (DBE, 2015, p. 160). The worst section was question 10, a calculus application in optimisation (22%).

Functions have been studied from various perspectives, such as the concept image of Tall and Vinner (1981), the APOS (action, process, object, scheme) theory of a function (Breidenbach, Dubinsky, Hawks and Nichols, 1992; Asiala, Brown, DeVries, Dubinsky, Matthews and Thomas, 1996; Asiala, Cotrill, Dubinsky and Schwingendorf, 1997; Dubinsky and McDonald, 2001) and a function as covariational reasoning (Carlson and Oehrtman, 2005; Castillo-Garsow, 2010).

The terms concept image and concept definition were coined by Vinner in 1981. A concept image is defined to be the cognitive structure associated with a concept in the individual's mind and includes the individual's interpretations of characteristics and processes that the individual connects to the concept. The concept image also includes intuitive ideas, mental images and associated properties and processes. These aspects are not always coherent and may differ from the formal concept definition. The cognitive structure is continually adjusted after successive exposure to the concept, during which different aspects of the concept image can be activated (Tall and Vinner, 1981). A part of a concept image or concept definition that may conflict with another part of the concept image or concept definition is called a potential conflict factor. If such a factor is evoked, it becomes a cognitive conflict factor.

The concept image theory of Tall and Vinner has proved to be a useful framework for investigations of students' mathematical behaviours and also when analysing their mathematical concepts (Breen, O'Shea, Larson and Pettersson, 2016). Tall and Vinner (1981) have investigated the concept image and concept definition as these relate to limits and continuity. Examples of other topics that have been studied using this framework, are increasing and decreasing functions, (Ian and Vinner, 1998), quadratic functions (Vaiyavutjamai, 2009), even and odd functions (Rasslan and Vinner, 1997) and inverse functions (Breen et al., 2016).

Based on Piagetian constructivism, a theory termed the APOS theory of functions was developed by Breidenbach et al. (1992). Dubinsky and McDonald (2001) further refined it and postulated a hierarchy of concept development, in which the student starts from actions, proceeds to processes, then to objects and ultimately to mental schemas. An action is a transformation of objects. An example of a student who entertains an action view of a function is when the student has a narrow view of a function as an equation to calculate output values for a given set of input values. The student's view of a transformation is therefore limited to an action.

A process view of a function develops when the action is repeated, the results collectively reflected upon, and internalised as a process. With a process conception of a function, a student will be able to link processes in order to construct a composition, and will also be able to reverse a process to obtain an inverse.

Continual reflection upon a function as a process may result into its eventual encapsulation as an object (Breidenbach et al., 1992). Not all students progress to an object view of a function, but when they do, their understanding of functions becomes stronger and more sophisticated (Vrabel, 2014).

> When an individual reflects on operations applied to a particular process, becomes aware of the process as a totality, realizes that transformations (whether they be actions or processes) can act on it, and is actually able to construct such processes, then he or she is thinking of this process as an object. In this case, we say that the process has been *encapsulated* as an object (Asiala et al., 1996, p. 8).

Transitions through the stages of action→process→object→schema (APOS in short) are usually not linear, but involve moving between stages (O'Shea, Breen and Jaworski, 2016). Carlson, et al., (2010) assert that a process view is also necessary for students to develop covariational reasoning, regarded as a critical component to a student's calculus readiness. Several researchers have claimed that students need at least a process view in order to develop a strong understanding of functions (Asiala et al., 1996; Dubinsky and McDonald, 2001; Carlson, Oehrtman and Engelke, 2010b; Maharaj, 2010; Judd and Crites, 2013) and to perform reasonably well in a calculus course.

The APOS theory has not been devoid of criticism. (Tall, 1999) claims that in most cases concept development does not follow a strict APOS sequence and therefore APOS is not always an appropriate theory. He suggests using APOS as a valuable tool but not as a general template (Kimani, 2008).

Sfard (1991) developed a conceptual framework that describes how students perceive functions. This framework is somewhat similar to object-process framework discussed under APOS, and is called the structural-operational framework. According to (Harel and Dubinsky, 1992), many mathematical concepts such as number, variable and function can be viewed from two complementary perspectives, as processes (operational view) and as objects (structural view).

A student with an operational view of a function has either a correspondence conception (e.g. Sfard, 1991) or a covariational conception (Confrey and Smith, 1995; Carlson, Jacobs, Coe, Larsen and Hsu, 2002). The correspondence conception is consistent with the traditional definition of a function as a correspondence between two sets, the domain A and the range B such that for every element in A, there corresponds only one element in B.

Various researchers, such as Confrey and Smith (1994) and Saldanha and Thompson (1998), have studied functions from a covariational perspective. Covariational reasoning involves awareness of two quantities that are changing concurrently. This concept is required to construct formulas and graphs when modelling relationships in applied contexts. Carlson, Larsen and Jacobs (2001, p. 354) define covariational reasoning as “the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other”. A covariational approach thus entails moving between successive values of one variable and coordinating this with moving between corresponding successive values of another variable, thus moving operationally from y_m to y_{m+1} as x moves from x_m to x_{m+1}.

The Covariational framework for learning functions has been reported to have more potential than the traditional correspondence position (Strom, 2006) and is particularly useful in understanding rate of change in calculus (Confrey and Smith, 1995; Carlson, 1998). Covariational reasoning is regarded as important for understanding the function concept (Confrey and Smith, 1994; Strom, 2006; Castillo-Garsow, 2010) and is thus critical for success in calculus. However, prospective mathematics teachers and even in-service teachers have difficulties with the concept (Koklu and Jakubowski, 2010; Zeytun et al., 2010).

Various covariational frameworks have been developed. The framework developed by Carlson (1998) and colleagues (Carlson et al., 2002) contains five distinct developmental levels: Coordination; Direction; Quantitative Coordination; Average Rate and Instantaneous Rate (Zeytun et al., 2010). Saldanha and Thompson (1998) also proposed a developmental model of covariation. Their model makes a distinction between the earlier and later stages. During the earlier stages, a learner should be able to coordinate the values of two quantities, such as one value of the independent variable of a function with the value of the dependent variable. During the later stages, time becomes the central focus:

> Later images of covariation entail understanding time as a continuous quantity, so that, in one's image, the two quantities' values persist. An operative image of covariation is one in which a person imagines both quantities having been tracked for some duration, with the entailing correspondence being an emergent property of the image (Thompson, 1994). In the case of continuous covariation, one understands that if either quantity has different values at different times, it changed from one to another by assuming all intermediate values (Saldanha and Thompson, 1998, p. 2).

Various software packages such as Geogebra (Hohenwarter, Hohenwarter, Kreis and Lavicza, 2008; Andraphanova, 2015), Autograph, Geometer's Sketchpad (Iyer, 2009) and Cabri tools (Falcade, Laborde and Mariotti, 2007) lend themselves to effective and appropriate teaching of the covariational concept. Falcade et al. (2007, p. 15), who employed Cabri tools in their study, concisely described the experience of students: "students grasped variability as motion, while the idea of covariation, incorporated in the coordinated movement of points on the screen, was experienced through the coordination between eyes and hands".

Research on the learning and teaching of functions has focused on various student interpretations of function and the differences between these and the formal notions of function (Brendefur, Hughes and Ely, 2015). Leinhardt et al. (1990) found that misconceptions and difficulties related to various aspects of functions namely

(i) identification of functions - students possess inaccurate ideas of what graphs of functions look like.
(ii) correspondence within a function – one source of confusion if the belief that functions must embody a one-to-one correspondence. Another area of confusion involves distinctions between many-to-one and one-to-many functions.
(iii) linearity – students display a tendency to gravitate toward linearity in a variety of situations and also over-generalise some properties of linear functions, which is ascribed to the fact that linear functions are the first functions that students are exposed to. De Bock, Van Dooren, Janssens and Verschaffel (2002a, p. 313) mention examples of over-generalisation such as "the square root of a sum is the sum of the square roots" or "the logarithm of a multiple is the multiple of the logarithm".
(iv) continuous versus discrete graphs - students often err by representing or interpreting discrete data as continuous and vice versa. An example is when students connect discrete points when it is inappropriate to do so. A function's graph is seen as continuous and smoothly connected. Piecewise functions are therefore not seen as functions (Stroumbakis, 2010). Koffka (1935) labelled the tendency towards continuity *the Law of Closure* (as cited in Vernon (2013, p. 60).
(v) multiple representations of functions - functions can be represented by ordered pairs, equations, graphs, and verbal descriptions of relationships. Knuth (2000) found that the translation from a graph to its equation is a more problematic concept than the translation from the equation to the graph. Studies by Greenes, Chang and Ben-Chaim (2007) and Knuth (2000) revealed student difficulties with the connection between presentations, particularly the Cartesian connection.

(vi) relative reading and interpretations - difficulties relate to attempts to construct and interpret graphs that represent situations. Students tend to focus on an individual point or group of points as opposed to the more global features of the graph. The main categories of difficulties include interval/point confusion, slope/height confusion, and iconic interpretations. Iconic representations refer to the interpretation of a graph as a literal picture of the related situation. An example is a travel graph that is interpreted as the paths of an actual journey.

(vii) the concept of variable within the equation - the inadequate understanding of the concept of variable is indicated as a source of misconceptions, particularly when such an understanding includes functions defined as the relationship between an independent and a dependent variable.

(viii) notation within the graph of a function itself – students experience difficulties with the construction of scale. One misconception relates to the construction of different scales for the positive and negative parts of the axes.

Stroumbakis (2010) also lists some common misconceptions of functions that are described and investigated in the literature: a function is a named object, (such as quadratic); a function has a unique formula; functions are always expressed in terms of two variables, typically x and y ($y = 4$ is therefore not identified as a function since the equation does not contain a variable); functions exist to operate on numbers and not as objects in their own right and lastly, different representations are not seen as a unified theme (e.g. graphical, symbolic).

This study focused on three groups of functions, namely linear functions, exponential and logarithmic functions and trigonometric functions.

The general definition of a linear function refers to a function that graphs to a straight line. The function therefore has either one or two variables with no exponents or powers. In calculus and related areas, a linear function is a polynomial function of degree zero or one, or is the zero polynomial. A more formal definition follows:

A linear function can be defined as a function that is homogenous and additive, in other words, $f(mx) = mf(x),$ and $f(x_1 + x_2) = f(x_1) + f(x_2)$.

> The function $f: \Re \to \Re, f(x) = mx, m \neq 0$ (or $y = mx$) fulfils both conditions and can be seen as a relationship of direct variation with a constant rate of change m, or a relationship of proportionality with the constant of proportionality m (Postelnicu, 2011, p. 3).

A linear function is often presented as an equation in the form $y = mx + b$, and is referred to as a linear equation of two variables, the slope-intercept form of the equation of a line, or simply the equation of a line (Postelnicu, 2011). The Cartesian connection can be described as follows:

A point $(x_0; y_0)$ is on the graph of the line f if and only if its coordinates satisfy the equation of f, $y = mx + c$ (Knuth, 2000; Postelnicu, 2011). Some researchers

> … consider all mathematical conventions associated with graphic representations in a Cartesian system of coordinates to be part of the Cartesian connection, as for example, the association between the change in y and the change in x from the slope formula, and their graphic representations as line segments with oriented magnitudes, rise and run (Postelnicu, 2011, p. 4).

The source of student and teacher difficulties with linear functions have been ascribed to the Cartesian connection (Leinhardt et al., 1990; Knuth, 2000; Van Dyke and White, 2004). In order to overcome these difficulties, functions should not be treated as separate concepts to graphs. Both together define the mathematical concept of function. “They are communicative systems, on the one hand, and a construction and organization of mathematical ideas on the other” (Leinhardt et al., 1990, p. 3).

Traditional textbooks tend to present linear functions in algebraic ways and ask students to create graphs of functions from the algebraic equation. This type of task may create a dependence on one type of representation, rather than an understanding of how the representations are related (Reiken, 2008).

Proportional reasoning is considered to be equivalent to linear reasoning. It was mentioned earlier that students find proportional reasoning difficult (Cartwright, 2014) and such difficulties encompass linear reasoning. These difficulties persist throughout secondary education and are not necessarily resolved before the start of tertiary education (Coetzee and Mammen, 2016). Despite finding proportional reasoning difficult, students tend to gravitate toward linearity when defining, describing, or graphing functions (Leinhardt et al., 1990; De Bock, Verschaffel and Janssens, 2002b; Van Dooren, De Bock, Janssens and Verschaffel, 2008).

Technology may assist students and lecturers to overcome difficulties. Leng (2011) found that the appropriate use of technology, specifically the advanced graphical calculator TI-NspireTM, allowed Singaporean students to visualise concepts and enabled them to generalise mathematical properties. Students were furthermore able to link multiple representations, especially algebraic and graphical representations, thereby improving their conceptual understanding and problem-solving skills. The calculator was employed by the students in various ways, such as an exploratory tool, a graphing tool, confirmatory tool, problem-solving tool, visualisation tool and calculation tool.

Several studies have pointed to students' tendency to gravitate toward linearity (De Bock et al., 2002b; Van Dooren, De Bock, Hessels, Janssens and Verschaffel, 2004; Van Dooren, De Bock, Janssens and Verschaffel, 2005; Van Dooren et al., 2008). The inappropriate use of linearity in non-linear situations, also referred to as *the illusion of linearity*, is one of the oldest misconceptions in the literature of mathematical thought. De Bock et al. (2002a) believe that a number of factors support students' tendency toward linearity such as its intrinsic simplicity and self-evidence along with regular reinforcement of linearity in school mathematics. De Bock et al. (2002a) elaborate on what they call the most well-known case of students' improper application of linearity in the domain of geometry and measurement. “In responding to questions about the effect of halving or doubling the sides of a figure to produce a similar figure, most students - and even prospective teachers - claim that the area and volume will be halved or doubled too” (De Bock et al., 2002a, p. 4).

Leinhardt et al. (1990) cite a few other studies that also reported on students' strong linear tendencies. One such study by Markovits, Eylon and Bruckheimer (1986) required students to generate examples of graphs that would pass through two given points. Students produced mostly linear graphs. When students were questioned on whether their answers were unique or whether other possibilities existed, they confidently indicated that a straight line was the only possible solution. This tendency is an overgeneralisation of linear graphs which are uniquely determined by two points. Karplus (1979), as cited by Leinhardt et al. (1990), reported that students' preference for straight lines may be linked to their perception of straight lines as being *more accurate* than other graphs.

The second group of functions reviewed in this study is the exponential and logarithmic functions. An exponential function is a mathematical function of the form $f(x) = a^x$ where x is a variable, and *a* is a constant, called the base of the function. The most commonly encountered base is the transcendental number *e*. Webber (2002) emphasises that exponential and logarithmic functions are vital concepts that play an important role in mathematical courses such as calculus, differential equations, and complex analysis. Furthermore, the perceived importance of these functions are based on their wide applicability in real-world situations (Makgakga and Sepeng, 2013). Castillo-Garsow (2010) lists at least fourteen fields of study that regard exponential functions as relevant to their field of work: Biology, Chemistry, Computer Science, Chemical Engineering, Civil Engineering, Electrical Engineering, Mechanical Engineering, Health-Related Life Sciences, Interdisciplinary Core Mathematics, Physics, Statistics, Teacher Preparation, Environmental Technology, and Electronics, Telecommunications and Semiconductor Technology. Furthermore, "exponential functions offer a unique opportunity to explore the relationship between mathematics and nature and, in doing so, can make mathematics relevant and accessible" (Confrey, 1994, p. 294).

Ferrari-Escolá, Martínez-Sierra and Méndez-Guevara (2016) discuss two covariational approaches to the exponential function. The first approach juxtaposes the varying amounts of one quantity in an arithmetic progression with varying amounts of another quantity in a geometric progression. These different, yet simultaneous variations, affect each other mutually. Confrey and Smith (1994) explains exponential covariation as the coexistence of variations in constant rates and in constant differences, also referred to as a "counting world" and a "splitting world" respectively. An additive or counting world is based on an arithmetic sequence, revealing a constant difference between successive elements, whereas a multiplicative or splitting world is based on a geometric sequence, which encompasses a constant multiple between successive elements (Ferrari-Escolá et al., 2016).

The second covariational approach to exponential functions focuses on rate of change. In a plenary paper titled “Conceptual analysis of mathematical ideas: some spadework at the foundations of mathematics education”, Thompson (2008) asserts that Confrey’s splitting approach is insufficient for explaining the idea that an exponential function’s rate of change is proportional to the value of the function. He proposes a different approach, which employs the concept of interest calculations. The characteristic property of exponential functions, namely that an exponential function always changes at a rate that is proportional to the function’s value, emerges naturally from the idea of compound interest. Thompson (2008) remarks that his approach accentuates the characteristic property of the exponential function at the expense of the intuition of doubling, tripling, etc. that comes from the idea of splitting and hence warns that although both approaches are contingent on a multiplicative conception of comparison and growth, the two do not tie together neatly.

In a study on the covariational approach to functions that included exponential functions, Castillo-Garsow (2010) identified five different ways of understanding exponential growth: geometric, compound, differential, harmonic, and stochastic. The harmonic and stochastic exponential models assume discrete population, while the geometric, compound, and phase plane exponentials assume a continuous population. The compound model is similar to Thompson’s model discussed earlier, whilst the phase plane model also has similarities to Thompson’s model, in that it begins with the assumption that rate is proportional to current amount. The geometric exponential assumes that intermediate values are calculated by the geometric mean; the harmonic exponential assumes deterministic waiting times, while the stochastic model assumes random waiting times. Castillo-Garsow (2010) asserts that powerful understandings of exponential growth do not necessarily result from mastery of a particular perspective, but rather from a rapid and fluent switching amongst perspectives as necessitated by context.

Research studies reveal that students have difficulties with exponential studies and that even mathematics and science teachers struggle with concepts such as decay and half-life (Strom, 2006). According to Confrey and Smith (1994), traditional textbook presentation minimises the underlying multiplicative operation, which decreases the likelihood that students will recognise the need to apply a particular function type in a contextual situation. Furthermore, Mulqueeny (2012) alludes to difficulties that students experience with irrational exponents. Participants in her study were reluctant to use a graph to evaluate the function at the point where an exponent was irrational, such as $\sqrt{3}$. She asserts that the use of irrational numbers in exponents constitutes a perturbation (Campbell, 2010) which is not resolved naturally, but instead has to be cultivated.

Logarithms and their applications are ubiquitous in scientific and real-life phenomena, such as sound intensity, earthquake intensity, star brightness, mineral hardness, acidity, the spacing of frets on the fingerboard of a guitar and between musical notes:

> Musical notes vary on a logarithmic scale because progressively higher octaves (ends of a musical scale) are perceived by the human ear as evenly spaced even though they're produced by repeatedly cutting the string in half (multiplying by ½). Between the neck and the mid-point of a guitar string, there will be 12 logarithmically spaced frets (Coolman, 2015).

According to historians, Napier coined the term logarithm. Napier, in an attempt to simplify computations, introduced a new notion of numbers which he initially called artificial numbers, but later called logarithms, from the Greek roots *logo*s (ratio) and *arithmos* (number). Napier's logarithms differed from the logarithms known today as Naperian (or natural, or hyperbolic) logarithms. His construction was wholly geometrical and based on the theory of proportions (Roegel, 2012). Napier established a correspondence between a geometric series and an arithmetic one. Napier defined the distance travelled by the arithmetically moving point as the logarithm of the distance remaining to be travelled by the geometrically moving point. "Napier constructed two independent worlds, a particle moving arithmetically and one moving geometrically, and by using time as a basis to visualize their cogeneration, created a relationship that we now call a log function" (Confrey and Smith, 1994, p. 338) . This juxtaposition of a geometric sequence onto an arithmetic sequence is equivalent to the operation discussed in the previous paragraph on exponents. This stands to reason, since the two concepts are inverses of each other.

Students typically have little if any intuition about exponential functions and their connections to logarithms, possibly because of difficulties with interpretation of symbolic notation (Mulqueeny, 2012). When students are required to solve a logarithmic equation such as $log_2 x = 4$, the equation first has to be converted to exponential form, which requires the precise interpretation and manipulation of each symbol. Students' inability to distinguish the role of each letter may explain why students are unsure of the meaning of the expression. Furinghetti and Paola (1994) have identified difficulties encountered by students as *syntactic-manipulation* and *semantic-conceptual.* They further observed that students, even if able to manage a satisfying *syntactic manipulation* of formulas containing parameters, did not grasp the underlying *semantics*. Lack of conceptual understanding of symbols and their meaning results in misconceptions. Working with symbols that are poorly understood, results in inappropriate manipulation of these symbols (Mulqueeny, 2012).

Various approaches have been forwarded as a way to overcome the problems mentioned above. Results from a study conducted by Mulqueeny (2012) indicate that exposure to information on the historical development of logarithmic concepts can support the understanding of logarithmic concepts and hence advise that students be introduced to the historical processes used by Napier and Briggs. In yet another study, Makgakga and Sepeng (2013) reported that a transformational approach to teaching and learning of exponential and logarithmic functions was effective and improved students' academic performance in these topics.

The third group of functions discussed in this study, is the trigonometric functions. A trigonometric function is a function of an angle expressed as the ratio of two of the sides of a right triangle that contains that angle. These are expressed as one of six ratios, namely the sine, cosine, tangent, cotangent, secant, or cosecant of the particular angle. Trigonometry and trigonometric functions have been an important part of the high school and undergraduate mathematics curriculum for the past century. In addition to numerous mathematical topics (e.g., Fourier series and integration techniques), various topics of science are reliant on trigonometric functions (such as projectile velocity and modelling wave behaviour). Trigonometry and trigonometric functions also offer one of the earlier mathematical experiences that combine geometric, symbolic, and graphical reasoning about functions that cannot be calculated through algebraic computations.

Studies show that students have difficulties understanding the ratio of two varying quantities (Cheng, 2010; Livy and Herbert, 2013). Recognising proportionality of quantities and using proportional reasoning is vital to understanding and using the idea of angle measure in trigonometry. According to Ronda (2015), angle measure and trigonometric function concepts are under-developed in pre-calculus students and even in-service and preservice teachers. Furthermore,

> Angle measures are often not conceptualized as an amount of openness between two rays with a common endpoint. Also, the need to use an arc of a circle to measure an angle's openness is usually not recognized by students. Studies show that when students are able to reason about how an angle's measure and the vertical coordinate of the arc's terminus covary, they are better able to understand the sine function and use it meaningfully to model periodic motion. This image of the sine function was also found useful for students in connecting their unit circle conceptions of the sine and cosine function functions to their conceptions of these functions in the triangle trigonometry context.

A process conception of function, as well as an understanding of covariational reasoning are required for understanding and using the sine and cosine functions. A content framework for studying sine and cosine was developed by Brown (2006) for a portion of coordinate trigonometry.

Research has revealed that students and teachers have difficulties with trigonometric functions (Siyepu, 2015). Both groups were also found to hold weak understandings of ideas underpinning trigonometry such as angle measure and the unit circle. Disconnected conceptions of the various contexts of trigonometry such as the unit circle and right triangle were found to be common (Moore, 2010). A study by Brown (2006) revealed that many students had an incomplete or fragmented understanding of the three major ways to view sine and cosine: as coordinates of a point on the unit circle, as the horizontal and vertical distances that are the graphical entailments of those coordinates, and as ratios of sides of a reference triangle. Cognitive obstacles included a fragile conception of rotation angle and unit, and a failure to connect a rotation on the unit circle to a point on the graph of the cosine or sine function. Students struggled to view sine and cosine as both ratios and numbers.

Although few studies have investigated the reasoning abilities needed to understand and use trigonometric functions, a study by Siyepu (2015) showed that the nature of errors displayed by students were both conceptual and procedural. Conceptual errors revealed a failure to appreciate the underlying relationships, while students also struggled with manipulations and algorithms, even if concepts were understood. “Linear extrapolation errors occurred when students over-generalised the property $f(a+b) = f(a) + f(b)$, which applies only when f is a linear function, to the form $f(a * b) = f(a) * f(b)$, where f is any function and * any operation” (p. 1). The main source of errors was identified as prior learning of mathematics, which had been dominated by rote learning of routines or procedures without their having made sense of these. Also, students tended to overgeneralise mathematical rules.

It is also important to note that functions are often combined and/or transformed. Geometric Transformations have been termed one of the most interesting and most beautiful themes of geometry and allows the development of visualisation, spatial perception and geometric literacy of learners (Andraphanova, 2015). Pre-calculus curricula cover transformations as horizontal and vertical shifts, horizontal and vertical stretches/shrinks, reflections over the coordinate axes, inverses as a reflection over the line $y = x$, and function composition (Kimani, 2008). However, researchers have been critical of various aspects of the curriculums. Bonn (2015), referring to a study by Dick and Childrey (2012), laments curriculums presenting transformations as an isolated topic, independently of other topics in mathematics, in particular geometry. Also, enabling students to visualise transformations helps the learner to appreciate families of functions and their relations to each other (Ninness, Barnes-Holmes, Rumph, McCuller, Ford, Payne, Ninness, Smith, Ward and Elliott, 2006). However, visualisation as an alternative approach to transformations is not favoured by students, partly because the school curriculum reinforces non-visual approaches (Presmeg, 2006). Despite the benefits of visualisation, research has shown students to be reluctant to use visual thinking (Eisenberg, 1994; Van Dyke and White, 2004; Rösken and Rolka, 2006).

Transformation of functions has been a major component of many levels of algebra as well as of more advanced courses in mathematics (Ninness et al., 2006). Ada and Kurtulus (2010) assert that conceptual understanding of geometric transformations advances learners' ability to learn other mathematical topics. Smith (2009) agrees that studying transformations improves learners understanding of function.

However, both these studies revealed learners' limited understanding of transformation of graphs. Kimani (2008) also reported that the students involved in his study had a limited understanding of the concepts of function transformation. Students seemingly memorised knowledge about these concepts that became accessible when familiar concept names were encountered. This knowledge seemed shallow and was influenced by function representation and notation. Students commonly viewed transformations as unrelated to the initial functions (Kimani, 2008). Furthermore, when the subjects in their study were asked to explain transformations, they offered memorised procedures as explanations and showed no desire to understand or explain the phenomenon (Zazkis, Liljedahl and Gadowsky, 2003) as cited by Kimani (2008). Yet another study revealed student struggles with generalising patterns, particularly with moving from one family of functions to another (Confrey and Smith, 1994). Likewise, Ada and Kurtulus (2010) found that students seemed to know the algebraic meaning of translation and rotation but struggled with the geometric interpretation. Ninness et al. (2006) found that students struggled more with horizontal transformations than vertical transformations, which run contrary to expectations - a horizontal transformation shifts the function to the left (negative direction) following the addition of a positive constant inside of the argument, whereas the subtraction of a constant inside of the argument produces a shift to the right (positive direction).

A study by Chiu, Kessel, Moschkovich and Munoz-Nunez (2002) reveals a strong preference amongst students for viewing non-vertical, non-horizontal parallel straight lines as horizontal translations of each other rather than vertical, even following a period of instruction that supported the vertical view. These researchers advise that instructors and curriculum designers take this partiality into account and even encourage students to develop and refine strategies associated with this conception rather than ignoring or trying to eradicate them. Also, educators should take note that a structural view of a function is a prerequisite to the effective understanding of function transformations (Kimani, 2008; Carlson, Madison and West, 2015) though other researchers support the concept of covariational reasoning to assist learners to transform images Smith (2009), as cited by Makgakga and Sepeng (2013).

Unfortunately there seems a reluctance amongst teachers to spend sufficient time on the topic of transformations. This averse attitude could be ascribed to a lack of deep understanding of geometric transformations and how they can be applied to multiple concepts (Bonn, 2015). Bonn laments teachers' preference for calculations in the coordinate plane where lengths of sides can be determined using the distance formula, to the exclusion of transformations which is a more accurate method. Bonn suggests that a lack of understanding leads teachers and students to believe that transformations are not integral to the teaching of geometry.

The concepts of a function and its inverse are essential for representing and interpreting the changing nature of a wide array of situations (Carlson and Oehrtman, 2005) such as describing the relationships between logarithms and exponentials. Studying the inverse of a function presents an additional advantage – various researchers (Attorps et al., 2013; Breen et al., 2016) claim that a study of the inverse function concept could be used to reinforce students' understanding of function in return. It is however worth noting that the process view of a function supports the notion of an inverse. Carlson, Oehrtman and Engelke (2010) point out that students who are unable to conceive of a function as a process (rather than taking an 'action' view) have difficulties inverting functions.

Carlson and Oehrtman (2005) categorise three different conceptions of inverse function: inverse as algebra (swap x and y and solve for y), inverse as geometry (the reflection in the line y = x) and inverse as a reversal process (the process of 'undoing'). In general and under certain conditions, $\boldsymbol{f^{-1}(f(x)) = x.}$ In other words, the inverse function undoes whatever the function does (Bayazit and Gray, 2004). This notion of 'undoing' captures the essence of the inverse function (Even, 1992). Various studies support this view and point to the value of the conception of the inverse as a reversal process. Carlson et al. (2010a) found that understanding the concept of an inverse as a reverse process empowered learners, enabling them to answer a wide variety of questions about inverses. A study by Even (1992) confirmed these findings. She investigated prospective secondary mathematics teachers' knowledge and understanding of inverse functions and found that several students preferred to use calculations and did not appear to have conceptual knowledge of the inverse as a reverse process of undoing. However, she claims that a naïve understanding of inverse that is limited to undoing is insufficient and may result in misconceptions such that all functions have inverses. Conversely, Breen et al. (2016) did not find any evidence supporting this claim.

The concept of an inverse is often not approached from the undoing perspective. In a number of countries such as Ireland and Sweden, inverse functions are taught as algebra by swapping *x* and *y* (Breen et al., 2016). This is also the common approach used in secondary schools in South Africa and is applied jointly with the geometric approach. Regretfully, the inverse as a reversal process is neglected.

Wilson, Adamson, Cox and O'Bryan (2011) argue that the common procedure of swapping x and y to find the inverse confuses students and can lead to problems with interpreting answers, especially for contextual or real-world problems. They contend that both swapping the variables and drawing the graph as a reflection in the line $y = x$ do not take into account the important aspect of the domain of the inverse function being the range of the function and vice versa. They propose using the alternate approach of solving for the dependent variable instead, rather than literally swapping x and y and contend that this approach will reduce confusion and enhance students' conceptual understanding of inverse functions. Van Dyke (1996) also expresses concerns about the exclusive use of the procedure of switching *x* and *y* and claims that the idea of undoing what the function does gets lost in the mechanics of switching the *x* and *y* and then solving for the newly named *y*.

Functions can, and should be, represented in various ways. Multiple representations provide various perspectives of mathematical concepts and therefore aid student understanding of concepts (Huang and Cai, 2011). Tall (1993, p. 9) summarises the strength of various representations concisely: "Graphics give qualitative global insight where numerics give quantitative results and symbolics give powerful manipulative ability". Tall, like numerous others, stresses the need for flexible movement between representations. Research on multiple representations has suggested that students who engage in tasks that involve multiple representations show an increased ability to translate between the different representations (Confrey and Smith, 1994; Amoah and

Laridon, 2004; Balyta, 2007; Reiken, 2008; Brendefur et al., 2015) and hence displayed improved problem-solving abilities (Brenner, Herman, Ho and Zimmer, 1999). Moschkovich (1998), as cited by Reiken (2008), refers to this ability as cognitive flexibility. More specifically, multiple representation tasks may help students develop connections between the different representations of function (Brenner et al., 1999). When students encounter different representations of functions, their cognitive flexibility improves, which is a gauge of competence in the function domain. Although some practitioners and researchers argue that struggling learners will be confused and overwhelmed by multiple representations, interviews with students indicated a preference for this approach, adding that it reduced their confusion (Lynch and Star, 2013).

Despite the clear value of multiple representations for learning and teaching calculus, Tall cautions that the best approach is to focus on the most useful representation, rather than multiple representations at the same time (Tall, 1993). Also, Postelnicu (2011) cites various studies linking multiple presentations to student difficulties. These studies found that student difficulties occurred most often when students were attempting to connect among various representations of linear relationships and functions.

The introduction of algebraic and graphical representations of functions can be seen as “one of the earliest points in mathematics at which a student uses one symbolic system to expand and understand another” (Leinhardt et al., 1990, p. 2). A graph can be thought of as a lens through which to explore a phenomenon. Graphs offer an alternative way to represent functions. Graphs connect formal static definitions of function with “the metaphor of motion” (Falcade et al., 2007, p. 3). Multiple representations of functions, also referred to as one of the *big ideas* of algebra, play an important role in students’ mathematical development (Lacampagne, Blair and Kaput, 1995; Amoah and Laridon, 2004; Lim, 2011). A graph has multiple potential meanings, which complicates conceptual understanding of graphical representations, hence inappropriate responses to visual attributes of a graph are the most frequently cited student errors (Garrett, 2010).

Four mental actions in the context of drawing or interpreting a graph of two quantities that change concurrently are associated with covariational reasoning, namely identifying the related quantities, imagining the direction of the related change; picturing the magnitude of change of one quantity with respect to the other quantity and lastly, visualising how the rate of change of the output variable with respect to the input variable is changing on a small contiguous interval of the input variable (Carlson et al., 2015; Ronda, 2015).

Brendefur et al. (2015, p. 3) cite a number of articles that report on difficulties with the representations of function as experienced by students and teachers. According to their study, students often *appear* to understand connections between equations and graphs, but the understanding seems superficial, a finding confirmed by Knuth (2000). Other studies report on students' problems with coordinate graphing conventions (Schoenfeld, Smith and Arcavi, 1993).

Reiken (2008) cites a number of studies which examined the approach taken by traditional textbooks to graphs. Some of the problems mentioned in these studies are that graphs are not necessarily integrated with other mathematical topics, and are treated as the ultimate end result to a problem. Yet, it can be said that a graph does not reach its full potential until it is used to make meaning (Garrett, 2010). Cartesian graphs are useful for examining extrema, convexity, and asymptotes (Garrett, 2010). Graphs can visually aid the understanding of a function as one-to-one or many-to-one, yet it has to be noted that textbooks tend to over-emphasise the translation from function to graphical representation and sometimes neglect the reverse process (Leinhardt et al., 1990) Stressing the importance of graphs, Reiken hence cites a few studies which mentioned the importance of viewing the graphical representation of a function as an equally valuable entity, also because graphs aid in the development of a concept image as described by Vinner and Dreyfus (1989). Yet, Falcade et al. (2007) assert that this dynamic interpretation is often neglected in textbooks.

Past research has shown that students are reluctant to use graphs (Knuth, 2000; Van Dyke and White, 2004). Van Dyke and White (2004) ascribes this disposition to a lack of understanding in three areas: the concept of function, the Cartesian connection, and the mathematical description of change. Brendefur et al. (2015) concurred that graphical representations were not a natural first choice of presentation for students and concluded that students were not accustomed to describing realistic situations using graphical representations. Brendefur et al. (2015) recommend that teaching practice should not be restricted to the procedural aspects of constructing coordinate graphs or to the technicalities of creating a mathematical rule that can accommodate a functional relationship. Students need to appreciate the potential of a representational scheme such as the coordinate graphing system. Gagatsis, Christou and Elia (2004), as cited by Trigueros and Martínez-Planell (2010), argue that representations constitute different entities . They concur with Brendefur et al. (2015) that the skills to translate between various representations do not ensue automatically but require explicit instruction. It therefore stands to reason that Balyta (2007) strongly endorses the use of technology to enhance student access to multiple representations and assert that appropriate use of technology presents an opportunity to revolutionize the teaching and learning of functions.

2.3.2.3 Differentiation

The derivative of a function of a real variable measures the change of a quantity (a function value or dependent variable) with respect to another quantity (the independent variable). The derivative of a function of a single variable at a chosen input value, when it exists, is the slope of the tangent line to the graph of the function at that point. The tangent line is the best linear approximation of the function near that input value. For this reason, the derivative is often described as the "instantaneous rate of change", the ratio of the instantaneous change in the dependent variable to that of the independent variable. Two distinct notations are commonly used for the derivative, one deriving from Leibniz and the other from Lagrange. In Leibniz's notation, an infinitesimal change in x is denoted by dx, and the derivative of y with respect to x is written $\frac{dy}{dx}$, suggesting the ratio of two infinitesimal quantities (Cajori, 1928). In Lagrange's notation, the derivative of a function $f(x)$ is denoted $f'(x)$ or simply f' (MIT, 1998). Derivatives can also be defined in a number of different ways, such as using the epsilon-delta definition, graphically in relation to steepness, symbolically as a formula, numerically as a gradient, and visuo-spatially as velocity (Tall, 1997).

Zandieh (2000) describes a layered framework for the concept of the derivative based on Sfard's process-object views (Sfard, 1991). According to Zandieh, Sfard's perspectives differ from that of previous authors in that Sfard regards processes and objects as two aspects of a single phenomenon, and not as mutually exclusive opposites (Von Korff and Rebello, 2012). Zandieh identified three layers of the student's understanding of a derivative, namely the ratio, limit, and function layers. Each layer can be viewed as a process or an object and each process-object pair can be represented in the form of an equation, graph, verbalization, or kinematic motion. Von Korff and Rebello (2012) describe the connections between the three layers as follows:

The ratio layer is the most basic of the three concepts. Students should understand the connection between formulas such as $\frac{\Delta y}{\Delta x}$, the procedure for finding the slope of a graph and average velocity as the ratio $\frac{\Delta x}{\Delta t}$. In the limit layer students should be able to imagine the denominator of the ratio approaching zero. In the third and final layer, the student should be able to conceive of the derivative as a function.

Orton (1983) conducted research on student errors in differentiation and reported adequate procedural knowledge but inadequate conceptual understanding of the derivative. He indicated that one in five of participants in his study confused the derivative at a point with the *y*-coordinate of the point of tangency. Stroumbakis (2010) lists the following difficulties with derivatives that are describe and investigated in the literature: overemphasis of positive and constant rates of change leading to a reluctance accepting or difficulty interpreting a variable rate; negative or zero rates of change; failure to distinguish between the derivative at a point and the derivative of a function and lastly, being unable to progress from a sequence of secants to the tangent and consequently from a sequence of average rates to the instantaneous rate.

When dealing with differentiation, rate of change and slope are the two of the core concepts. It has been suggested that conceptual understanding of topics such as rate of change and proportion are dependent on conceptual understanding of fractions (Norton and Hackenberg, 2010). The connection can be justified mathematically. Bressoud (2016) laments students' difficulties with proportional reasoning, which affect their understanding of constant rate of change. Conceptual understanding of constant rate of change is a prerequisite for understanding average rate of change, which in turn is fundamental to understanding the meaning of the derivative. He refers to a study by Carlson et al. (2010b) who administered the rain-gauge problem of Piaget, Blaise-Grize, Szeminska and Bang (1977) depicted in Figure 2.8 to 1205 students at the end of a pre-calculus course. Only 43% identified the correct answer

($4\frac{2}{3}$). Many students preserved the difference rather than the ratio, giving 5 as the answer.

> In the figure below are drawings of a wide and narrow cylinder. The cylinders have equally spaced marks on them. Water is poured into the wide cylinder up to the 4th mark (see A). This water rises to the 6th mark when poured into the narrow cylinder (see B). Both cylinders are emptied, and water is poured into the narrow cylinder up to the 7th mark. How high would this water rise if it were poured into the empty wide cylinder?

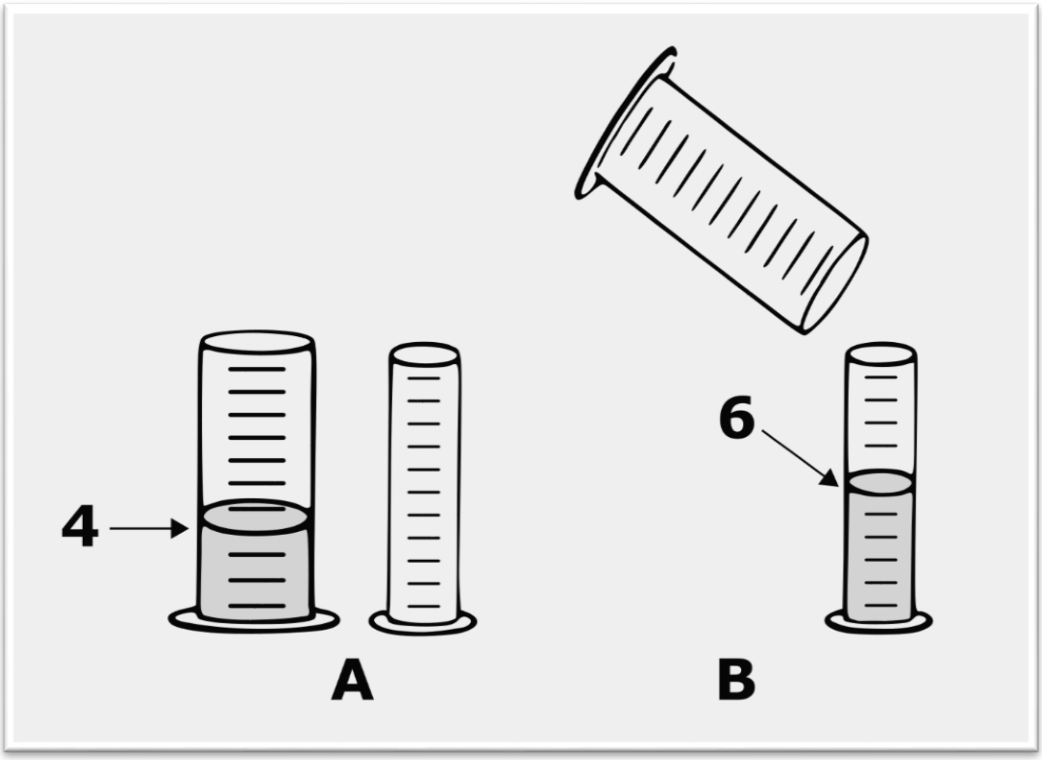

FIGURE 2.8: The rain-gauge problem of Piaget (Carlson et al., 2010b)

Studies show that students find problems that involve the ratio of two varying quantities difficult (Cheng, 2010; Livy and Herbert, 2013) which leads to problems with related rates. Swanagan (2006) ascribes students' difficulties with related rates to their apparent inability to visualise rates of change with two different quantities. Rate of change can be interpreted as concomitant changes in two quantities. Thus both covariational reasoning and proportional reasoning are considered as supportive constructs for the notion of rate (Confrey and Smith, 1994). Proportional reasoning is related to covariational reasoning as both involve two varying quantities. Two varying quantities are related proportionally if their ratios remain constant or if they are related by a constant multiple (Carlson et al., 2015).

Confrey and Smith (1994) considered rate of change as a primary point of entry when teaching exponential functions. They attempted to avoid a context-independent interpretation of rate, and tried to rather use a description of rate which could demonstrate its use across various contexts. They started out with problems based on discrete variables. They identified three ways in which students described rate of change, namely additive rate of change, multiplicative rate of change and a "proportional new-to-old" rate of change. Their analysis found that the multiplicative rate of change to be a legitimate approach and advised using this approach to support student understanding of constant doubling times and constant half-lives. They believe that the multiplicative approach has the potential to enrich concepts and language of mathematics education.

Confrey and Smith (1994) argue that rate of change should be introduced much earlier in the curriculum, and not delayed until the introduction of calculus, where it is studied as a 'property' of functions. They refer to studies such as those done by Kaput and West (1994). The latter has shown that even young children can use rate of change as a way to explore functional understanding.

Slope is regarded as one of the core concepts in most secondary school mathematics curriculums (Greenes et al., 2007). Much of the research that has been done on slope has examined ways of understanding slope. The three primary representations of function (algebraic, graphical, and tabular) all contain information about the Cartesian connection and slope (Reiken, 2008). Slope between two points is often conceptualised as the ratio of the change in y-coordinates to the change in the associated x-coordinates. The phrase *rise-over-run* is a popular mnemonic for the algorithm $\frac{\Delta y}{\Delta x}$, not only in the United States, but also in South Africa. Walter and Gerson (2007) posit that students who view slope quantitatively as a ratio, has an instrumental understanding of slope, struggle to interpret the number graphically and also struggle with understanding rate of change. They therefore perceive slope as a formula and resort to algebraic representations for its calculation (Reiken, 2008).

Reiken (2008, p. 210) described five different ways in which students understood slope, namely

> … slope as a number from a formula, slope as a number from counting, slope as a number in front of x, slope as number that is a relator, and slope as a number that is a measure of rate of change. Of the five ways of thinking about slope, the number as relator and number as rate of change were the only ways of thinking about the slope number that implied a deeper mathematical understanding of the number itself.

Slope can be understood in qualitative and quantitative ways (Stump, 2001). Some students view slope in terms of tangible objects such as a ramp or the steepness of a hill and others furthermore view slope as a ratio of two numbers. Students were reported to be quite proficient in determining whether graphs had positive or negative slopes, yet were unable to link the sign of the slope found by calculation to the graph of the line. Students also struggled with the difference between zero and undefined slope (Reiken, 2008).

Stump (2001) investigated the importance of visual or graphical representations of slope and advised that students be exposed to multiple representations of slope. Multiple representations have been found to support students' conceptual understanding of slope. Lynch and Star (2013) report that students in the multiple-representation group in their study displayed more advanced understanding of slope on some problems when compared to students in the traditional group. Tasks involving multiple representations of a function may be a good way to facilitate the connections between the different representations of slope as well as an understanding of the Cartesian connection. These connections are essential for understanding slope as a concept.

2.3.2.4 Integration

The Riemann integral is the simplest definition of an integral and the one usually encountered in elementary calculus. The Riemann integral of the function $f(x)$ over x from a to b is written $\int_a^b f(x)dx$ (Wolfram, 2017) and denotes the precise value of the area under the graph of f between a and b if $f(x) \geq 0$ (Rasslan and Tall, 2002). The fundamental theorem of calculus links differentiation to the definite integral: if f is a continuous real-valued function defined on a closed interval $[a;b]$, then, once an antiderivative F of f is known, the definite integral of f over that interval is given by $\int_a^b f(x)dx = F(b) - F(a)$ (Wolfram, 2017). An antiderivative can be defined as follows: Let f be a function of x. A function $F(x)$ is called an antiderivative of $f(x)$ if $\frac{d}{dx}\left(F(x)\right) = f(x)$, that is, F is an antiderivative of f if the derivative of F is f (Bajracharya, 2012).

An analysis of the integration process was conducted by Von Korff and Rebello (2012) along the same lines as Zandieh's framework for the derivative (Zandieh, 2000). Their framework describes integration using a network of processes and objects, with multiple routes to understanding. Their framework furthermore addressed infinitesimal quantities and infinitesimal products, whereas Zandieh's framework considered ratios of finite quantities, and limits of those ratios, but not ratios of infinitesimal quantities. However, they do caution that awareness of these layers is insufficient, and that a student has to be able to relate each layer of the integral to several different representations, including graphical, verbal, equation, and concrete physical representations. In other words, "a student needs to know what is being added up and why" (Von Korff and Rebello, 2012, p. 12).

Concept image (Tall and Vinner, 1981) is yet another framework used by researchers (Bezuidenhout and Olivier, 2000; Rasslan and Tall, 2002) to frame their study of student understanding of definite integrals (Bajracharya, 2012).

Ghazali et al. proposed an alternative framework of integration, which measured students' understanding by their ability to translate between symbolic, graphical, and verbal representations of concepts. They explained that "understanding refers to the acquisition of basic definitions, basic operations and the ability to translate and interpret between the different representations, that is symbolic, graphical, numerical and applications" (Ghazali, Abdullah, Ismail and Idris, 2005).

Fisher, Samuels and Wangber (2016) studied common student conceptualisations of the definite integral and found three central conceptions: antidifferentiation, area, and accumulation. The researchers graded understanding in each of these categories as shallow or deep.

> Shallow understanding of antidifferentiation might include only procedural knowledge for basic functions (e.g. polynomial, trig, exponential). Deep understanding might include a full conception of the Fundamental Theorem of Calculus. A shallow understanding of area might include only the nominal notion itself. A deep understanding might allow reasoning about the bounds which maximize the integral given the graph of a function (Olivier, A., 2000); it might encompass approximations through Riemann sums as well as exact answers from a definite integral. Shallow understanding of rate-based accumulation might only include a restatement of the Fundamental Theorem of Calculus. Deep knowledge might include the ability to set up integrals for applied problems (as in (Sealey, 2006) or the ability to represent it with the graph of the antiderivative (Tall, 1991).

A further structure to measure student understanding of the integral, is the pseudo-structural understanding of Sfard (1991). When a student can manipulate an object without comprehension of its internal structure, the student is said to have *pseudo-structural* understanding of a pseudo-object (Sfard, 1991; Zandieh, 2000). A pseudo-object is any object that the student understands in a pseudo-structural way (Von Korff and Rebello, 2012). Fisher et al. (2016) reported student conceptualizations of integration that indicate pseudo-structural thinking.

One of the key concepts in calculus is the definite integral of a function. This concept encompasses notions such as Riemann sums, limits, derivatives and area. A number of studies have investigated student understanding of the definite integral. Since conceptual understanding involves the ability to assimilate conceptual and procedural knowledge, students who display conceptual understanding should be able to make connections between all of these concepts. Serhan (2015) however found in his study that students relied mostly on procedural knowledge and that they had limited understanding of the definite integral, a result support by the study conducted by Mahir (2009). Similar results were reported by Rasslan and Tall (2002). Only nine out of 41 students involved in their study knew how to use the definite integral to determine the area between the graph of a function and the x-axis. Kiat (2005) likewise reported that learners from a secondary school in Singapore preferred to focus on procedural aspects and lacked procedural as well as conceptual understanding of integration. He cited Thomas and Ye (1996) who conducted an earlier study on integration, investigating student thinking and misconceptions associated with the Riemann integral. Students involved in this particular study were found lacking in conceptual understanding and preferred to engage in procedural tasks.

Research reveal some common difficulties that students experience when working with the definite integral. One of these is when students are confronted with situations when a function changes sign and an area has to be computed, part of which may be below the x-axis (Orton, 1983; Rasslan and Tall, 2002). A similar problem was contained in a study conducted by Kiat (2005), and the mean score for this question was only 20%. Many students had no idea that they needed to sketch the curve of the function and simply integrated the function mechanically using the given domain as the limits. Bezuidenhout and Olivier (2000, p. 78) claim that this misconception, which they term "an inappropriate *area-conception* of the integral", results from a generalisation of the special case $f(x) \geq 0$. In a study that explored students' understanding of symbolic and verbal definitions of the definite integral, Grundmeier, Hansen and Sousa (2006), as cited by Serhan (2015), found that most students were able to integrate correctly but had difficulty with both the symbolic and verbal definitions of a definite integral . Another common mistake involving integration occurs when students confuse the concepts related to integration with that of differentiation (Bajracharya, 2012).

Transfer from mathematics to other fields such as engineering and physics has always been problematic, even if students have the necessary mathematical skills. Integration is not exempt from this difficulty. Research on students' application of calculus in physics suggested that students might not conceptually understand mathematical processes although they were often able to carry out appropriate calculations (Nguyen and Rebello, 2011). Engineering students are required to use integration in contexts ranging from mechanics to introductory electricity and magnetism. Nguyen and Rebello (2011) found that most students in calculus-based physics courses recognised the need for an integral in solving the problem, yet failed to set up the desired integral. They ascribe this difficulty to students' inability to understand the infinitesimal term in the integral and/or failure to understand the notion of accumulation of an infinitesimal physical quantity. Bajracharya (2012) ascribes some of the difficulties that students have with the definite integral in physics to a lack of coherence between mathematics and physics.

It is furthermore important for mathematics as a service discipline to note that in some disciplines such a physics, the direction of integration (where x decreases instead of increases) plays a significant role, whereas mathematics instruction puts very little emphasis on this concept (Bajracharya, 2012). Many students in the latter study failed to see the significance of the role of direction of integration in determining the signs of integrals.

Stroumbakis (2010) asserts that difficulties with integration seem to stem from misconceptions about infinity and limits, and that these concepts should be strengthened. Serhan advised that multiple representations and their connections with the concept definition be emphasised. Furthermore, more attention should be paid to the Riemann sum and its applications.

2.3.3 Theoretical and Conceptual Frameworks

Students struggle with calculus and its inherent complexities. Students especially struggle with conceptual understanding of calculus, and have difficulties applying their knowledge in the various related fields. Students' learning problems are however not only due to the complexity of the material, but could be ascribed to other factors (Confrey and Kazak, 2006), such as outdated pedagogies.

The theoretical framework of this study was based on constructivism. Constructivism became prevalent in mathematics education because it addressed the major concerns of mathematics educators (Confrey and Kazak, 2006). Integrated in the study are aspects of constructivism such as AL, cognitive dissonance, scaffolding, knowledge for learning and teaching and formative assessment.

Constructivist theorists highlight the crucial role that activity plays in mathematical learning and development (Cobb, 1994). Piaget asserted that "knowledge arises from the active subject's activity, either physical or mental, and that it is goal-directed activity that gives knowledge its organization" (Von Glasersfeld, 1995, p. 56). Various other sources from the literature confirm the value of using AL approaches at university

(Drake and Battaglia, 2014). AL was therefore employed in this study in order to improve students' conceptual understanding of calculus.

Formative assessment was utilised to provide evidence about learning, in order to adapt the teaching and to facilitate improved learning. Formative assessment should ideally be entrenched in the teaching and learning process, and not as a separate add-on. One way of accomplishing seamless formative assessment, is by using ARS. The software allows statistical analysis of each students' individual response, as well as the groups' responses. GQ combined with clickers allow continuous, almost effortless formative assessment. GQ were also employed to generate cognitive conflict, since cognitive conflict stimulates conceptual change. The conflict gets resolved with the aid of PDs. Knowledge is constructed socially, according to "Vygotsky's principle that ideas appear first in the external 'social' plane, then become internalised by the individual" (Black and Wiliam, 2009, p. 20). Thought processes are organised when students verbalise a problem and relevant solution strategies. The learner is thus enabled and the process brings about a better understanding of the concepts. Dissonance is resolved when students abandon the misconceptions and adapt correct interpretations.

Chien et al. (2016) posit that the effectiveness of PD may be attributed to the scaffolding that ensues when students assist one another to 'advance in the Zone of Proximal Development (Vygotsky, 1934/1986)' (Samuelsson, 2008, p. 240). Instructors need PCK and TPACK to scaffold learning and to determine which pre- and misconceptions exist in the mind of the learner. Technology Knowledge is "knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face (Koehler and Mishra, 2009, p. 66). TPACK is the basis of effective teaching with technology, requiring an understanding of the representation of concepts using technologies.

A literature review covered functions, differentiation and integration. Multiple representations of these topics were stressed, since calculus education reformers advocate a multi-perspective approach. This approach, often called The Rule of Three, states: 'Every topic should be presented geometrically, numerically, and algebraically' (Hughes-Hallett et al., 2005, p. vii). ARS enable effective multiple representations of functions, especially graphical representations. Multiple representations provide various perspectives of mathematical concepts and therefore aid student understanding of concepts (Huang and Cai, 2011).

A more recent perspective on constructivism proposes a reconciliation of radical and social constructivism (Cobb, Yackel and Wood, 1992). Voigt (1992) maintains that both cultural and social processes are integral to mathematical activity. Considering this viewpoint, and after reviewing and evaluating the pertinent literature, I have aligned myself with Voight. I value aspects of both Piaget's and Vygotsky's theories. I believe in the significance of cognitive conflict, but unlike Tall (1977), I do not believe that resolution of conflict is the teacher's sole responsibility. Instead, conflict is better resolved in a social setup, where PD (discussions in small groups) requires students to verbalise thoughts and defend their stance. I have therefore employed the notion of cognitive conflict in this study, by using AL. Using formative assessment, I challenged students with GQ, followed by PD and immediate feedback. Despite aligning myself with constructivism and after reading the warnings issued by Kirschner et al. (2006) and Brodie's views on the teacher's role as mediator, I decided to also make use of direct instruction. Despite criticism of teacher-centred teaching methods, it is still the preferred teaching method of many teachers. Much success has been achieved with direct instruction in the Far East. It "also has many advantages, such as the ability to deliver a large amount of information quickly. It is an easy and safe way for teachers to teach, most students are accustomed to it, and so on" (Zhang, 2003, p. 101). Concern about covering the contents of the prescribed syllabus is common amongst mathematics instructors and it is often cited as a reason for not implementing RBAs in teaching (Kay and LeSage, 2009; Johnson, Ellis and Rasmussen, 2014). Braun et al. (2017) assert that the implementation of new pedagogies is a long-term process, and that new instructional techniques cannot be effectively implemented rapidly. Because of time pressures and concerns about my own lack of TPACK and experience with IE, I decided on a balanced approached, and aimed for the best of both worlds. I decided to use RBAs in 25% of the allocated contact time, and direct instruction in 75% of the time.

2.3.4 Summary

Chapter Two described the theoretical framework that the study was based on and reviewed the literature related to the research questions. Aspects of constructivism such as AL, PCK, Threshold Concepts, Cognitive Dissonance and Formative assessment were all linked to the research questions. The literature review covered the technologies and methodologies used in RBAs in teaching calculus, as well as the topics included in the ConcepTest, namely functions, differentiation and integration. Functions were discussed along the NCTM's framework of five *Big Ideas*: the Function Concept; Covariation and Rate of Change; Families of Functions; Combining and Transforming Functions and lastly Multiple Representations of Function (NCTM, 1989, 2000). However, the second of the categories, Covariation and Rate of Change, was expanded to include other research perspectives of the function concept, namely the Concept Image of Tall and Vinner, the APOS theory of Breidenbach et al. (1992), Dubinsky and McDonald (2001), the Structural-Operational Framework of Sfard (1991) and lastly the covariational approach mentioned by the NCTM and advanced by researchers such as Carlson et al. (2001). Three groups of functions were discussed, namely linear functions, exponential and logarithmic functions and trigonometric functions. Hence a literature review was presented on function transformation and the inverse concept. The section was concluded with an exposition of the importance and value of multiple representations of functions.

The discussion of differentiation covered slope, rate of change and graphical representations, whereas the section on integration focused on the definite integral.

The common misconceptions and misunderstandings related to each topic form an important part of PCK and were therefore referenced in the discussions.

Chapter Three will describe the research paradigm, the research approach and hence the research design. The population, sampling methods and data collection instruments will be described. Issues such as validity and reliability; data collection procedures, data analysis and ethics will be addressed.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Introduction

The previous chapter discussed the main attributes of the theoretical framework of this study, and how the theory informed the study. The chapter also presented a review of the literature relevant to the research questions.

Chapter Three will describe the research paradigm, approach and design. The population, sampling methods and data collection instruments will be discussed. Issues such as validity and reliability, data collection procedures, analysis and ethics as well as the protocol used in the study will be addressed.

3.2 Research paradigm

> The term paradigm was first termed by Thomas Kuhn (1962)**.** A paradigm is a collection of understandings (explicit or implicit) on the part of an individual or group of individuals about the kinds of things one does when conducting research in a particular field, the types of questions that are to be asked, the sorts of answers that are to be expected, and the methods that are to be employed when searching for those answers (Asiala et al., 1996, p. 2).

The Oxford English dictionary online (OED, 2017) defines a paradigm as "a world view underlying the theories and methodology of a particular scientific subject". The Merriam Webster Dictionary elaborates on this definition and defines a paradigm as "a philosophical and theoretical framework of a scientific school or discipline within which theories, laws, and generalizations and the experiments performed in support of them are formulated" (Merriam-Webster, 2017).

A research paradigm is thus an established model, accepted by a substantial number of people in a research community. A paradigm consists of important constructs: ontology, epistemology, methodology and methods (Scotland, 2012). Ontology refers to the study of being and the nature of the reality (Crotty, 1998). Epistemology addresses "…the ways in which social reality ought to be studied" (Bryman, 1992) and concentrates on "…the origins and nature of knowing, the construction of knowledge and the relationship between the knower and the known (Maykut and Morehouse, 1994)", as cited by Assalahi (2015). Epistemology is thus concerned with how knowledge can be created, acquired and communicated, in other words what it means to know (Scotland, 2012). Every paradigm is based upon its own ontological and epistemological assumptions which cannot be proved or disproved. "Different paradigms inherently contain differing ontological and epistemological views; therefore, they have differing assumptions of reality and knowledge which underpin their particular research approach. This is reflected in their methodology and methods" (Scotland, 2012). Methodology is the strategy or plan of action which underpins the choice and use of particular methods, the specific techniques and procedures used to collect and analyse data (Crotty, 1998).

Each academic discipline may have its own research paradigms. Some of the main paradigms used in educational research are the Positivist paradigm, the Interpretive paradigm and the Critical paradigm. Some approaches make use of a multi-paradigmatic methodology. In this study, a positivistic paradigm was used.

3.2.1 Positivistic paradigm

Positivism is a philosophy which first appeared in Francis Bacon's writings in the 16th century (Crotty, 1998). The ontology of the positivistic paradigm is based on realism. Realism claims that objects exist, and can be researched independently of the observer and the researcher (Scotland, 2012). Realism, which is the epistemological underpinning of positivism, understands meanings to reside within entities as objective truth, independent of the human mind (Crotty, 1998).

The positivist epistemology is therefore one of objectivism. Positivists claim to be able to discover absolute knowledge about an objective reality. Reality according to Positivism is observable, replicable and verifiable (Karlsson, 2016). Objectivism is the thus the ontological tradition behind positivism (Assalahi, 2015).

These underpinnings influence the methods used by positivists. Since a positivist methodology is directed at explaining relationships, deductive approaches are usually followed, and verifiable evidence are collected. Commonly, empirical testing is employed with control groups, random samples, controlled variables and the use of statistical techniques to analyse data (Scotland, 2012). Positivists furthermore view their methodology as value neutral, thus any results obtained are regarded as value neutral (Scotland, 2012).

3.3 Research approach

A "research approach" has a less evaluative meaning: it refers to a way of doing research, which may or may not be accepted by a significant proportion of a research community. Approaches could refer to designs, methods of data collection or analysis. The main research approaches are the inductive (qualitative) and the deductive (quantitative) approaches.

The quantitative research approach makes use of empirical methods and empirical statements which are expressed in numerical terms. According to Cohen, Manion and Morrison (2013) empirical statements describe the real world (descriptive) as opposed to being prescriptive (what should be). Quantitative research designs are either descriptive (usually based on one measurement only) or experimental (Pre-test, Post-test design) (Babbie, 2010). A descriptive study establishes only associations between variables; an experimental study attempts to establish causality. Quantitative approaches collect data by experimentation or observation (Assalahi, 2015) and these are used for testing theories and hypotheses (Sukamolson, 2007). Quantitative methods employ numerical methods such as statistical or mathematical analysis of data. There are four main types of quantitative research approaches: descriptive approaches, correlational studies, quasi-experimental approaches and experimental approaches (Babbie, 2010). The differences between the four types primarily relates to the degree the researcher designs for control of the variables in the experiment.

The qualitative research approach is concerned with assessment of attitudes, opinions and behaviour (Kothari, 2004). The aim of qualitative approaches is to answer questions about experience, meaning and perspective, most often from the standpoint of the participant (Hammarberg, Kirkman and de Lacey, 2016). Qualitative research is important in the behavioural sciences where the aim is to discover the underlying motives of human behaviour (Kothari, 2004). Qualitative research often provides detailed narrative descriptions and explanations of phenomena investigated, and data are collected through methods such as in-depth interviews or observations of subjects. Other techniques include completion tests such as sentence and story completion tests (Kothari, 2004).

3.3.1 A comparison of the two main research approaches

The deductive approach is concerned with testing a theory, whilst the inductive approach is concerned with generating theory emerging from data. Deductive approaches are generally associated with quantitative research and inductive approaches with qualitative research. The quantitative approach is derived from positivism while the qualitative approach is derived from critical science and interpretive science (Eiselen, 2006). The main purpose of the quantitative approach is to describe and explain, while that of the qualitative approach is to understand. Although the quantitative approach's main advantage is related to its inherent ability to generalise, it is exactly in this regard where its shortcomings lie (Eiselen, 2006). The approach is criticised by proponents of the qualitative approach as being unconcerned with the social factors that concern human beings (Assalahi, 2015). The researcher's viewpoints and value judgements are isolated from the research, which is ideally objective and neutral. Likewise, no attention is paid to the individual differences between subjects (Hara, 1995). In educational research, factors such as "the complexity of the society, changes over time, and cultural differences" complicate the research process and make it difficult to take a neutral stance (Hara, 1995).

3.4 Research design

Research design refers to the overall scheme used to incorporate the different components of the study in a coherent and logical way. It includes the strategic plan for the collection and analysis of data. This study followed a quasi-experimental (non-equivalent groups) design using Pre-Post testing (Table 3.1). A quasi-experimental design is similar to a traditional experimental design, but differs in that assignment to experimental is not random. A quasi-experimental study can be viewed as a nonrandomized, Pre-Post intervention study. Quasi-experimental research designs test causal hypotheses and offer practical options for conducting impact evaluations in real world settings (White and Sabarwal, 2014). The independent variable in studies based on ConcepTests is the reform calculus course that has had a change in instruction mode and/or implementation and integration of technology. The dependent variable is conceptual knowledge of calculus (Kueffer and Latterell 2001).

This study was framed within the quantitative approach since its purpose was to collect data and analyse the data statistically. The approach was experimental, and involved Pre-testing and Post-testing. The design used in this study is called the Untreated Control group Design with a Pre-test and Post-test, and was diagrammatically proposed by Cook and Campbell (1979) as follows:

$$\frac{O_1 \quad X \quad O_{1,2}}{O_1 \qquad\quad O_{1,2}}$$

where the O represents an observation (i.e. Pre-test or Post-test) and the *X* represents an experimental or an intervention or event. The detailed design for this study can be found in Table 3.1.

TABLE 3.1: Non-equivalent Control group Design*

Experimental group first semester:	**N**	**O_1**	**X**	**$O_{1,2}$**
Control group first semester:	N	O_1		O_1
Experimental group second semester:	N	O_1	X	$O_{1,2}$
	N	O_1	X	$O_{1,2}$
Control group second semester:	N	O_1		O_1
	N	O_1		O_1

*N depicts non-equivalent groups, X denotes treatments and O denotes observations or measures (Trockim, 2006). Each group is depicted on its own horizontal line. Vertical alignment of O's denote tests that are measured at the same time.

3.5. Description of the population of the study and sampling

3.5.1. The population

The target population for this study comprised 461 students studying Mathematics towards a ND in Engineering or ND in Analytical Chemistry at a comprehensive university in the Eastern Cape. More specifically, the population was the mainstream science and engineering students registered for M2.

The purpose of the ND in Analytical Chemistry is to train technicians to provide solutions in almost all areas of chemistry and other industries such as pharmaceuticals, mining, water, forensics, petrochemicals, consumer products and the polymer industries (WSU, 2015). The purpose of the NDs in engineering is to train technicians who will meet the criteria for registration as professional engineering technicians by the Engineering Council of South Africa (ECSA). The technicians, once trained, are expected to be able to perform procedural design functions of limited context, and to diagnose and solve well-defined engineering problems (WSU, 2015, p. 99).

Engineering courses were offered at two campuses of the university where the study was conducted, whereas Analytical Chemistry was only offered at one of the campuses. Between 1600 to 1800 students register annually for first and second semester mathematics for diploma courses in Engineering and Analytical Chemistry. The first-year students are split into mainstream students and extended stream students. Most universities in South Africa apply a point score to NSC qualifications to assist with admission and placement of applicants. Admission Point Score (APS) applicants who have obtained an NSC qualification must meet the required APS that is set for each programme at the selected university. NSC results are translated into points for each NSC subject. Results of interviews and other assessments may also be taken into account in determining the total APS.

Scores applied by the Faculty of Science, Engineering and Technology at the selected university for placement of the 2015 intake of students are explicated in Table 3.2

TABLE 3.2: Admission Point Scores (APS) in 2015 for Science and Engineering applications at the selected university

Level	Percentage	Symbol	Status of Achievement	APS
7	90 – 100%	A+	Outstanding achievement	9
7	80 – 90%	A	Outstanding achievement	8
6	70 – 79%	B	Meritorious achievement	7
5	60 – 69%	C	Substantial achievement	6
4	50 – 59%	D	Moderate achievement	5
3	40 – 49%	E	Adequate achievement	4
2	30 – 39%	F	Elementary achievement	1
1	0 - 29%	G	Not achieved	0

Source: Hidden - to conform with anonymity (2015)

Applicants to the Department of Analytical Chemistry in 2015, the year the data was collected, needed an NSC achievement rating of at least 3 (40 % to 49%) in each of three subjects, namely English (as a home language or first additional language), Mathematics and Physical Science. Applicants to engineering studies needed a higher score, namely 4 (50% to 59%), in Mathematics and Physical Science. Furthermore, a total APS of 30 was required for both science and engineering courses. Other requirements, which are not relevant to this study, had to be met too. If a student had a rating lower than that required by the department, the student was allowed admission to the extended stream of the specific department. The assumption was that students placed in extended streams were not as well prepared for university studies as the students accepted directly into the mainstream. Students from the extended groups were allowed extra time to complete their diploma. The extra time ranged from one to two years. In certain diplomas, such as Analytical Chemistry and Electrical Engineering, the two streams were merged after the first semester of studies. In Mechanical Engineering and Civil Engineering, the two groups remained separate until they had completed their second semester successfully.

3.5.2. Sampling

Education research imposes restrictions on sampling. "Experimental design have limitations when applied to educational settings" (Goba, Balfour and Nkambule, 2011, p. 272). In the field of education research, random selection of participants is often not practical. Researchers are not allowed to disturb the normal running of the teaching institution and hence groups are selected to participate in research as opposed to individual participants. It is therefore not surprising that only 5% of the postgraduate educational research written in the English medium in South African institutions from 1995 to 2004, used random assignment of participants (Goba et al., 2011). They emphasised that the majority of the studies (56%) used convenience sampling and not random sampling.

Various methods exist for determining optimal sample sizes for research. The population size influences the eventual sample size (Delice, 2010). Based on tables compiled by (Krejcie and Morgan, 1970), researchers suggest a sample size of between 30 and 500 if parametric tests are used. The recommended sample sizes of three sets of researchers were compared in Table 3.3. The averages of their recommendations were tabulated in the last two columns.

TABLE 3.3: Comparison of suggested sample sizes

Population size	Suggested sample size (Stoker, 1989)	%	Suggested sample size (Krejcie and Morgan, 1970)	%	Approximate Average	%
20	20	100	19	95	20	100
50	32	64	44	88	38	76
100	45	45	80	80	63	63
200	64	32	132	66	65	49
500	100	20	217	43	159	32
800	120	15	260	33	190	24
1000	140	14	278	28	209	21
10 000	450	4.5	370	3.7	410	4.1

Sources: Stoker (1989), Krejcie and Morgan (1970), Veal (2006)

According to Stoker (1989), smaller samples are acceptable in the case of homogeneous population elements. Homogeneous populations share features such as culture, ethnicity and age. The student population involved in this study ascribed to that condition.

Hence, convenience sampling was applied to select experimental and control groups. Only mainstream students were included in the study. The researcher planned to have a sample size of between 120-200 students. The experimental group comprised of 119 volunteering students from a population of 461 registered for Mathematics as a service subject for ND in science or engineering at a South African university. Those not in the experimental group were taught through teacher-centred traditional approaches which have been the norm. However, only 71 out of those in the traditionally taught cohort volunteered to write both pre- and post-tests. As such, the total number of subjects in the study was 190, i.e., 119 from the experimental cohort and 71 from the traditional cohort. The average recommended ratio of a sample to a population of 500, is 159:500 or 32%. The ratio of the sample to the population in this study was 190:461, which amounted to 41.2%, which was sufficient.

In an attempt to improve the internal validity of the study, science students were included in the experimental group, since the entrance criteria for this diploma were lower, and the students were academically weaker at mathematics than the rest of the population. An additional problem was that the Analytical Chemistry diploma programme had Mathematics I scheduled during students' first study semester, whereas M2 was scheduled 18 months later. In contrast, M1 and M2 for the engineering groups were scheduled in consecutive semesters. Experience has shown that the 18 month time lapse between the two courses in the Analytical Chemistry diploma adversely affected students' academic performance in M2. On the other hand, when these students enrolled for M2, they were academically more mature compared to the other M2 students. The maturity of the science students may have counteracted the negative effects of the time gap between M1 and M2.

The sample selection was not random. Therefore, the researcher will not be able to generalise findings to other populations. However, studies show that students generally regard ARS as a fun approach to learning and teaching and instructors regard ARS as an efficient way of monitoring students' progress. The results of this study may therefore point to more effective ways of learning and teaching calculus and contribute to the knowledge base on RBAs in calculus classrooms.

3.6 Instrument of data collection

"Quality of the measurement tool should play a fundamental role in the analysis of the data it produces; however, this element is often overlooked" (Bradley, Sampson and Royal, 2006).

The instrument contained a background questionnaire which elicited biographic and demographic information as well as information regarding their Grade 12 mathematics achievement, their satisfaction levels with the latter and an evaluation of the value they attached to mathematics as a determinant for their chosen career path.

The instrument, the CCIT, was adjusted from the CCI, an internationally standardised instrument to make it more suitable for the purposes of this study. The CCI was in turn modelled on the FCI in physics. The CCI measures conceptual understanding of the fundamental principles of differential calculus. The test consists of 22 multiple choice questions, and distracters are designed to elicit misconceptions (Peterson, 2012). The researcher was granted permission to use the original test (CCI) after signing an agreement of confidentiality (Appendix A). Some adjustments were made to the test to make it suitable for diploma students at a South African university. The weakness of the CCIT is that it had not been tested before this study. The strength was that it was based on, and was similar to, the CCI, an iconic benchmark in Calculus ConcepTests, and the only such test available at the time the study commenced.

The very nature of calculus reform demands different methods of assessment than those used in traditional courses. The predicament of a researcher making use of quantitative comparisons between traditional calculus and reformed calculus courses, is: does one give a reform-taught group of students a traditional examination, or alternately, a traditional class a reform exam? No standardised reform tests in the form of Concept Inventories had been available internationally, until the advent of the CCI of Epstein at the end of 2013 (Epstein, 2013). This test, following the multiple successes of the Force Concept Inventories (FCI) used in Physics, was regarded as iconic, and a benchmark in Mathematics Education. Bressoud (2013) comments as follows:

> Epstein's Calculus Concept Inventory (CCI) represents a notable advancement in our ability to assess the effectiveness of different pedagogical approaches to basic calculus instruction. He presents strong evidence for the benefits of Interactive engagement over more traditional approaches. As with the older Force Concept Inventory developed by Hestenes *et al.* [2], CCI has a great deal of surface validity. It measures the kinds of understandings we implicitly assume our students pick up in studying the first semester of calculus, and it clarifies how little basic conceptual understanding is absorbed under traditional pedagogical approaches. Epstein claims statistically significant improvements in conceptual understanding from the use of IE, stronger gains than those seen from other types of interventions including plugging the best instructors into a traditional lecture format. Because CCI is so easily implemented and scored, it should spur greater study of what is most effective in improving undergraduate learning of calculus.

The development of a ConcepTest is a lengthy, arduous and iterative process. According to Madsen, McKagan and Sayre (2014), the development process should follow certain guidelines:

(i) Gathering students' ideas about a given topic, usually with interviews or open-ended written questions, and identifying patterns in these ideas.
(ii) Using students' ideas to develop questions where the responses cover the range of students' most common incorrect ideas using the students' actual wording.
(iii) Testing these questions with another group of students and ensuring that students choose the correct answer for the right reasons. Usually, researchers use interviews where students talk about their thinking for each question.
(iv) Testing these questions with experts in the discipline to ensure that they agree on the importance of the questions and the correctness of the answers.
(v) Revising questions based on feedback from students and experts.

(vi) Administering concept inventory to large numbers of students. Checking the reproducibility of results across courses and institutions. Checking the distributions of answers. Using various statistical methods to ensure the reliability of the assessment.

(vii) Revising again.

This rigorous development process should produce valid and reliable assessments that can be used to compare instruction across classes and institutions (Madsen et al., 2014).

Based on the steps to develop a good research-based assessment, Madsen (2016) additionally created a list of seven categories of research validation in order to assess the rigourr of the development process of concept inventories. In order to validate Epstein's test, I used the guidelines compiled by Madsen et al. (2014) and compared Epstein's CCI test to each of the stated validation categories (Table 3.4).

TABLE 3.4: The CCI measured against seven validation categories

Research validation categories of Madsen	**Epstein's CCI validated according to Madsen's seven categories**
Questions based on research into student thinking	A set of Cognitive Laboratories (analytic interviews with students) was done.
Studied with student interviews	A set of Cognitive Laboratories (analytic interviews) was done.
Studied with expert review	A team of developers, consisting of colleagues and professionals were involved in the development.
Appropriate use of statistical analysis	Reliability coefficient was measured as 0.7
Administered at multiple institutions	In 2008 the CCI was administered to all fifty-one sections of Calculus I at the University of Michigan (1,342 students). The test was also administered to students of the East China Normal University, Shanghai, P.R., China.
Research published by some other than developers	Thomas, M. (2014). Analyzing Conceptual Gains in Introductory Calculus with Interactively-Engaged Teaching Styles. (Doctor Of Philosophy), The University of Arizona, Arizona. Retrieved from http://hdl.handle.net/10150/299075
At least one peer-reviewed publication	The following articles appeared in peer-reviewed journals and these studies used CCI as an instrument: A cross-national study of calculus, International Journal of Mathematical Education in Science and Technology. *46, 4*, 481- 494. Three articles in Maxson, K. and Szaniszlo, Z. (Ed.) (2015). Special Issue on the Flipped Classroom: Reflections on Implementation [Special Issue]. PRIMUS, 25(8), of which one is: Ziegelmeier and Topaz (2015). Flipped Calculus: A Study of Student Performance and Perceptions. Primus, 25 (9-10), 847-860.

Epstein's test certainly appeared to pass the various tests of validation set by Madsen et al. (2014). Considering the importance of the CCI test and its iconic status, I endeavoured to use the test in my research. Another important consideration was my inexperience with ConcepTests. I was a novice in the area of using and applying ConcepTests, and also had no experience in compiling this type of test. When the CCI was scrutinised, the test however appeared to be too difficult for diploma students and some questions were not found to be suitable for the specific curriculum. Experts were consulted, and the test was adjusted to make it more suitable for diploma students. After a pilot study, further adjustments were made, again in consultation with experts. One question in particular was phrased in a word-rich and lengthy manner, and one of the experts was of the opinion that students who study in a language other than their mother tongue would have difficulties understanding and interpreting the question correctly. That question was replaced with one which was deemed similar, yet phrased more simply. Three other questions yielded a zero score for all participants in the pilot study. These questions were also replaced with questions which were deemed similar, but slightly easier.

It was clear that test development was very time-consuming and technical, and resources were not available to go that route. Because of these reasons and the hype around Epstein's benchmark test, I decided to use four questions from Epstein's test. These questions did fit in with the curricula of the relevant academic departments serviced. Experts from these departments opined that these questions were of a suitable degree of difficulty. The rest of the questions were replaced by similar questions which were deemed to be better aligned with the curriculum of the departments and with students' prior knowledge. A decision was made to include integration as one of the topics, since the test was going to be used after students had already passed a first course in calculus.

ConcepTests are notoriously difficult, and because of this reason, students can guess answers without having a robust understanding of the concepts being tested, yet student scores are usually much lower than one would expect if they were guessing (Redish, 2003), as cited by Madsen et al. (2014). This is because well-designed distractors entice students to pick other options than the correct one (Madsen et al., 2014). Even if students guessed, the results remain valid:

> Further, instructors use the *differences* in scores between Pre- and Post-test (e.g. raw gain, normalized gain or effect size), not the value of the scores themselves, to determine the effectiveness of their teaching. If

scores were slightly inflated by guessing, looking at differences in scores cancels out this effect, and the comparisons remain valid (Madsen et al., 2014, p. 2)

3.7 Validity and reliability

3.7.1 Validity

Validity is the extent to which a test measures what it is supposed to measure (CIRT). Validity refers to the appropriateness, meaningfulness, and usefulness of the specific inferences made from test scores. Validity can further be categorised as internal validity and external validity.

3.7.1.1 Internal validity

Internal validity addresses the question whether a study has generated accurate and valid findings of the specific phenomena which have been studied. A study has internal validity if the constructs were measured in a valid manner, the collected data are accurate and reliable, the analyses are relevant for the type of data, and the final conclusions are adequately supported by the data. Internal validity is concerned with controlling extraneous variables and outside influences that may impact the outcome. “This is important in quasi-experimental studies that are attempting to demonstrate causation to ensure that the experimental treatment (X) is, in fact, responsible for a change in the dependent variable (Y)” (CIRT, p. 1).

3.7.1.1.1 Threats to internal validity

The following threats to internal validity have been identified:

- **History –** Events that occur during a study that may impact the results. If an event influenced one of the groups and not the other, differences between groups may be ascribed to the event, and not necessarily to the intervention.
- **Maturation –** Natural changes, biological or psychological, within the participants over the time of the study may impact the results. Generally, maturation is not a threat to the two group design, assuming that participants in both groups mature at the same rate. Also, the maturation threat in this study

was minimal, since the study was conducted over a relatively short period, with Post-tests conducted three months after Pre-tests.

- **Testing –** Pre-tests may influence the performance of subjects on subsequent tests simply due to the fact that participants have already seen or completed the test before. Since both experimental and control groups in this study were exposed to the Pre-test and the Post-test, any differences between groups should not be ascribed to testing influences.
- **Instrumentation –** Changes in testing instrumentation during a study may affect what is being measured and how it is measured. In this study, both experimental and control groups wrote the same Pre- and Post-tests which minimised the threat to internal validity from this particular source.
- **Statistical Regression –** Statistical regression, or regression to the mean, can be a concern in studies with extreme scores, either particularly high or low. Scores are typically less extreme in subsequent testing in most situations, making meaningful Pre-test and Post-test comparisons more difficult. If the control group's scores however remain similar to the Pre-test scores such as was the case in this study, then statistical regression should not be regarded as a threat. Also, in a study on the FCI, Henderson (2002) found that the Pre-test did not bias Post-test results.
- **Selection –** If the subjects placed into the groups are selected in a non-random manner or are not equivalent at the beginning of the study, the results of the study could be biased when making comparisons between the groups at the end. A quasi-experimental design by definition lacks random assignment, which could be viewed as a threat to internal validity. Assignment to treatment versus no treatment is by means of self-selection or administrator selection. In this study, pre-existing groups were selected by the researcher to participate in the study. "Quasi-experimental designs identify a comparison group that is as similar as possible to the experimental group in terms of baseline (pre-intervention) characteristics. The comparison group captures what would have been the outcomes if the programme/policy had not been implemented (i.e., the counterfactual)" (White and Sabarwal, 2014). To counter-act a potential threat to internal validity, the group which presented the lowest academic ability, as measured by the entrance criteria to their studies, was selected as an experimental group.

Experimental Mortality – Test subjects drop out of studies for a variety of reasons. The loss of participants from control groups may impact the study if the withdrawal or mortality rate is higher in one group or if it is particularly high in both groups. The experimental group had a mortality of 15% and the control group a mortality of 41%, which is substantially higher. The remaining number of participants was however still satisfactory.

- **Selection Interaction –** The selection method may interact with one or more of the other threats and impact results. Compensatory rivalry may affect the outcomes of the intervention. When subjects in the control group perceive the teaching of the experimental group to be of higher quality or more interesting because of the use of technology, social competition may motivate the latter to attempt to reverse or reduce the anticipated effects of the desirable treatment levels. The opposite may occur. The subjects of the control group may experience feelings of resentment because of missing out on the intervention and become demoralised. In this study, the experimental group and the control group were located at two remote campuses approximately 90 km apart, or at two delivery sites, approximately 30 km apart, which reduced the possible contact between the groups and therefore the threat to internal validity.
- The Hawthorne effect is a psychological phenomenon that produces an improvement in human behaviour or performance as a result of increased attention from superiors, clients or colleagues. The John Henry Effect refers to the tendency for people based in a control group to perceive themselves at a disadvantage to the experimental group and work harder in order to overcome the perceived deficiency. These effects can produce short-term benefits associated with the special attention rather than the intrinsic worth of the treatment given to a research test group (Hawthorne effect) or the desire of a control group to exceed the performance of a competing test group (John Henry effect). Although students may benefit from IE in the long term, they were not however singled out for special attention. This benefit is intrinsic to the pedagogy and should not be classed as a Hawthorne effect. Furthermore, such benefits should diminish when the ‘treatment’ is selected as the strategic, regular pedagogy.

3.7.1.2 External validity

External validity addresses the issue of whether findings of a study can be generalised to similar and usually larger populations. External validity is therefore synonymous to generalisability (Mouton and Marais, 1996). In true experimental studies, the random selection of participants and random assignment of the study participants into groups ensure that the members of the study are truly representative of the larger population. The best evidence for external validity is whether or not the research findings can be reproduced with different populations, settings, treatment variables or times. Random selection ensures that results can be generalised. Quasi-experimental studies present challenges because participants are not randomly selected. Selection of a reasonably similar control group is therefore important.

3.7.1.2.1 Threats to external validity

- Interaction Effects of Testing – The Pre-test may make the participants more aware of the issues to be studied and may therefore influence the response to the treatment. However, in this study a control group counteracts this concern, since both experimental and control groups wrote the Pre- and Post-tests. Awareness of research themes or issues under consideration will therefore affect both groups, not only the experimental group.
- Selection Bias – This occurs when subjects are selected in a manner that does not ensure that they are representative of the overall population. The random selection of subjects is a critical factor in determining external validity. In educational research, this is however not always possible.

3.7.2 Reliability

An instrument is reliable if similar results are achieved when the test is repeated on the same sample. Several procedures exist for establishing the reliability of an instrument, such as the test-retest, the split-half technique, calculations of internal consistency (Cronbach alpha) and inter-rater reliability (Delice, 2010; Agbatogun, 2013; Vosloo, 2014). Inter-rater reliability is a measure of reliability used to assess the degree to which assessments by experts correspond. It measures the extent to which test results are consistent when measurements are taken by different people using the same instrument. Test-retest is a measure of reliability obtained by administering the same test twice over a period of time to a group of individuals. One of the most popular reliability statistics in use today is Cronbach's alpha which determines the internal consistency or average correlation of items in an instrument (Santos, 1999). If the items are strongly correlated with each other, their internal consistency is high and the alpha coefficient will be close to one. In this case, some of the items may be redundant. On the other hand, if the items are poorly formulated and do not correlate strongly, the alpha coefficient will be close to zero. The following guidelines for the interpretation of Cronbach's alpha coefficient are generally accepted by researchers: 0.90-high reliability; 0.80-moderate reliability and 0.70-low reliability (Vosloo, 2014). For the purposes of the current study, a pilot study was conducted, the test-retest procedure was followed, Cronbach's alpha calculated and t-tests and Chi-squared tests conducted.

3.7.3 Validity and reliability of the instrument used in the study

In order to evaluate the face validity and the content validity, the instrument used in this study was subjected to scrutiny by experts in the field. Face validity denotes whether a test measures what it claims to measure. Face validity was improved by conducting pilot tests to verify the relevance and representativeness of the items in the various categories. Content validity is the degree to which a test matches a curriculum and accurately measures the specific outcomes objectives on which a program is based. When a test has content validity, the items on the test represent the entire range of possible items the test should cover. Assessments by subject experts are used to determine if a test is accurate, appropriate, and fair (McCowan and McCowan, 1999). The test was subjected to the scrutiny of experts.

The construct validity of an instrument refers to the extent to which the set of items measure what they purport to measure. Construct validity is the extent to which a test measures a theoretical construct (McCowan and McCowan, 1999). Construct validity can be established in various ways (Eiselen, 2006). In this study PBI-scores were calculated to determine the construct validity for the subsections of the questionnaire

3.7.3.1 Validity and reliability of the CCI

Epstein and Yang obtained funding in 2004 to develop a calculus concept inventory. Since 2004, a team of developers consisting of colleagues and professionals such as consultants and companies specialising in the standardisation of tests, assisted Epstein and Young to develop and refine the CCI. The test developers included items on functions, derivatives, and a third section on limits, ratios, and the continuum.

An interviewing technique called Cognitive Labs, also referred to as analytic interviews, was utilised from 2006 to 2007 to eliminate items with poor discrimination. The researchers also checked that the test items were addressing the intended misconceptions. Finally, psychometric analysis was conducted by consultants. The reliability coefficient was found to be 0.7. The test has subsequently been used internationally and has yielded similar results in diverse countries (Epstein, 2007; Peterson, 2012).

3.7.3.2 Validity and reliability of the CCIT

The CCIT is based on the CCI and similar to the CCI, but changes were made to make it more suitable for diploma students at a South African university. The CCIT further differs from the original CCI in that it contains questions on anti-derivatives, whereas the CCI did not.

It is important to refine the instrument to fit the target cohort to be tested. The original CCI targeted Calculus I students studying for degrees at international universities. The CCIT targeted diploma students taking calculus as a service subject for science and engineering diplomas. Diploma students differ from bachelor degree students in various ways, the most important being lower entrance level skills and academic performance prior to their university studies. The researcher had to take cognisance of this fact. The material for the intervention had to be pitched carefully in order to stimulate conceptual thinking, but care had to be taken that it was not be beyond the capabilities of diploma students, since students could get discouraged and despondent. Furthermore, the quality of the analyses based on the instrument could be compromised.

> It is a well-known principle of educational measurement that the difficulty of the items used to assess student achievement should match the ability of the students taking the assessment. In the context of assessing mathematics achievement, measurement is most efficient when there is a reasonable match between the mathematics ability level of the student

> population being assessed and the difficulty of the assessment items. The greater the mismatch, the more difficult it becomes to achieve reliable measurement. In particular, when the assessment tasks are much too challenging for most students, to the extent that many students are responding at chance level, it is extremely difficult to achieve acceptable measurement quality (Mullis, Martin, Ruddock, O'Sullivan and Preuschoff, 2009, p. 39).

Although the researcher took this statement into consideration, she did not want to deviate too much from the original nature of Epstein's CCI test, since the latter was, and still is, an iconic benchmark in calculus concept testing. Also, "quality assessment must be aligned with curriculum and teaching to have any effect" (Epstein, 2007). The curriculum for mathematics as a service course for diploma programmes differ quite substantially from curricula for the degree courses in South Africa. Adapting the test was therefore a challenge, because it involved judgements about how the targeted group would respond to the items, both individually and collectively, but doing so prior to having any corroborative information. A balance had to be found between the high difficulty levels of a ConcepTest, and the needs of the targeted cohort. This challenge required subject expertise, teaching experience and PCK particular to the subject and the context in which it is taught (Dunne, LONG, Craig and Venter, 2012). Input was thus obtained from experienced subject experts on the suitability of items and item structures. The test was hence piloted on students who had already passed M2, and afterwards more adjustments were made to the test. As already mentioned elsewhere, persuading students to participate in the pilot study proved difficult. Not a single person pitched for the pilot test when the first call was made. After making incentives available, eight students who had already passed M2 agreed to write the pilot test in February 2015.

The CCIT consisted of 19 questions and covered functions, derivatives and anti-derivatives. Almost all the questions were multiple choice questions. In one question though, students were expected to identify two units of measurement and write these down. The CCIT was used as a Pre- and Post-test for M2 students (Appendix B). Some M2 groups were selected to be in the experimental group who were exposed to the intervention designed for this research, and other groups were selected as control groups. All were tested using the same CCIT test to evaluate the effectiveness of the intervention. The questions were divided into three categories, namely functions, differentiation and antiderivatives (Table 3.5).

TABLE 3.5: CCIT: Three categories of questions

Category	Topic	Question numbers
1	Functions	2, 11, 13, 15.1, 16, 18
2	Derivatives	3, 4, 5, 6, 8, 9, 10, 14, 15.2, 17
3	Antiderivatives	1, 7, 12

Criterion validity is regarded as the ability of a measure to correlate with other standard measures of similar constructs or established measures. The CCIT was based on the only Calculus Concept Inventory developed to date, namely the CCI. However, only four of the questions in the CCI were retained. The other questions were replaced with similar questions which were deemed more suitable to diploma students. A confidentiality agreement was signed between the researcher and the author of the CCI (Appendix A). The four questions from the CCI that were used in the CCIT, can therefore not be made public. Only their topics are mentioned. The sources of the other questions are depicted in Table 3.7. The reliability of the test was low, as depicted by Cronbach's alpha in Table 3.6.

TABLE 3.6: Cronbach alpha of the CCIT

Reliability Statistics	
Cronbach's Alpha	**No of Items**
0.434	19

TABLE 3.7: Sources of questions of the CCIT

	Topic	Source
1	Integral and constant of integration	Question 12 of "Constructing Antiderivatives Analytically" (Terrell, 2005, p. 3)
2	Exponential growth	Epstein
3	Limit	Question 15 (Terrell, 2005, p. 121) This type of problem links differentiation rules with the limit definition of derivative
4	Slope	Gibson (2011, p. 96) Gibson, L.R.
5	Velocity	Epstein
6	Instantaneous rate of change	Question 24 of the section The Complete Set of Differential Calculus Questions, (Terrell, 2005, p. 8) This type of problem prepares students for related rates problems.
7	The definite integral and area calculation	Question 10 of Section 5.2, "The Definite Integral" (Terrell, 2005, p. 6)
8	The derivative of a function	Epstein
9	Maximum of a function read from a graph of the derivative of a function	Adapted from Question 55 Section 6.1 Antiderivatives Graphically and Numerically (Terrell, 2005, p. 27)

10	Fourth derivative of a trigonometric function	Question 121 Classroom Voting Questions: Calculus I The Trigonometric Functions Q4 (Terrell, 2005, p. 32)
11	Reflection and Horizontal and Vertical shifts	Question 140 from " The Complete Set of Differential Calculus Questions " (Terrell, 2005, p. 39)
12	Anti-derivative of a function determined from a graph of the function	Question 20 from "The Complete Set of Differential Calculus Questions", (Terrell, 2005, p. 7)
13	Amplitude and period determined from a graph of a trigonometric function	Question 209 from the "Pre.student.edition.pdf" (Terrell, 2005, p. 57)
14	Testing graphically the signs of the function, first derivative and second derivative	Question 38 of the section "The Complete Set of Differential Calculus Questions", (Terrell, 2005, p. 12) Also Question 11, (Hughes-Hallett et al., 2005, p. 91)
15	Units of first derivative and of the inverse of a function	Adapted from Question 8, (Hughes-Hallett et al., 2005, p. 88)
16	Use a graph to read values of a function at two points, and the inverse of a function at two points	Adapted from Question 11, Calculus Readiness Test, Department of Mathematics, Simon Fraser University (SFU)
17	Tangents	Epstein
18	Linear functions	No 13 of the section on Functions and Change (Terrell, 2005, p. 3) . Also similar to question 13.2, from Schlatter (2011, p. 89).

The CCIT was also analysed by experienced mathematics lecturers, and their recommendations were implemented. They were hence asked to rate the expected performance of the students in each question in the questionnaire as either very easy, easy, moderate, difficult or very difficult (Figure 3.1). Most of the questions were rated as difficult or very difficult (73.7%), in line with the questions contained in the CCI test. A further 21.1 % of the questions were rated as moderate and only one question was rated as easy. None of the questions was rated as very easy.

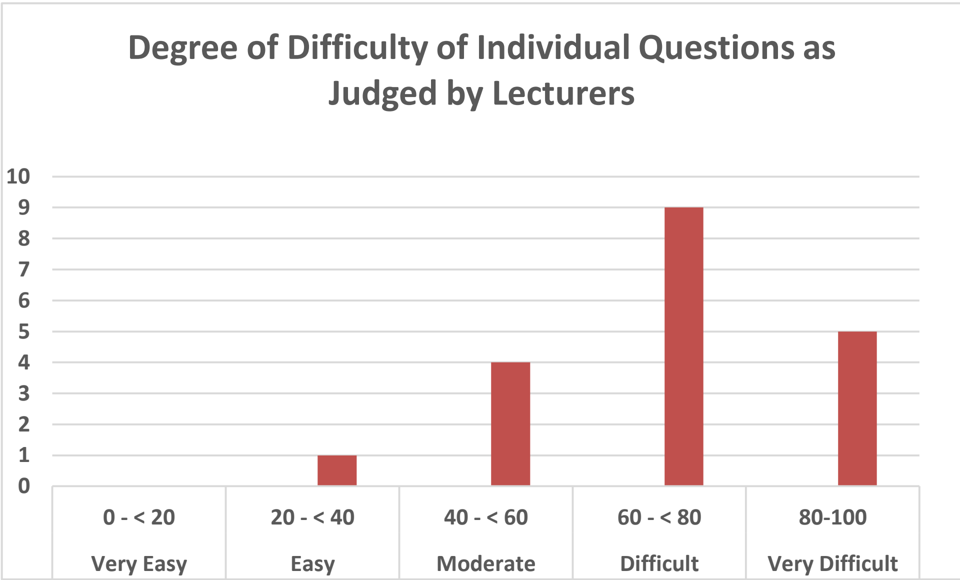

FIGURE 3.1: Difficulty level of CCIT as judged by experts

3.8 Data collection procedures

A quasi-experimental model was employed with Pre-Post-testing and control groups. The intervention was tested on selected groups of Calculus II students from the university using the CCIT instrument. Only mainstream students were selected to participate in the study. The experimental group was lectured using RBAs and the control group was lectured using traditional approaches. The Pre-tests were given in the second week of lectures in a time-slot allocated to tutorial periods, which had not commenced at the time. Students wrote the Post-tests during the last week of lectures, also in a tutorial time-slot. The testing was conducted at various times and venues which suited the particular participating groups. Field workers supervised the process to ensure that no undesired lecturer-researcher influence was exerted on students.

Researchers such as Couch and Knight (2015) allocate participation points (reward for participation in research). In this study, no participation points were awarded and tests scores did not contribute to promotion scores. This format encourages participation, yet questions remain regarding students' commitment to the assessments. ConcepTests are difficult and differ from the assessments that students are familiar with, in that little procedural activities are required, but more mental deliberation. Students may therefore get demotivated during the test and may not spend a sufficient amount of time and effort to complete the assessments which may affect student scores (Couch and Knight, 2015).

3.9 Pilot study

Pilot studies are conducted for two reasons, one as a feasibility study or trial run, and the other in order to Pre-test the instruments. A pilot study is a small scale test of the methods and procedures that will be used on a larger scale in the main study (Hazzi and Maldaon, 2015). "Moreover, pilot studies are undertaken to identify possible practical challenges (ethical, political, procedural or policy issues) the researcher may likely encounter and eventually affect the research process" (Agbatogun, 2013, p. 139).

Vosloo (2014) summarised some of the useful attributes of a pilot study:

(i) A pilot study may reveal possible flaws in the logistics (such as ambiguous instructions, or inadequate time limits);
(ii) A pilot study may reveal unclear or ambiguously formulated items;
(iii) A pilot study affords researchers or assistants an opportunity to study non - verbal behaviour or discomfort on the part of participants. These may point to previously unidentified anomalies in the measurement process.

The main advantage of a pilot study is that it could give advance warnings about potential problems with the main study. The main aim of the pilot study was to test and refine the selection of material for the instrument. Difficulties were experienced when trying to persuade subjects to take part in the pilot test. Since all the M1 students were targeted for the main study, the students who had already passed M1 were approached to take part in the pilot test. None of those students pitched on the day of the test. Eventually, an incentive in the form of a lucky draw for an amount of R 250.00

was used to entice students to participate and hence eight students wrote the pilot test in February, 2015. They took an average of 41 minutes to complete the test. The lucky draw was done by an impartial colleague, with a representative from the students who wrote the test in attendance. The draw was won by student number 214153312 and the amount paid to him in cash, for which he duly signed a receipt. The average score for the pilot test was 39.8%. Students clearly experienced three of the questions as very difficult, since none of the students got these questions correct. These questions were hence adjusted or replaced.

3.10 The protocol employed in the study

Although there is no general agreement yet amongst researchers on what activities constitute IE, Thomas (2014a) compiled a protocol of interactions, which included organising activities into categories, such as public or private, and then according to the initiator of the episodes. Without going into such detail, it is generally assumed that during traditional lecturing, students passively listen to lectures, whereas in IE classes, students are actively engaged with the material and interacting with the instructor and each other. Furthermore, knowledge is usually measured in traditional classes with questions that are largely algorithmic (Hake, 1998a), whereas knowledge measured in the IE classes are more conceptual. These are the two extremes, and in between these extremes are of course a spectrum of levels of IE. It is generally measured by the proportion of class time allocated to IE.

Students enrolled for mathematics as a service course at this campus had time slots allocated to mathematics on their timetable four times a week for 1.5 hours at a time. The researcher in this study decided to use one of these time slots for IE, and to continue with traditional lecturing during the other three time slots. This arrangement is generally classified as a low level of IE. During the IE time slot, clickers were distributed to students, and GQ and PD were utilised. Examples of GQ used in the IE lessons can be found in Appendix F.

3.10.1 Awarding marks to clicker activities

Awarding marks to clicker activities has become a controversial practice (White et al., 2011). White et al. (2011) recommend rewarding students for giving correct answers or for participating in high-value constructivist learning activities. They further advice against the use of clickers for high-stakes testing because of the ease of cheating with clickers.

When using ConcepTests, researchers have to consider a variety of variables which could possibly influence response rates and the degree to which students take the assessment seriously (Couch and Knight, 2015). One issue is whether clicker assessments should be used for low-stakes or high-stakes assessments. ConcepTests usually result in low scores, at least in the Pre-tests, and it is therefore not advisable to use these in high-stake testing. It has become common practice to reward participation points in order to encourage students to participate in the Pre- and Post-tests and to attend the clicker classes. Although many researchers reward students for correct answers and/or participation, this researcher decided against it, mainly because she did not want to disadvantage the students. The lecturer was a novice in the use of the clicker technology and the pedagogy that accompanies it, and lacked TPACK. The Pre-and Post-tests were low-stakes assessments and were not used in any way for promotion purposes. Also, only 40 clickers were available and clickers had to be shared in the bigger classes (one class had 93 students), which made individual assessment impractical.

3.10.2 Reform-Based text books

No Reform-Based text was used in the intervention, which may have influenced the results. This decision was taken by the researcher since she did not want to disadvantage the students and introduce a new text, not knowing if the new pedagogy would be received positively. A new text is costly, since students are not able to buy second hand textbooks from the previous years' students. Also, the reform text books are not readily available in South Africa, and would have had to be imported at great cost from the international publisher.

3.11 Data analyses procedures

The normality of the data was investigated. Student t-tests were used to compare scores. The Pearson correlation coefficient was calculated to gauge correlations. The raw gain and normalised gains were calculated. The Cronbach alpha and the KR-20 were calculated for reliability purposes. A Difficulty Index and Item Discrimination PBI were calculated to investigate the correlation between the score on particular items and the overall test score.

Hence the gain and the normalised gain of a class was calculated using the following formula:

$$G = <g> \frac{gain}{possible\ gain} = \frac{(mean\ post{-}test\ \%\ score) - (mean\ pre{-}test\ \%\ score)}{100 - mean\ pre{-}test\ \%\ score}$$

Hake (1998) views the gain <g> to be high if $G \geq 0.7$, medium if $0.3 \leq G > 0.7$ and low if $G < 0.3$. The normalised gain score measures the fraction of previously unknown material that is learned throughout the course. If a group improves from an average of 40% in a Pre-test to 70% in the Post-test, the normalised gain would be G = 0.5, meaning that class, on average, correctly answered half of the 60% of the material they answered incorrectly at the beginning of the semester (Thomas, 2014a).

3.12 Ethical considerations and requirements

The instrument contained a cover letter / letter of introduction where the aims of the study and the instructions for completion of the tests were provided, as well as section consent in which students provided written consent to form part of the study (Appendix E). This research subscribed to the guidelines of the university that was used as a research site and the University of Fort Hare's ethics committees (Appendix C), and adhered to the predetermined guidelines in their policy documents. Prior permission for this study was obtained from the university where the researcher was registered (Appendix C), as well as the university where the study was conducted (Appendix D). All participants were assured of anonymity and were informed that their participation was voluntary**.** Participants were informed that they were free to withdraw at any point without explanation or any negative consequences. Signed informed consent (Appendix E) was obtained from each participant. No underage students participated in the study. The researcher signed an assurance of no conflict of interest. The instruments displayed the name and contact details of the researcher. To overcome possible conflicts of interest that may arise from the dual role of instructor-researcher, outsiders were employed to provide letters of information, collect consent forms and administer the assessment process.

Furthermore, the dual role of the instructor-researcher was not regarded as an issue in this research project, since the methods that were used, namely RBAs and in particular the use of GQ (GQ) and PD, are generally accepted methods in the context of mathematics instruction, and may form part of regular formative practices.

Pre-existing groups were used in this quasi-experimental study. Usually, the use of pre-existing groups diminishes the ethical concerns that are associated with random assignment – for example, the withholding or delaying of a potentially effective treatment or the provision of a less effective treatment for one group of study participants (White and Sabarwal, 2014). In this study, a case could be made for ethical concerns, since not all the groups were subjected to the treatment or intervention which could be potentially advantageous to the students in the experimental group. However, since each lecturer at the university is autonomous regarding the pedagogical methods chosen for his/her teaching, and the control groups were selected from remote campuses/delivery sites, the potential negative ethical effects were minimised.

3.13 Summary

Chapter Three described the research paradigm, approach and design. The population, sampling methods and data collection instrument was discussed. Issues such as validity and reliability; data collection procedures, analysis and ethics were addressed.

The next chapter will provide a detailed report of the results obtained during the collection of the data. Data will be presented, interpreted and analysed.

CHAPTER FOUR

DATA ANALYSIS, INTERPRETATION AND PRESENTATION

4.1 Introduction

The previous chapter described the research paradigm, approach and design. The population, sampling methods and data collection instruments were discussed. Issues such as validity and reliability; data collection procedures, analysis and ethics were addressed.

This chapter presents the analyses of the data obtained during the collection of the data for the research question. The Null and Alternative Hypotheses were:

H_0: Reform-Based Approaches, as opposed to Traditional Approaches, used in teaching calculus as a service subject for science and engineering diploma students have no effect on their conceptual understanding of calculus.

H_a: Reform-Based Approaches, as opposed to Traditional Approaches, used in teaching calculus for science and engineering diploma students, have a positive effect on their conceptual understanding of calculus.

4.2 Data presentation

4.2.1 Analysis techniques

The normality of the data was examined, first with a histogram and hence with a Chi-squared Goodness-of-fit-test. Student t-tests were used in comparison of scores. The Pre-test scores of the experimental group was compared to the Pre-test scores of the control group to investigate whether the two groups started out from an equal footing. Hence the Post-test scores were compared with Pre-test scores. The Pearson correlation coefficient was calculated to determine whether there was any correlation between self-reported Grade 12 Mathematics scores and CCIT scores. The raw gain and normalised gains were calculated. The Cronbach alpha and the KR-20 were calculated for reliability purposes. The Difficulty and Item Discrimination PBI were calculated to investigate the correlation between the score on particular items and the overall test score.

4.2.2 Biographical data

Participants in the study were registered for one of four diplomas in science and engineering. The frequency distribution according to diploma type is given in Table 4.1.

TABLE 4.1: Frequency distribution of students according to diploma type

Diploma	n	Percentage (%)	Cumulative Frequency (%)
Electrical Engineering	34	17.9	17.9
Mechanical Engineering	31	16.3	34.2
Civil Engineering	54	28.4	62.6
Analytical Chemistry	71	37.4	100.0
Total	190	100.0	

There were 140 students who wrote the Pre-test. However, 21 out of them did not write the Post-test although they were part of the experimental group at the start. Hence the effective sample size was reduced to 119. From the control group, 132 students wrote the Pre-test, but only 71 chose to write the Post-test (Table 4.2). The size of that group was therefore reduced to 71.

TABLE 4.2: Number of students who wrote the Pre- and Post-tests

	No of students	
Group	**Pre-test**	**Post-test**
Experimental Group	140	119
Control group	132	71

The gender distribution was balanced in the experimental group (49.6% males and 49.6% females). The gender distribution was however skewed in favour of males in the control group (Table 4.3).

TABLE 4.3: Gender distribution of experimental group and the control group

	Pre-test	
Gender	**Experimental**	**Control**
Male	59 (49.6%)	50 (70.4%)
Female	59 (49.6%)	21 (29.6%)
Total	118	71
Not indicated	1	0

The results indicated that 50 (26.3%) of the respondents reported Grade 12 Mathematics marks below 50% (n = 190) and these students were therefore considered to be at academic risk of failing Mathematics. A further 75 (39.5%) indicated scores of between 50% and 59% and were considered to be in need of academic support. Therefore almost two thirds (65.8%) of the participants in this study (sample combined with control group) reported Grade 12 Mathematics marks which were deemed too low for unsupported university studies in Mathematics. Only 65 (34.2%) of the respondents reported a Grade 12 Mathematics mark of above 60% (Table 4.4).

TABLE 4.4: Grade 12 self-reported Mathematics scores of all participants

Score Bands	Frequency (n = 190)	Percentage	Cumulative %
0 - 29%	0	0	0
30 - 39%	5	2.6%	2.6%
40 - 49%	45	23.7%	26.3%
50 - 59%	75	39.5%	65.8%
60 - 69%	48	25.3%	91.1%
70 - 79%	9	4.7%	95.8%
80 - 100%	7	3.7%	99.5%
Missing/spoilt	1	0.5%	100%

4.2.3 The CCIT test scores

4.2.3.1 Normality of data

The normality of a data set needs to be tested as a prerequisite for using parametric tests. When the sample size however exceeds 30, parametric tests can be used with safety (Pallant, 2007), provided the sample data is reasonably symmetrically distributed about the mean and the mean is approximately equal to the median. Although the sample size did exceed 30, the normality of the data was tested, first by a visual test and hence with the aid of calculations. A histogram of the distribution of the test scores revealed a shape that approached the bell-shaped curve of the normal distribution (Figure 4.1). The histogram was reasonably symmetrical about the mean. A Chi-squared Goodness-of-fit-test was hence conducted in Excel to test the Null hypothesis that the data were normally distributed. The Null hypothesis was rejected (p = 0.05, df = 5, calculated $\chi^2 = 38.5$, critical value = 11.07). The distribution was therefore not normal. The distribution was positively skewed, with skewness of 0.44. The mode was 6, the median was 7 and the mean was 7.35. The rule of the thumb is that in right skewed distributions, mode < median < mean, which was the case here. The kurtosis was 0.23, compared to 3, the kurtosis of a univariate normal distribution. Acceptable limits for skewness and kurtosis are ±2 (Gravetter and Wallnau, 2014). Both the skewness and the kurtosis of this set of data was therefore not in the acceptable range for a normal distribution. Kurtosis is used to measure the peaked-ness versus flat-topped-ness of a curve (Kothari, 2004). Distributions with kurtosis less than 3 are said to be platykurtic, which means the distribution has fewer outliers and thinner tails than the normal distribution.

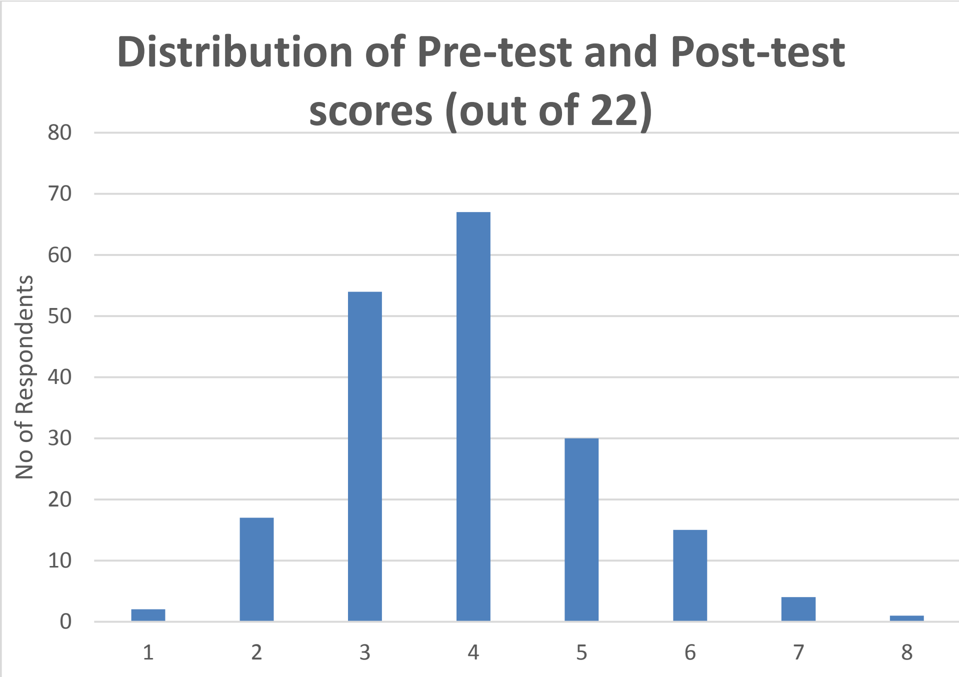

FIGURE 4.1: Distribution of scores in the combined Pre- and Post-test

4.2.3.2. F-test used to compare variance of Pre-test Experimental group to variance of the Pre-test Control group

The F-test was used to compare the variance of the Pre-test experimental group to the variance of Pre-test control group (Table 4.5). This comparison is required before the Student t-test can be used to compare groups. The results indicated that the variances of the two groups did not differ significantly (p = 0.01). The t-test for equal variances should therefore be used when testing the two independent groups.

TABLE 4.5: Results of F-test to compare variances of Pre-test Experimental group to Pre-test Control group

Two-Sample F-Test for Variances ($\alpha = 0.01$)		
	Control	**Experimental**
Mean	7.82	7.08
Variance	6.18	5.27
Observations	71.00	119.00
df	70.00	118.00
F	1.17	
P(F ≤ f) one-tail	0.22	
F Critical one-tail	1.63	

4.2.3.3 Pre-test mean scores of Experimental group compared to Pre-test mean score of Control group

The experimental group had a lower Pre-test mean score than the control group. This difference was significant (t = -2.09, df = 188, p = 0.05) as revealed by a t-test. This result was to be expected, since the access criteria for the ND in Analytical Chemistry (AC) were lower than those of the engineering diplomas. Almost 60% (71) of the 119 students from the experimental group were AC students.

4.2.3.4 Pre-test mean scores compared to Post-test mean scores

The Pre-test mean scores of the experimental group were compared to their Post-test mean scores by doing a two-sample paired t-test (Figure 4.8). The following hypotheses were tested at the 5% level of significance, where $\bar{X}_1$ was the Pre-test mean and $\bar{X}_2$ the Post-test mean:

H_0: $\bar{X}_2 - \bar{X}_1 = 0$

H_a: $\bar{X}_2 - \bar{X}_1 > 0$

The experimental group's mean improved from 7.08 in the Pre-test to 8.40 in the Post-test (Table 4.6). The Null hypothesis was rejected. The Post-test mean differed significantly from the Pre-test mean at a 5% level of significance.

TABLE 4.6: Results of a Two Sample Paired t-test conducted on the Experimental group (*t* = -16.38, df = 118, p = 0.05)

t-Test: Paired Two Sample for Mean		
	Pre-tests	**Post-tests**
Mean	7.08	8.40
Variance	5.27	7.99
Observations	119	119
Pearson Correlation	0.96	
Hypothesised Mean Difference	0	
Df	118	
t Stat	-16.38	
P(T ≤ t) one-tail	0	
t Critical one-tail	1.66	
P(T ≤ t) two-tail	0	
t Critical two-tail	1.98	

In comparison, the control group's mean score dropped from 7.82 in the Pre-test to 7.56 in the Post-test (Table 4.7). When comparing the Pre-test scores to the Post-test scores of the control group, the following hypotheses were tested, with $\bar{X}_1$ the Pre-test mean and $\bar{X}_2$ the Post-test mean.

H_0: $\bar{X}_2 - \bar{X}_1 = 0$

H_{a2}: $\bar{X}_2 - \bar{X}_1 \neq 0$

The Null hypothesis could not be rejected. The Pre-test mean was no different to the Post-test mean (p = 0.05). The intervention therefore had a positive effect on the experimental group, but the control group did not improve.

TABLE 4.7: Results of a Two Sample Paired t-test conducted on the Control group (t = 0.93, df = 70, p = 0.05)

t-Test: Paired Two-Sample for Mean		
	Pre-test	**Post-test**
Mean	7.82	7.56
Variance	6.18	5.42
Observations	71	71
Pearson Correlation	0.55	
Hypothesized Mean Difference	0	
Df	70	
t Stat	0.93	
P(T ≤ t) one-tail	0.18	
t Critical one-tail	1.67	
P(T ≤ t) two-tail	0.35	
t Critical two-tail	1.99	

The detailed results per diploma and experimental/control groups are depicted in Table 4.8. The group that scored the lowest mean in the Pre-test was the Analytical Chemistry group, which formed part of the experimental group. The means of all the control groups dropped from Pre-test to Post-test.

TABLE 4.8: Summary of Pre-test Post-test Means of all groups

Group	**Diploma**	**Pre-test (%)**	**Post-test (%)**
Experimental	Civil Engineering	37.22	46.46
	Electrical Engineering	33.14	41.13
	Analytical Chemistry	29.7	34.2
Control	Civil Engineering	36.14	34.51
	Electrical Engineering	35.28	33.94
	Mechanical Engineering	34.46	35.61

From Figure 4.2 it is clear that all the means of the experimental groups increased, whereas all the means of the control groups dropped from Pre-test to Post-test.

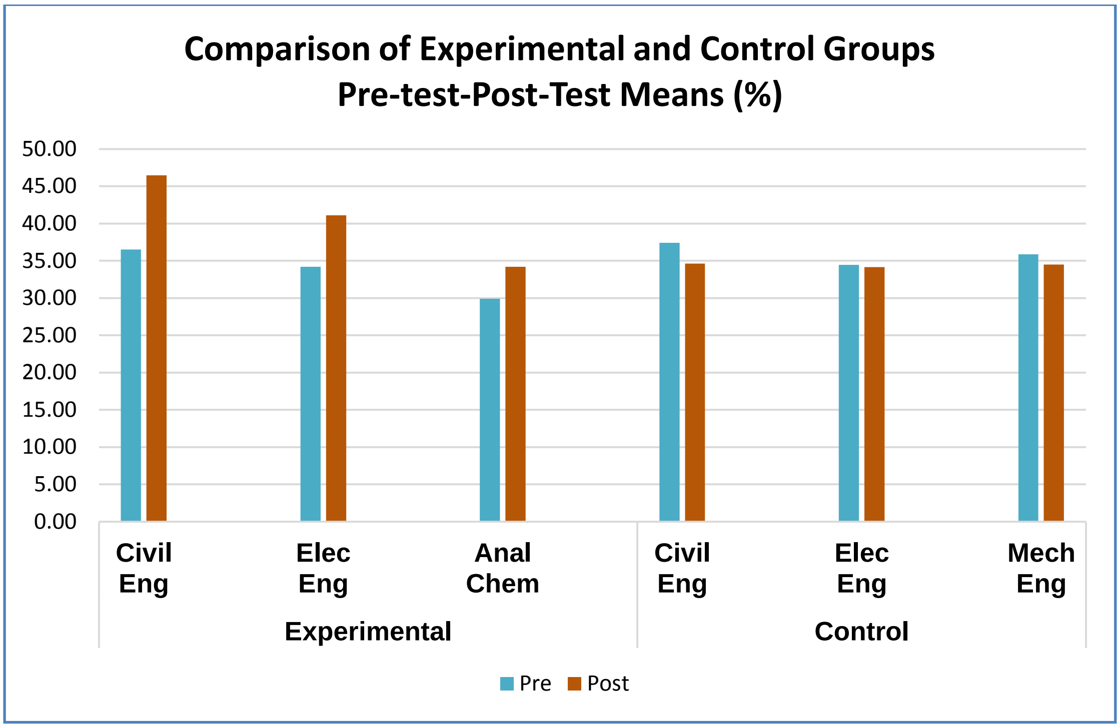

FIGURE 4.2: Comparison of Pre-test to Post-test means of Experimental versus Control groups.

Figure 4.3 depicts the shift to the right in the distribution between the Pre-test and the Post-test scores of the experimental group. The mode moved to the right and the number of low scores dropped.

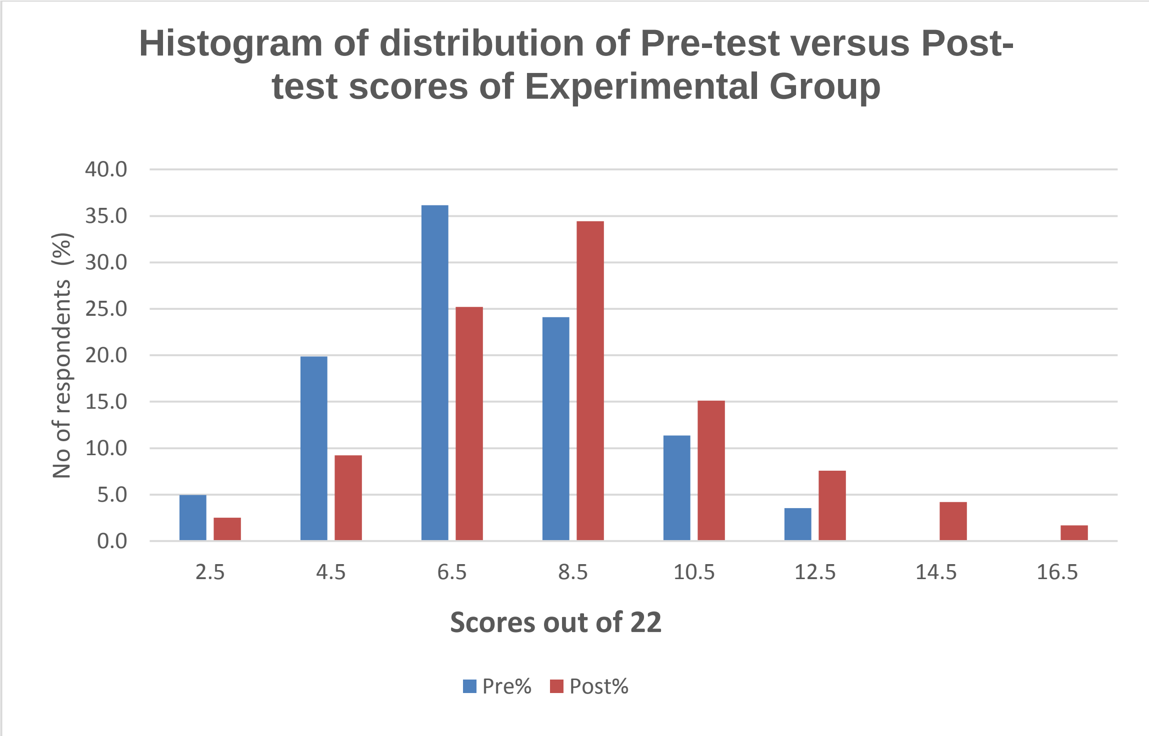

FIGURE 4.3: Histogram depicting distribution of Pre-test versus Post-test scores of the Experimental group.

A continuous graph of the distribution is useful to inspect the normality of the distribution and whether the curve tends towards a bell shape. The Histograms reveal approximate bell-shaped curves for the Pre- and Post-test scores of the experimental group. The Histograms are reasonably symmetrical about the mean. They also clearly depict the shift to the right in the scores of the experimental group (Figure 4.4).

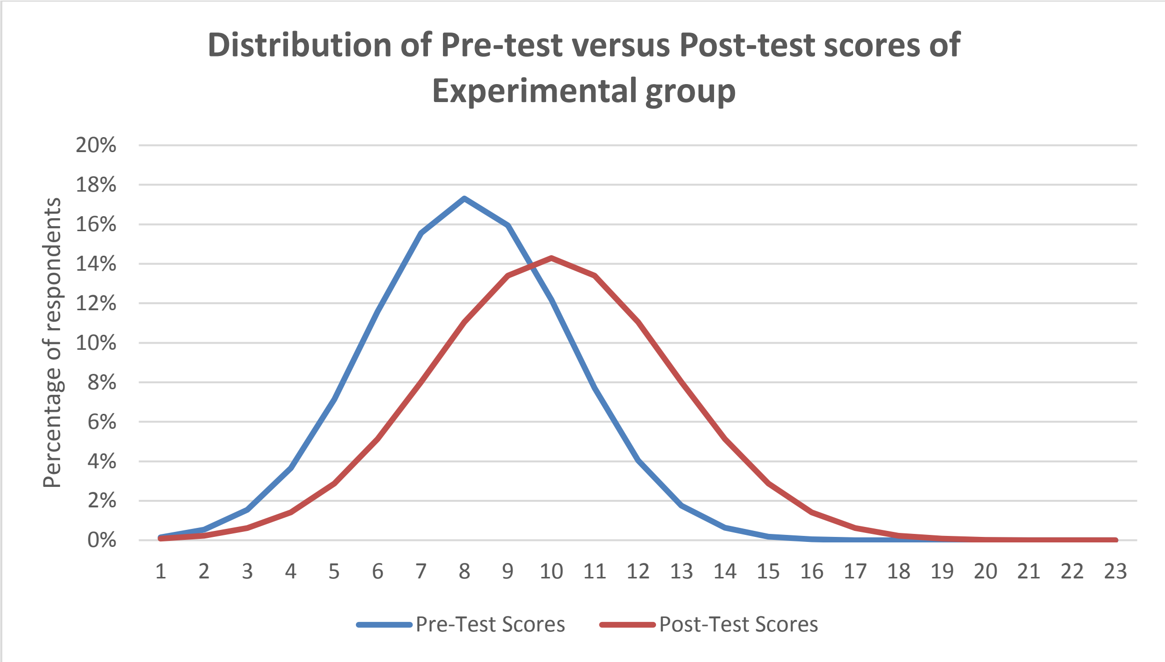

FIGURE 4.4: Bell-curves depicting approximate distribution of Pre-test versus Post-test scores of the Experimental group

Figure 4.5 depicts the absolute difference between the Pre-test and Post-test scores of both the experimental group and the control group in the form of straight lines. The slope of the straight lines indicate the size of the increase from Pre- to Post-test. The biggest absolute improvement was attained by the Civil Engineering experimental group (9.93%). The smallest improvement in the experimental group was attained by the Analytical Chemistry group (4.29%). AC had the lowest Pre-test mean score. It is however noteworthy that the mean of the AC group improved to a level where it almost equalled the weakest Pre-test mean of the Mechanical Engineering group. As already mentioned, the AC group comprised of almost 60% (71) of the 119 students from the experimental group. The lines representing the control groups have negative slopes, indicating a drop in the means from Pre-test to Post-test..

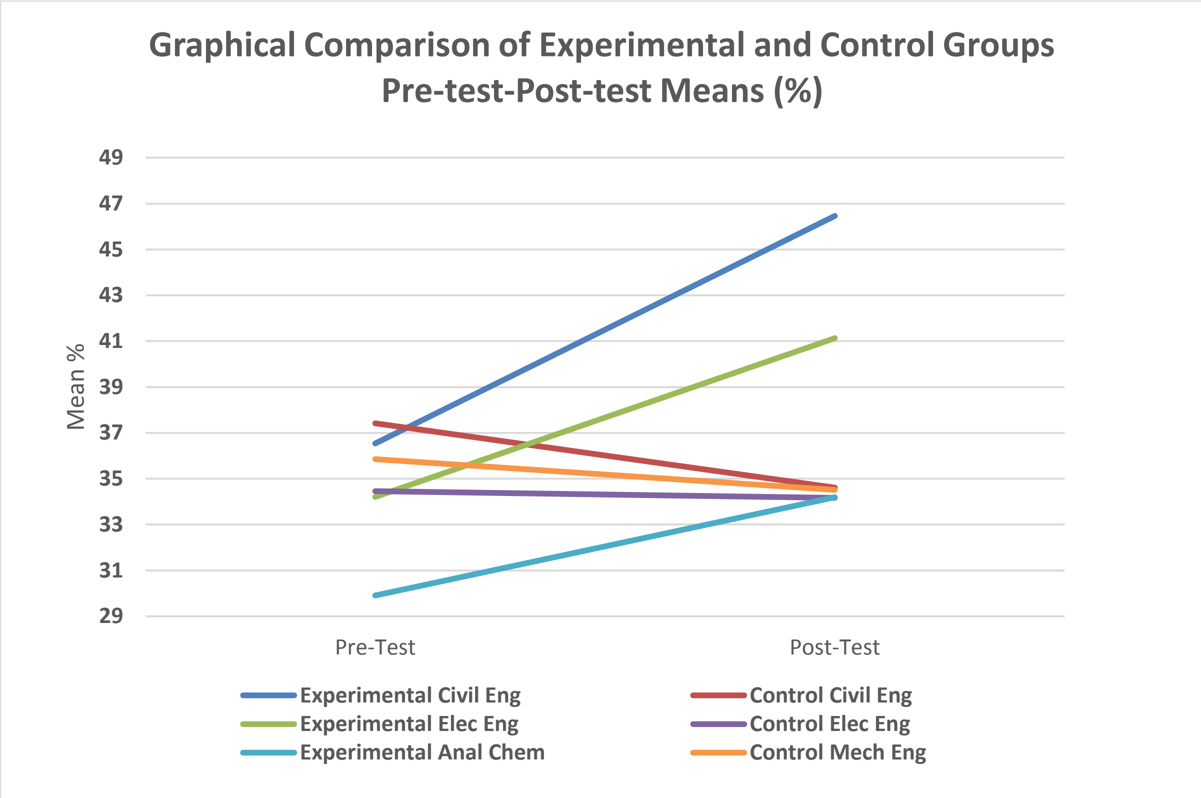

FIGURE 4.5: Mean scores (%) of Experimental groups versus Control groups from Pre-test to Post-test

4.2.3.5 Correlation of test scores to Grade 12 scores

The Pearson correlation between self-reported Grade 12 Mathematics scores and the Pre-test scores was low, at 0.28. The correlation between the self-reported Grade 12 Mathematics scores and the experimental group's post test scores was slightly higher, at 0.38.

4.2.3.6 The normalised gain

The average normalised gain G = <*g*>, was calculated as the ratio of the actual average gain to the maximum possible average gain. The class performance was measured by the normalised gain, defined to be:

$$< g > = \frac{\text{gain}}{\text{possible gain}} = \frac{\text{class mean posttest \% score} - \text{class mean pretest \% score}}{100 - \text{class mean pretest \% score}}$$

$$= \frac{\bar{X}_f - \bar{X}_0}{100 - \bar{X}_0}$$

The <> brackets around the g-value indicate a group score as opposed to an individual's score. The raw gain achieved by the experimental group was 6% and the normalised gain <g> achieved by the experimental group was 0.09. That means that in the Post-test, this group was able to correctly answer 9% of the material that they did not answer correctly in the Pre-test. The raw gain in terms of the standard deviation was 0.59 of a standard deviation. In contrast, the control group's mean score dropped. The gain was negative (-0.01), which represents -0.05 of a standard deviation.

A summary of the gains achieved by each class group is presented in Table 4.9. The *t*-test scores revealed that the difference between the Pre-test and Post-test gains for the experimental group was significant ($p = 0.01$), and that there was no significant gain for the control group (Figure 4.6). The gains achieved by the experimental groups were however in the low range and corresponded to the low use of IE (a maximum of 25% of class time). The biggest absolute improvement was achieved by the Civil Engineering experimental group (<g> = 0.16, g = 9.93%, 0.92 of a standard deviation).

TABLE 4.9: The Gain (g) and Normalised Gain (<g>) of Experimental versus Control Groups

		Pre-test Mean (%)	**Post-test Mean (%)**	**Raw Gain (%)**	**<g>**	**Std dev (out of 22)**	**Std dev (%)**	**Gain i.t.o. std dev**
Exp	Civil Eng	36.53	46.46	9.93	0.16	2.38	10.81	0.92
Control	Civil Eng	37.41	34.62	-2.80	-0.04	2.82	12.82	-0.22
Exp	Elec Eng	34.20	41.13	6.93	0.11	2.36	10.72	0.65
Control	Elec Eng	34.46	34.16	-0.29	0.00	2.52	11.46	-0.03
Exp	Anal Chem	29.90	34.19	4.29	0.06	2.13	9.68	0.44
Control	Mech Eng	35.86	34.51	-1.35	-0.02	2.20	10.01	-0.13

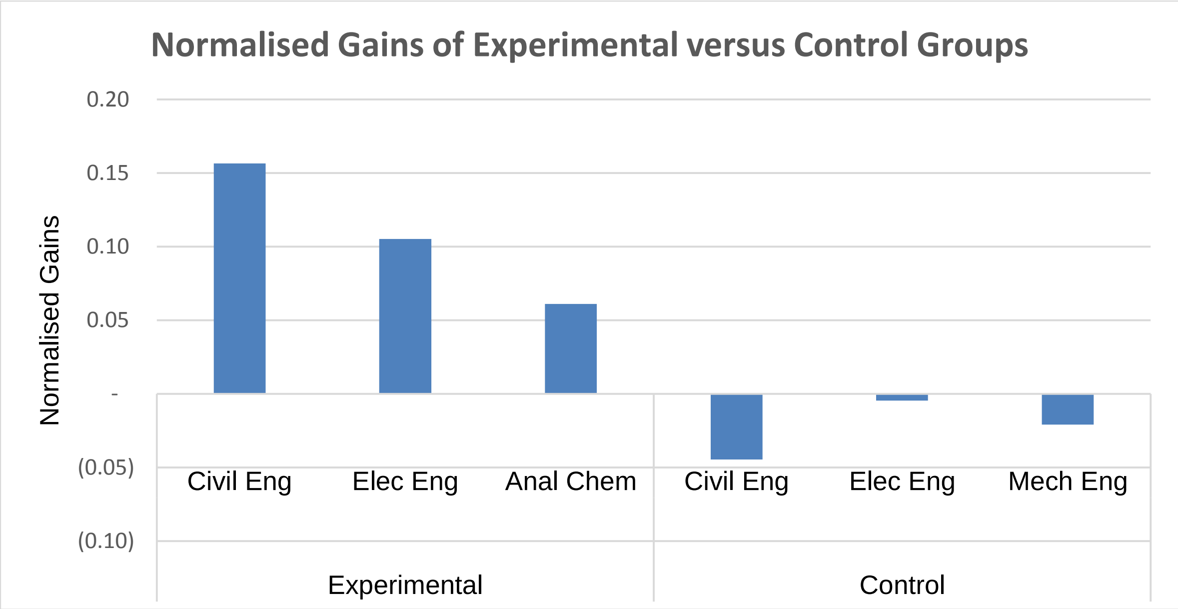

FIGURE 4.6 Normalised Gains of Experimental versus Control Groups

4.2.4 Validity and reliability

4.2.4.1. Validity

Criterion-Related Validity, and more specifically, Concurrent Validity, was examined by testing how the items were related to other variables. The researcher attempted to establish whether a relationship existed between the scores in the assessments and the scores on other concurrent variables such as question A4 – Students' self-reported score in Grade 12 Mathematics. Students who scored high in Grade 12 Mathematics should be getting higher scores in the assessments. A statistically significant relationship, at the 10% level of significance, was found between the students' self-reported Grade 12 Mathematics score, and nine of the questions: B6, B8, B9, B10, B11, B12, B13, B15.2 and B18 (Table 4.10).

TABLE 4.10: Results of Pearson Chi-Squared Test, calculated after cross-tabulating Question and Grade 12 Score

Pearson Chi-Squared	Value	Asymptotic Significance (2-sided)
B6	18.217	0.006
B8	14.634	0.023
B9	11.509	0.074
B10	35.797	0.000
B11	16.113	0.013
B12	29.184	0.000
B13	26.207	0.000
B15.2	11.512	0.074
B18	29.540	0.000

4.2.4.2 Reliability

The Cronbach Alpha score, which reflects the reliability of the test, was in the low range at 0.434 (Table 4.11) and the average inter-item correlation was also low (0.0271). Inter-item correlation gives an indication of the strength of the relationship between the items; the extent to which items on a scale are assessing the same content. Ideally, the average inter-item correlation for a set of items should be between 0.20 and 0.40, suggesting that items are reasonably homogenous yet contain sufficiently unique variance. When values are lower than 0.20, then the items may not be representative of the same content domain (Piedmont, 2014). The results point to a unitary test construct instead of a three-factor construct. B1, B10, B12 and B17 displayed correlation scores of above 0.35. The other items had low correlation.

TABLE 4.11: Cronbach Alpha of the CCIT

Reliability Statistics	
Cronbach's Alpha	No of Items
0.434	19

4.2.5 Item analysis

The difficulty of an item in a test (the P-value) is the proportion of respondents that answered an item correctly. It ranges from 0.0 to 1.0. High values indicate that the item is easy, while low values indicate that the item is difficult. Items 4 and 6 were the easiest in the CCIT, with Difficulty indices of above 50%, calculated from the Post-test experimental CCIT scores (Table 4.13). Items 14 and 15.1 had indices between 40% and 50%. Items 1 and 2 were the most difficult for this cohort of students, as they scored below 20% for these two questions. The average difficulty index was 30.8%, which falls in the difficult range (Table 4.12).

P-values above 0.90 are very easy items and should possibly be omitted. P-values below 0.20 are very difficult items and should be revised and/or identified as an area for re-instruction. If almost all of the students get the item wrong, there is either a problem with the item or students were not able to learn the concept. However, if an instructor is trying to determine the ratio of students who mastered a certain concept after an intervention, highly difficult items may be deemed necessary (UT, n.d.). The ideal P-value is slightly higher than midway between chance (1.00 divided by the number of choices) and a perfect score (1.00) for the item. For example, on a four-alternative, multiple-choice item, the random guessing level is 1.00/4 = 0.25; therefore, the optimal difficulty level is 0.25 + (1.00 - .25) / 2 = 0.62 (UT, n.d.). Almost 80% of the questions in the CCIT were therefore either difficult or very difficult for this cohort of students.

TABLE 4.12: Interpretation of the Difficulty Index

Range	Difficulty Level	f	% of Questions
Below 20	Very difficult	2	15.8%
20 to below 40	Difficult	13	63.1%
40 to below 60	Average	4	21.1%
60 to below 80	Easy	0	0.0%
80 and above	Very easy	0	0.0%

Source: PPUKM (2015)

The item discrimination index or Point Biserial Index reflects the correlation between the score on a particular item and the overall test score. PBI is a measure of how well an item distinguishes between those who have high test scores and those who are less skilful. Scores range from -1 to 1, where a positive score indicates that those who scored well on the test answered the particular item correctly (McGahee and Ball, 2009). The PBI for distractors should be negative. Scores near -1 indicate that the item is answered correctly by those who perform the worst on the overall test and incorrectly by those who perform the best on the overall test, an undesirable result (McGahee and Ball, 2009). Different approaches are taken when calculating the discrimination index. One approach is to select the top 50% of subjects and assign them to the high skill group, and the others to the low skill group. Another approach is to divide the group into three, a top group (27%), middle group (54%) and bottom group (27%) (Kelley, 1939), as cited by PPUKM (2015). Subjects whose scores fall in the bottom 27 to 33 percent are assigned to the bottom group. The discrimination index is calculated as the percentage of subjects in the high skilled group who answered the item correctly minus the percentage in the low skilled group who answered the item correctly (PPUKM, 2015).

Kuder-Richardson Formula 20, or KR-20, is a reliability measure for a test with dichotomous data. This is a measure of the likelihood of obtaining similar results if the test is re-administered to another group of similar students. Reliability refers to how consistent the test results are, or how well the test is actually measuring what it is intended to measure (PPUKM, 2015). The KR-20 is used for items that have varying difficulty. For example, some items might be very easy, others may be difficult. The scores for KR-20 range from 0 to 1, where 0 is no reliability and 1 is perfect reliability.

The closer the score is to 1, the more reliable the test. An acceptable KR-20 score depends on the type of test. In general, a score of above 0.5 is usually considered reasonable for high-stakes assessments (EAC, 2017). The score on this particular test was 0.48, which is close to 0.5.

TABLE 4.13: Item difficulty and item discrimination of the CCIT

Question	Difficulty Index (D)	Discrimination Index (R) or Point Biserial Correlation
1	7.6%	0.03
2	14.3%	0.09
3	38.1%	0.06
4	53.8%	0.06
5	37.0%	0.13
6	51.3%	0.09
7	24.4%	0.09
8	37.8%	0.13
9	26.1%	0.16
10	21.8%	0.13
11	22.0%	0.03
12	36.1%	0.19
13	21.2%	0.06
14	43.2%	0.16
15.1	47.9%	0.13
15.2	19.3%	0.13
16	21.2%	0.09
17	27.7%	0.13
18	33.6%	0.16

The PBI scores in the CCIT were low, and ranged from 0.03 to 0.19 (Question 12). PBI scores can be interpreted along the guidelines given in Table 4.14 (McGahee and Ball, 2009; PPUKM, 2015). When a test consists of mostly very difficult items as was the case for the CCIT, then the variability is low and the PBI's will also be low.

TABLE 4.14: Guidelines for interpretation of item discrimination scores

Score	Interpretation
0.09 to 0.19	Revise item
0.20 to 0.29	Fair
0.30 to 0.39	Good
0.4 and above	Very good

4.3 Summary of Chapter Four

This chapter presented the results obtained during the collection of the data for the research question. A summary of the biographical data was given, after which the CCIT test scores were presented. The normality of the data was investigated, first in visual form and then with a Chi-squared Goodness-of-fit test. The Pre-test scores were compared with the Post-test scores and it was shown that the experimental group improved from Pre-test to Post-test while no such improvement was detected for the control group. Correlation between self-reported Grade 12 Mathematics scores and CCIT was shown to be low. The raw gain and normalised gains were presented and discussed. Issues with validity and reliability were addressed. Item analysis was done in terms of Item Difficulty and Item Discrimination. In the next chapter, the findings will be discussed and interpreted. The limitations of the study will be explained, and recommendations for further research will be made.

CHAPTER FIVE

DISCUSSION OF FINDINGS, IMPLICATIONS, RECOMMENDATIONS AND CONCLUSIONS

5.1. Introduction

In this chapter, the findings of the study are discussed and interpreted. The limitations of the study are explained, and recommendations are made for further research.

5.2 Findings

5.2.1 The positive effect of RBAs on the conceptual understanding of calculus concepts

The main finding of this study was that RBAs in the teaching of calculus made a significant impact ($p < 0.01$) on the conceptual understanding of calculus of the experimental group as measured by a ConcepTest. The Post-test mean score of the experimental group was significantly higher than the Pre-test mean score. In contrast, the mean score of the control group did not improve from the Pre-test to the Post-test. The improvement in the mean score of the experimental group occurred despite an unequal start. The experimental group's Pre-test mean was significantly lower than the mean of the control group.

5.2.2. Conclusions from the review of the literature

Some responses to calculus reform have been negative, possibly even defeatist:

> I have many times seen the demands of a calculus course overwhelm the academic preparation and maturity of many enrollees. In this respect, at least, I see calculus reform as an attempt to solve a problem that may have no solution—calculus by its very nature may always be destined to have lower retention and completion rates than other, less challenging courses, regardless of how innovative or student-centered the reforms we introduce may be (Windham, 2008, p. 33).

However, according to Thomas (2014a), there does seem to be evidence that at least some of the measures of IE predict student gains. In the current study, IE was implemented at low levels (less than 25% of contact time). Despite these low levels of IE, the experimental group gained significantly as opposed to no gain by the control group.

5.2.3 Findings based on the empirical research

The average normalised gain *G* was used to gauge the effectiveness of an intervention designed to promoting conceptual understanding in calculus. The gain *G* is defined as the ratio of the actual average gain to the maximum possible average gain:

$$G = \frac{\text{gain}}{\text{possible gain}} = \frac{\text{mean posttest \% score} - \text{mean pretest \% score}}{100 - \text{mean pretest \% score}} = \frac{\bar{X}_f - \bar{X}_0}{100 - \bar{X}_0}$$

The *t*-test scores revealed that the gain for the experimental group was significant ($p < 0.01$), but that there was no significant gain for the control group. The gain achieved by the experimental group was however in the low range and corresponds to the low use of IE (a maximum of 25% of class time). The results are similar to the results reported by Mantini et al. (2014). They reported a course-wide net gain of 10.15 percent after two consecutive semesters. The gain in this study was 6.13 percent after only one semester of IE. Epstein (2013) reported gains in the range 0.08—0.23. The gains achieved by a study conducted at the University of Michigan in 2008, were however much higher:

(i) The average gain *G* over all fifty-one sections was 0.35.
(ii) Ten sections had a normalised gain of 0.40 to 0.44.
(iii) The range of the normalised gain scores was 0.21 to 0.44 (Epstein, 2013).

Their level of IE was however much higher. Interactivity levels were rated as 2.7 on a four-point scale where 1 represented not interactive and 4 represented very interactive. This translates to a score of almost 70%.

The biggest raw gain in the current study was achieved by the Civil Engineering group (9.93%), which corresponds to the lower scores in Epstein's studies on IE (Epstein, 2013). Rittle-Johnson et al. (2001) claim that the pace of conceptual learning may be too gradual for Pre-test and Post-test measures to be useful. This may be the case, especially since the interventions in this study lasted for only one semester.

5.2.3.1 Instructor and campus as potential causal factors

It is significant to note that one of the classes (Class EC) which formed part of the control group, was taught with Traditional methods by the lecturer who also conducted the intervention with the experimental group. All the other classes were taught by other lecturers. The decision to include class EC in the control group was taken during the planning phase to control the teacher-effect variable. The mean of Class EC dropped marginally from the Pre-test to the Post-test, resulting in a negative gain (Table 5.1). Since the average score of Class EC dropped from Pre-test to Post-test, the experimental group's improved scores should not be attributed to teacher-effects, since the instructor was shared by class EC and the experimental group. This step accounts for the possibility of the instructor as a potential causal factor in the outcome of the study. It also accounts for possible differences in instruction at the two campuses as a casual factor, since the EC group was located on the same campus as the experimental group at a different delivery site. The EC group was taught by the same instructor as the experimental group, and yet, similar to the other control groups, showed no gains on the Post-test.

TABLE 5.1 Pre-test and Post-test means of the class EC

Pre-test mean	Post-test mean	Gain
34.46%	34.16%	-0.29

5.2.4 Reliability and validity of Concept Inventories

Since the completion of this study, a number of articles have appeared in the press on Epstein's Calculus Concept Inventory. Some of these articles were based on a re-examination of the test and found some significant issues with the instrument. Thomas (2014a) found that one particular item was not following expected patterns. He was not able to pinpoint a reason for this behavior, but posited that there may be some reason the specific student population in his study was misinterpreting the question and suggested interviewing students in order to resolve the issue.

Another issue wih the CCI refers to the internal structure validity, which indicates how subscales may relate to each other. The items within the subscales should be highly correlated and items between the different subscales may also be correlated, though likely to a much lesser extent. The goal of a validity study regarding the internal structure of an instrument is to determine if the items are measuring distinct constructs or just a single underlying construct in order to justify the usage of subscores. Internal factor analysis conducted by Bagley, Gleason, Rice and Thomas (2016) on the CCI revealed a unitary factor, contrary to the three-factor model (functions, derivatives and limits/ratios/the continuum) that Epstein and colleagues suggested. The same results were found after analysing the CCIT used in this study. The results pointed to a unitary model. Comparable to the findings reached by Bagley et al. (2016) when analysing the CCI test, the item responses in the CCIT test explained only one factor, knowledge of calculus. Neither the CCI nor the CCIT therefore generates information about conceptual understanding of different components of calculus, but instead only measures overall calculus knowledge.

The reliability of the CCIT was calculated as 0.434. One guideline states 0.3 as at the threshold, 0.5 or higher as adequate, and 0.7 or higher as high (Griffee, 2012), cited by Hazzi and Maldaon (2015). Epstein's CCI was found to have a Cronbach alpha of around 0.7 (Epstein, 2007). Although the Cronbach of the CCI is higher than that of the CCIT, it may still be regarded as unsatisfactory for a high-stakes test. According to Bagley et al. (2016), an alpha of 0.8 is the minimum requirement for use in educational research. However, some researchers differ from them and argue that a low alpha is acceptable in low-stakes situations. During the development of a Function Concept Inventory, O'Shea et al. (2016) reported a Cronbach alpha coefficient of 0.531. Even after removing some items from the test the Cronbach alpha coefficient remained of similar magnitude. The researchers argued that the concept inventory was therefore not suitable for high-stakes assessments but had discrimination value. O'Shea et al. (2016) quoted studies such as the one by Adams and Wieman (2011), who contended that low values of Cronbach's alpha on a concept inventory were acceptable considering the restrictions on the size of such a test and the possible uses of the resulting data. They further argue that high values of such an index could indicate redundant questions. Jarrett et al. (2012) quoted Miller (1995), who said that for instruments such as concept inventories, tests of internal consistency such as the person reliability index or the Cronbach alpha coefficients could under-estimate reliability and that the actual reliability could be higher:

> Miller described three methods for estimating reliability: test-retest, alternative-forms and internal-consistency. The author pointed out that the less homogenous a test is, the lower the estimate of reliability an internal-consistency measure will give. Concept inventories typically measure understanding of a number of concepts so a good concept inventory could not be expected to be homogenous, as good understanding of one of the concepts may not imply good understanding of others. Therefore measures of internal consistency can 'badly under-estimate reliability' (Miller, 1995; p. 270) of non-homogenous tests such as concept inventories (Jarrett, Ferry and Takacs, 2012, p. 5).

Whilst developing The Calculus Concept Readiness (CCR) Instrument, Carlson et al. (2010a) also encountered a relative low reliability alpha of 0.54 for their test, implying that the test would not be effective as a placement test. They ascribed the lower reliability to extraneous factors such as low motivation. According to Finn (2015), unmotivated students and their subsequent low-effort behaviour can substantially impact low-stakes testing scores as well as the validity of interpretations that are based on these scores. Reduced performance introduces construct-irrelevant variance to the resulting scores.

Various studies are quoted by Finn to support the claims that low motivation and low effort substantially impact performance. Wise and DeMars (2005) reviewed 12 studies that experimentally manipulated test-taker motivation to examine the validity of test scores under low-motivation test scenarios. The researchers found that students who were motivated outperformed unmotivated students, with differences up to as large as 0.59 standard deviations. In another study, a meta-analysis of more than 50,000 test scores from 200 schools found that students who reported having put forth at least reasonable effort scored between 0.25 and 1.5 standard deviations higher than students who reported expending no effort (Schiel, 1996), as cited by Finn (2015). Yet another study cited by Finn (2015), was the one by Liu et al. (2014). They agreed with the previous finding that learning gains may be more attributable to students' lack of motivation than to their lack of learning. Their study indicated that self-reported motivation showed a significant relationship with scores, even when controlling for college admission scores and placement test scores. Finn hence refers to the work done by Wise and colleagues (Wise and DeMars, 2005; Wise and DeMars, 2006; Wise and Smith, 2016) when he claims that low-motivation behaviours like rapid guessing distort the psychometric properties of test scores and result in reduced convergent validity and erroneous increases in the internal consistency of the scores. Although the current study did not include a question on self-reported motivation and effort, the researcher observed that the AC students, a substantial part of the experimental group, rushed through the Post-test in a remarkably short time period. They had all left the venue after 20 minutes, which is not enough time to complete a difficult ConcepTest effectively. When confronted, participants reported that they experienced academic pressures and had to rush to prepare for a practical examination in another subject. These pressures had a negative influence on their test motivation and affected the level of effort expended on the test.

Finn (2015) mentioned expectation as yet another factor that may influence students' motivation. Expectancy theory predicts that a discrepancy between individual ability and test difficulty (when the test is harder or easier than the student expected it to be), may impact test-takers' motivation. The difficulty of test items similarly influences test-taker motivation (Wise and Smith, 2016). Performance differences between lower and higher motivation students become apparent on the most difficult items (Wolf, Smith and Birnbaum, 1995). Also, Wolf, Smith, and Birnbaum (1995) found that students without a motivational factor of consequence or benefit would not exert maximum effort on items that were more than moderately difficult. Thus, to overcome low motivation, Finn (2015) advised that tests should have consequences, because low-stakes tests without consequences result in low motivation and reduced effort. In the current study, only 40 clickers were available for use in instruction. Some class sizes were more than 80. It was therefore difficult to allocate individual marks to answers, although group scores could have been recorded. Also, assessments had no consequences, and results may have been affected by low motivation. No participation points were awarded and tests scores did not contribute to the promotion scores. Furthermore, the Post-test may have been affected by negative expectations, since the students had already written the Pre-test and were therefore familiar with the high level of difficulty of the test. This factor may have negatively influenced expectations.

A way to monitor motivation levels, is to employ global self-report measures, such as inserting a question at the end of the test to probe motivation levels. Wolf et al. (1995) found that students' ratings on a global motivation questionnaire taken at the end of a standardised testing session correlated with their test scores. These self-report measures however also cause concerns. It is not always easy to adjudge the value of the answers, which may comprise response biases. Also, low effort during test taking may be matched by equally low effort expended for the self-evaluation (Finn, 2015). In my opinion, all possible measures should be put in place to overcome the problems mentioned above. Tests should have consequences, even though these may be limited. Also, self-report questions should be included, although these may have some intrinsic limitations.

5.3 Implications

The instructor in this study did not find it easy to incorporate mathematics equations in the Turning Point software. Preparation of lectures was found to be time-consuming and at times frustrating. Valuable class time had to be used to distribute the clickers at the start and end of the lectures. The researcher found that RBAs in calculus teaching required more effort from the instructor than the traditional lecture format. According to Knill (2009), teaching with technology is challenging and always poses a risk. These issues need to be considered when introducing RBAs to a curriculum. Social culture, as well as the learning and teaching culture of a specific group or institution, strongly influence learning and teaching (Leung, 2005; Shin, Lee and Kim, 2009). It implies new methodologies cannot necessarily be introduced by an institution without careful planning and without the necessary support structures in place. Because of the ease of teaching with traditional approaches, people may prefer to continue teaching in traditional ways, even if these methodologies are outdated.

When introducing RBAs, one has to consider a couple of factors. Learning and teaching culture may be resistant to change and proposals for improvement in the learning and teaching of calculus will have to take cognisance of this reality. The personality, qualifications and experience of the lecturer as well as the culture of the university and its faculties and departments may additionally influence teaching practice. One of the mathematics lecturers interviewed by Windham in his study, laments that the innovative elements of a reformed calculus curriculum are difficult to implement whilst simultaneously having to cover the prescribed material mandated by the curricula (Windham, 2008). Inexperience with RBAs and lack of support may complicate such difficulties. Finding a balance between the needs of the instructor, the available resources and the needs of the student population will continue to be an issue that colleges and universities must face (Judd and Crites, 2013).

It was mentioned earlier that Hake (1998) has alluded to the possibility that IE teaching may be more beneficial for some students than for others. In this study, high-performing students may have gained more from RBAs than low-performing students. The instructor was informally told that six of the top AC students from the experimental group had decided to change careers in favour of a career in mathematics. To the best of our knowledge, this has never happened before, and may be attributed to the successful application of RBAs in the learning and teaching of Mathematics.

5.4 Summary of the study

This study tested whether RBAs in the teaching of calculus, utilising IE, PI and clickers, would lead to higher levels of conceptual understanding of calculus as measured by a ConcepTest. RBAs in the teaching of calculus have been developed to counteract the problems that are generally seen as the result of traditional teaching. High failure rates in first-year university mathematics have been reported internationally, also in South Africa. First year mathematics courses, containing mostly calculus topics, act as a gateway to various programmes at universities. Failure in calculus, and therefore in first year mathematics courses, influences throughputs, and retention and graduation rates (Engelbrecht and Harding, 2015). Researchers ascribe these problems to the under-preparedness of entry-level students combined with outdated pedagogies used by lecturers (Judd and Crites, 2014).

Reformers support active, hands-on, student-centred approaches to learning and teaching, utilising computers and graphical methods in order to facilitate understanding and connection of complex mathematical concepts. In this study, the RBAs selected for the intervention were AL, IE, GQ, PD, and ARS, also referred to as clickers. The potential advantages of AL is evident from research, such as improved short-term and long-term recall of information, increased conceptual understanding, improved attendance of lectures, higher retention in academic programmes, enhanced critical thinking skills, interpersonal relationships and self-esteem, fewer misconceptions and improved teamwork skills (Drake and Battaglia, 2014). The role of GQ was to kindle perturbations. PD was encouraged to provide students opportunities to verbalise their thoughts, order their thought processes, explain their solution methods and defend their point of view; to collaborate with their classmates and to subsequently construct knowledge. Regular formative assessment was made possible by the use of ARS and clickers. Evidence point to the positive effect that formative assessment has on learning, and mention was made that formative assessment was especially effective for low-achieving mathematics students (Pengfei, 2007; Dibbs and Oehrtman, 2014). Formative assessment was therefore appropriate for this study, since the cohort encompassed a proportion of students who started off in the extended stream.

The theoretical framework for this study was based on the constructivist theories of Piaget, Von Glasersfeld and Vygotsky. Constructivism took hold in mathematics education because it addressed the major concerns of mathematics educators (Confrey and Kazak, 2006), such as students' limited conceptual understanding and mathematics instruction that was emotionally intimidating and alienating (Confrey and Kazak, 2006).

This study followed a quasi-experimental (non-equivalent groups) design using Pre-Post testing. The target population for this study comprised students studying mathematics towards a ND in Engineering or ND in Analytical Chemistry at a comprehensive university in the Eastern Cape. More specifically, the population was the mainstream Science and Engineering students registered for the service course M2. The instrument, the CCIT, was adjusted from the CCI, an internationally standardised instrument, to make it more suitable for the purposes of this study. It comprised 19 questions on functions, differentiation and integration. The CCIT was subjected to scrutiny by experts in the field. After a pilot study, the instrument was improved upon.

The intervention was tested on selected groups of Calculus II students from the university using the CCIT instrument. The experimental groups were lectured using RBAs in 25% of the allocated contact time, and direct instruction in 75% of the time.

The control group was lectured using traditional approaches. No participation points were awarded and tests scores did not contribute to the promotion scores.

The main finding of this study was that RBAs in the teaching of calculus made a significant impact ($p < 0.01$) on the conceptual understanding of calculus of the experimental group compared to the control group. The experimental group scored significantly higher in the Post-test compared to the Pre-test ($p < 0.01$) and also scored significantly higher in the Post-test than the control group. These results were achieved despite of the experimental group's significantly lower scores in the Pre-test. In contrast, the mean score of the control group did not improve from Pre-test to Post-test ($p < 0.01$). The gain achieved by the experimental group was in a low range and corresponded to the low use of IE (25% of contact time).

5.5. Limitations of the study

In line with research norms, the researcher identified what may be considered as limitations to this study. The instructor was inexperienced with ConcepTests, which may have influenced certain aspects of the procedures followed, such as not having any consequences attached to the test results. The instructor was also inexperienced with AL and the use of clickers and the related software, and lacked TPACK. It is therefore possible that teaching opportunities were not always utilised to the full. The equipment did not always cooperate and hence the teaching did not always continue smoothly. Only 40 clickers were available and students had to share clickers. Clickers were distributed at the start of each IE lecture and collected again at the end of the lecture. This was time-consuming. The laptop, data projector and clickers had to be carried to the venue together with lecture notes and lecturing tools required in class. No local on-campus support was available regarding the use of the clicker hardware or software.

The instrument used in the study was found to be unsuitable for this cohort of students. The Cronbach alpha of the CCIT was 0.434 and the KR-20, a reliability measure for a dichotomous data, was 0.48. The researcher is of the opinion that the test was pitched at a level too high for this particular cohort of students. The students struggled with the level of difficulty of the test. A number of contributing factors have been identified. The first is that this cohort included a substantial number of students whose Grade 12 results were too low for direct access into their programme of choice. These students were placed into the extended stream when they first applied for access to the university. The extended stream is allowed extra time to complete the courses. Typically, they take a year to complete courses that the mainstream students do in a semester. After the successful completion of the extended first year of studies, these students are re-introduced into the mainstream. At the start of this research project, they were therefore regarded as mainstream students, but had an extended stream background. Hake (1998) has alluded to the possibility that students' backgrounds and prior knowledge may influence their performance on IE interventions. I would like to add that students' backgrounds and especially inefficient prior knowledge may influence their performance on ConcepTests negatively. It was mentioned elsewhere that the insufficient development of the concept of fractions in the lower grades may effect students' conceptual understanding at university level (Norton and Hackenberg, 2010; Bressoud, 2016; Coetzee and Mammen, 2016). Also, South African learners' performance on international studies have proved to be poor (Reddy, Zuze, Visser, Winnaar, Juan, Prinsloo, Arends and Rogers, 2011) and start early in their school careers. In assessments conducted from 1998 to 2002 by the JET, poor results were obtained on Grades 5, 6, 7, 9 and 11 (Bernstein et al., 2004) and the tests had to be simplified because learners could not cope with the degree of difficulty of the original tests. The poor mathematical background of the South African students, combined with lack in prior knowledge, may influence university students' performance on ConcepTests. Mathematical difficulties carried from previous years are not necessarily resolved before students enter university (Coetzee and Mammen, 2016).

Another contributing factor was that the assessments had no consequences, and this led to low motivation levels and low levels of effort exerted on the assessments. Low motivation levels and low effort usually lead to random selection of multiple choice answers, without the necessary reflection. Caldwell (2007) advises the inclusion of an option "I don't know" as an answer choice to prevent guessing.

All these factors may act as limitations when the researcher attempts to generalise findings. Also, the sample chosen was not random. Outcomes from this study will not necessarily be applicable to all universities in South Africa where diploma studies are offered.

5.6. Recommendations

Thomas (2014a) advises that instructors should take cognisance of their students' background and current level of understanding in order to tailor the instruction to their specific students. He asserts that prior mathematics knowledge may be a significant variable in IE research and may even influence whether IE instruction is effective for a specific student population. The background of the students may determine how effective IE instruction will be for that particular group of students. Peterson further ponders the possibility that the effects of IE on students' understanding may not be immediately clear:

> It may take a longer amount of time than one semester to see the effects of IE instruction, and the potential effect it has on student understanding. By conducting a longitudinal study over multiple years, it may be possible to find additional effects of IE instruction. These effects might take the form of greater conceptual understanding which occurs later, or is retained for a longer time. Additionally, other variables such as attitudes towards mathematics, conceptual understanding in other sciences, or persistence in STEM fields could be studied (Peterson, 2012).

The RBAs used in this study were effective and valuable. However, their deployment was laborious and strenuous. The implementation of AL techniques involved a substantial learning curve for students and teachers alike, and care has to be taken when introducing these. Concerns include increased preparation time, and difficulties with time management and coverage of course contents within the allocated time (CBMS, 2016). Resources to support instructors will be required for wider implementation of these approaches.

I recommend the combination of RBAs with Traditional (direct) teaching. Brophy contends that direct instruction works best when it is used for teaching canonical knowledge (initial instruction establishing a knowledge base) and recommends social constructivist techniques for constructing knowledge networks (Brophy, 2006 p. 534).

I would further like to caution against rapid changes. Braun et al. (2017) agree and assert that the implementation of new pedagogies is a long-term process, and that new instructional techniques cannot be effectively implemented rapidly.

5.6.1 Recommended framework

A recommended framework is depicted in Figure 5.1. Two role players are involved in this scenario, the students and the facilitator. The facilitator is depicted in a smaller shape than the other role players, since contrary to traditional teaching approaches, the facilitator is not at the centre of the interaction. The facilitator's responsibilities include the selection and preparation of the material for the interaction and to initiate and facilitate the formative assessment sessions. The material includes GQ, and is selected based on its potential to stimulate perturbations and discussions, and to address and resolve misconceptions. The facilitator has to provide scaffolding during the resolution phase. The facilitator also has to analyse the results of the session and hence the iterative process is repeated – new material has to be selected based on the outcomes of the previous session. Using technology to teach is never easy, and the facilitator needs TPACK to do so effectively.

The student's role is to be an active participant in this process. Students have to verbalise their thoughts, discuss potential solutions with their peers and be prepared to defend their point of view. The student also has to take responsibility for the learning process, and has to be an active participant in the resolution phase when the instructor undertakes to resolve misconceptions.

The framework depicts the facilitator interacting with the students via the ARS. Although traditional teaching can function without technology, the latter creates more teaching and learning opportunities and its effective employment has a positive effect on conceptual understanding. The facilitator requires TPACK for the interaction with the technology, and also requires PCK to use formative assessment effectively to be able to positively impact students' conceptual understanding. The students experience calculus in a novel way by interacting with calculus concepts and with each other through the ARS.

Formative assessment provides evidence about learning, in order to adapt the teaching and to facilitate improved learning. It is a continuous, iterative process that informs both teacher and learners (Stull et al., 2011). IE combined with GQ and clickers allow continuous, almost effortless formative assessment, coupled with PD and immediate feedback. The combination of IE with PI afford the role players the ultimate formative assessment environment, as assessments are done continuously.

The immediacy and ease with which feedback is provided by ARS, make clickers the ideal formative assessment tool.

PD or “pair/share” is a learning opportunity during which small groups discuss the questions and defend their point of view. Peer interaction provides students the opportunity to collaborate with their classmates, explain their solution methods, and construct knowledge.

Conceptual understanding is depicted as a subsection of the Calculus shape, since it is not the only important part. Procedural ability is also vital and the two facets are interlinked and both are positively affected by RBAs. The output of this model is improved levels of conceptual understanding of calculus.

The researcher recommends the framework depicted in Figure 5.1 for the teaching and learning of calculus, using RBAs.

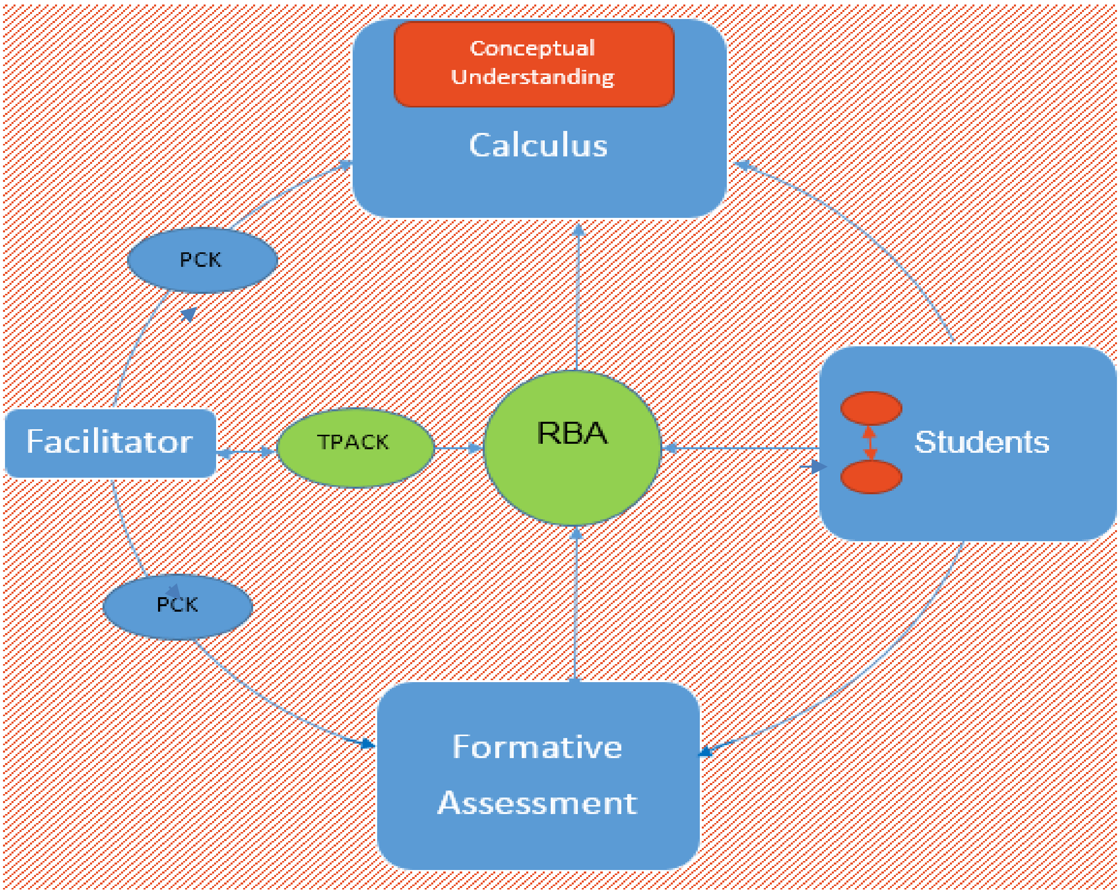

FIGURE 5.1: CONCAL: A Proposed Reform-Based Framework for CONceptual understanding of CALculus. Source: Researcher

5.7 Recommendations for further studies

Hake (1998) alluded to the possibility that IE teaching may be more effective for some students than others. He hinted that it would be useful to investigate individual student characteristics such as educational backgrounds, student attitudes towards the course, and prior knowledge. Further studies should take cognisance of this fact.

Also, ConcepTests are generally regarded as being of a higher degree of difficulty than the tests that students are familiar with, and the inherent nature of these tests may demotivate participants. I have concluded that the CCIT test used in this study was not pitched at the correct level, and was not altogether suitable for the students studying at this particular university. A more suitable test is therefore required for diploma students studying calculus in South Africa. Interventions using Pre-Post testing should be more extensive than one semester and last for at least a year, since changes in the levels of conceptual understanding occur gradually.

In retrospect and considering the latest research, I have come to the conclusion that an approach where attendance and performance points or marks are allocated to the Pre-tests and Post-tests, may yield more valuable research results. Students may be more motivated to exert effort when participation is graded. This point of view is supported in a study done by Henderson (2002), who found signs of lack of seriousness in answer patterns on the FCI. Cognisance should be taken of ethical considerations, however, since participation in a research study constructed along these lines will have to be compulsory, and not optional.

ConcepTests for all levels of Mathematics are currently being developed internationally by teams of researchers, such as the FCI developed by O'Shea et al. (2016) and these may assist other developers in their efforts. Bagley et al. (2016) argue for the creation of an item bank which should contain rigorously-developed and validated questions. The bank should be accessible by instructors for assessment purposes and by researchers to measure conceptual understanding gains associated with various instructional techniques. Such an item bank has the potential to significantly impact teaching and learning and also research on conceptual understanding in mathematics.

5.8 Contribution to new knowledge in the field of Mathematics Education

This study examined the effects of RBAs in teaching on the conceptual understanding of calculus in Science and Engineering diploma studies at a South African university. The finding was that RBAs in the teaching of calculus had a positive effect on the conceptual understanding of calculus. The gains were small, and corresponded to the low levels of IE employed in the study (25%).The results of this study contribute to the knowledge base of best practices that are associated with RBAs. Confirming results from other studies as cited by Kueffer and Latterell (2001), these results suggest that the traditional lecture format found in most university and college classrooms may not be the most effective method of instruction, but that care has to be taken when introducing RBAs in teaching calculus.

5.9. Conclusion

I have found clickers to be an effective instructional tool which enhanced a positive class atmosphere, AL, participation, and conceptual understanding. IE methods combined with PD were found to be an effective tool in raising levels of conceptual understanding of calculus. It is hoped that these results will prompt other STEM subject instructors to consider introducing RBAs in their teaching.

5.10 Summary of Chapter Five

In this chapter, the findings of the study were discussed and interpreted. A Reform-Based model called CONCAL, based on IE, PI, clickers and formative assessment was recommended. The implications of the study were discussed, the limitations of the study were explained, and recommendations were made for further research.

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LIST OF APPENDICES

Appendix A: CCI Confidentiality agreement

BASIC SKILLS DIAGNOSTIC TEST (BSDT) and CALCULUS CONCEPT INVENTORY (CCI) — Security Requirements.

Below should be the letter on the required non-disclosure agreement. I must have your specific agreement to all of the terms in order to send you the test. The easiest way is to paste the below agreement into an email, and follow the document. Please state which tests you are requesting, and that you agree that all non-disclosure terms apply to both tests. Usually the easiest way is to paste the below terms into an email, fill in your professional info, and provide a digital "signature". Obviously I have no ability to get the police after violators and I am dependent on the good will of people requesting the test. Since there is no monetary value here, I think this should not be a big deal. It is just to be sure everyone understands the stakes are huge, but not in money. I need you to provide the following information:

Name: J.(Hanlie) Coetzee

Institution: the university selected for the study.

Title: Mrs

Phone:+27(0)43-7271333_or +27(0)84-3944459

Email: kobus@pwrdata.co.za

I will use the test(s) in the following courses or programs:

S1, S2 and S3 (First, second and third semester) Engineering Mathematics for the National Diploma in Mechanical, Electrical and Civil Engineering at the university selected for the study.

Security is a major concern, thus you must agree to the following:

I, Johanna (Hanlie) Coetzee agree to and will be bound by all terms of this agreement.

I understand that the future utility of these tests, and the validity of the entire project is dependent on maintaining this security. I will contact Dr Jerome Epstein with any issues on my specific situation.

As the tests are likely to be published, and just for continued utility, security is a major concern.

I understand that I will be sent the test(s) subject to the following conditions:

I guarantee that:

- I will make only such copies as are needed to administer the tests to my students, I will retain all such copies, and return all copies and originals to Jerome Epstein, and delete all instances of the test items from all computers, if requested,
- I will not distribute any part of the test to any person or organization in any form whatsoever, whether verbally, in writing, or in electronic form or any other form, without express written consent from J. Epstein. I agree that I may show the test to colleagues I trust, but I may not transfer the tests to colleagues in any form. Refer any inquiries to Epstein
- I understand that unauthorized distribution of any portion of the test(s) represents possible financial or professional loss to Epstein and to the project, and that I am responsible if such unauthorized distribution results from my action or non-action. Many years of work and analysis (and NSF money) are involved in these instruments.
- Neither the tests as a whole, nor any portion of them, may be posted on the web, whether for internal access only or for public viewing.. Test administration must be via pen (or pencil) and paper, and students must not take away the document in any way that permits copying. I may post comments on the test or its results, I may post Epstein's email address. But the test items themselves must not be publicly accessible or copyable in any form.
- Administration of the tests via the web can present particular security problems. I agree to discuss with Epstein directly any plans to administer the test in that way, and I agree not to do so until I have received from Epstein explicit approval. .

I fully accept and will be bound by all of the above terms:

J. Caetzu.

____

With your agreement to the above, you are licensed to use the tests without charge until such time as either test is published. At any such time, all permission herein granted to use the test is void.

Please respond clearly indicating your agreement to all of the above.

Now that all that legal stuff is out of the way, I can proceed in a more friendly tone. . . With your agreement as above, I will send you the test(s) as a .pdf (for the CCI) or WORD document (for the BSDT), which you should have no problem printing and copying, as needed. The CCI is intended to be used as a Pre-test/Post-test pair for first semester calculus. There are no integrals in it. It is a "Concept Inventory", modeled on the Force Concept Inventory in physics. The BSDT has a similar purpose. So far the BSDT is not multiple choice. It is best to use it that way, you will learn the most. However, we are working on a multiple choice form.

Now that all that legal stuff is out of the way, I can proceed in a more friendly tone. . . With your agreement as above, I will send you the test(s) as a .pdf (for the CCI) or WORD document (for the BSDT), which you should have no problem printing and copying, as needed. The CCI is intended to be used as a Pre-test/Post-test pair for first semester calculus. There are no integrals in it. It is a "Concept Inventory", modeled on the Force Concept Inventory in physics. The BSDT has a similar purpose. So far the BSDT is not multiple choice. It is best to use it that way, you will learn the most. However, we are working on a multiple choice form.

To be useful in the ongoing data collection and analysis, The BSDT must be used in this way (not multiple choice). Please write for more details. . . The BSDT is entirely appropriate as a pair for a course in which such Basic Skills are to be developed. I have used it that way many times. As have now a great many others. The CCI is in no way a *comprehensive* test for calculus and should not be used alone as a final exam. Each test has been given to date to more than 7,000 students, as far as I know, at perhaps 250 schools, including about 50 other countries. Many users have put the results into an Excel spreadsheet which can readily do much worthwhile analysis. For the BSDT, there is also a sheet on when to assign half credit if you want a comparison with other schools to be as valid as possible.

Jerome Epstein

Polytechnic University

jerepst@att.net, 718-429-3437

Appendix B: The CCIT

The Calculus Concept Inventory for Technicians

This test is designed to help your instructor evaluate your needs and those of your fellow students. The test is completed anonymously. Respondents will not be identified. Participation in this research project is voluntary, and you are free to withdraw at any point without explanation or any negative consequences. This test will not affect your marks in any course. However, it is important that you take it seriously and do the very best you can on it.

University of Fort Hare
Together in Excellence

Section A: Please put a mark in the appropriate block:

1.1 Name of diploma you are registered for

ND Electrical Engineering	ND Mechanical Engineering	ND Civil Engineering	ND Analytical Chemistry

1.2 Mainstream/Extended S1 or S2

	Mainstream	Extended
S1		
S2		

2. Gender:

Male	Female

3. Age (completed years):

15-19	20-24	25-29	30-34	35-39	40-

4. What was your score for Mathematics in Grade 12?

Code 1: 0-29%	Code 2: 30-39%	Code 3: 40-49%	Code 4: 50-59%	Code 5: 60-69%	Code 6: 70-79%	Code 7: 80-100%

5. Please indicate to what extent you agree with the following statements:

	Strongly disagree	Disagree	Neutral	Agree	Strongly Agree
5.1 A good pass in Mathematics is important for my future					
5.2 I am satisfied with my Grade 12 Mathematics results					

Section B. **Time: 90 minutes**

Select the correct answer from the given list, and indicate your choice on the answer sheet. In some cases, more than one answer may be correct. In such a case, select the answer that is best in your opinion.

1. Which of the following could be antiderivatives (integrals) of $6x^2$?
   a) $2x^3$
   b) $2x^3 + 7$
   c) $2x^3 - 5$
   d) $2x^3 + c$
   e) All of the above

2. May not be published, according to confidentiality agreement with J. Epstein

3. $\lim_{h \to 0} \frac{\sin(x+h) - sinx}{h}$ is
   a) 0
   b) 1
   c) $\sin x$
   d) $\cos x$

4. Suppose you have a line with a slope of 3. If the *y*-value changes by 6, how much does the *x*-value change?
   a) 3
   b) 2
   c) 1

d) -2

5. May not be published, according to confidentiality agreement with J. Epstein

6. The radius of a snowball changes as the snow melts. The instantaneous rate of change in radius with respect to volume is

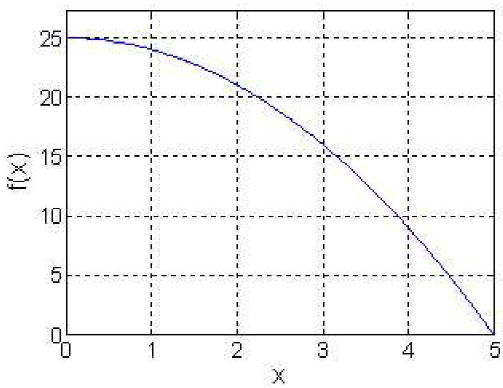
(a) $\frac{dV}{dr}$

(b) $\frac{dr}{dV}$

(c) $\frac{dV}{dr}+\frac{dr}{dV}$

(d) None of the above

7. Which of the following is the best estimate of $\int_0^3 f(x)dx,$, where *f* (*x*) is given in the figure below?



a) 13

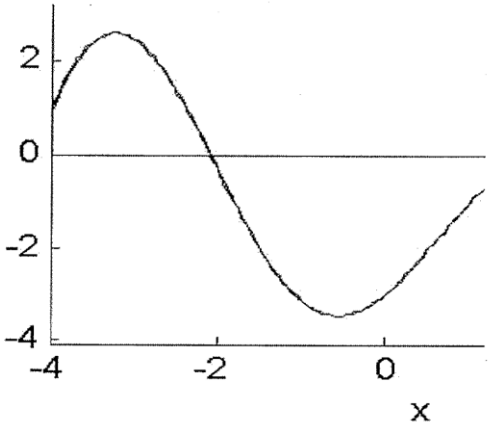
b) 17

c) 65

d) 85

8. May not be published, according to confidentiality agreement with J. Epstein

9. The figure below is the graph of $f'(x)$ (the derivative of the function f). Where is the maximum value of $f(x)$ (not $f'(x)$) on [-4; 0] ?



a) $x = -2$

b) $x = -3.2$

c) $x = 2$

d) $x = -4$

10. If $f(x) = sinx$ then

a) $f(x) = f''''(x)$ (four derivative signs)

b) $f(x) = -f''(x)$

c) $f'(x) = \cos x$

d) All of the above

11. Take the function $f(x)$ and shift the function right h units. Reflect the result across the y-axis. The end result is:

a) $f(x+h)$

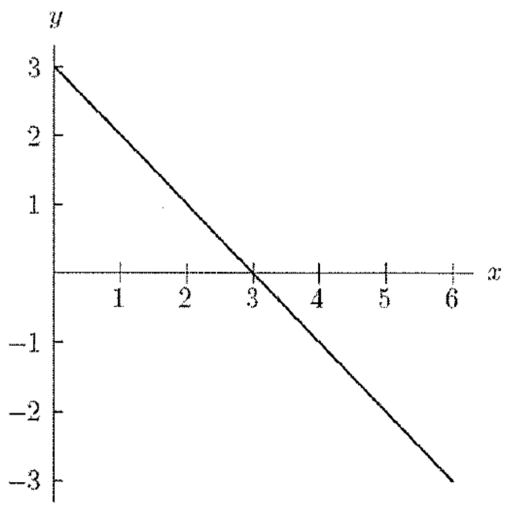
b) $f(x-h)$

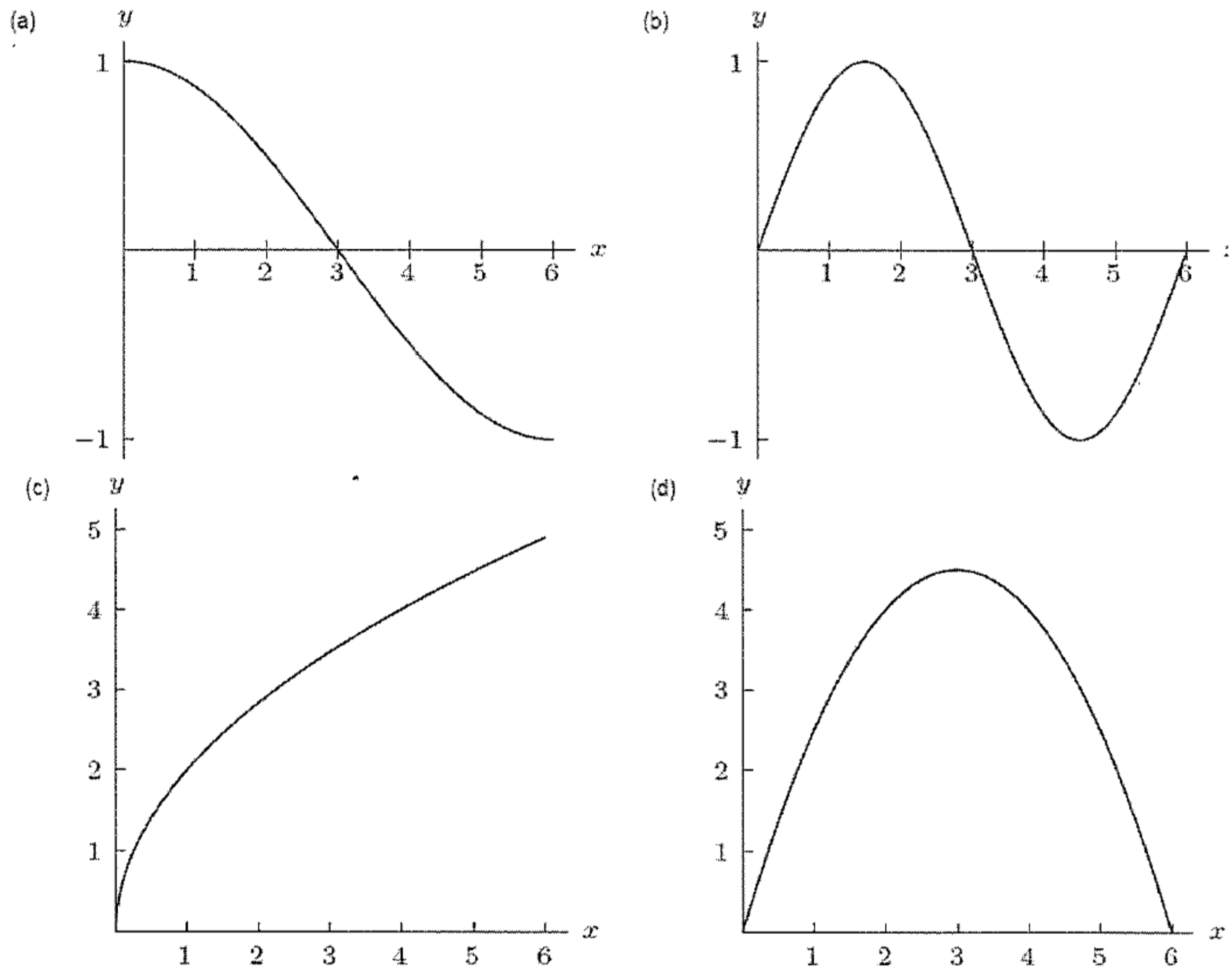
c) $-f(x-h)$

d) $-f(-x-h)$

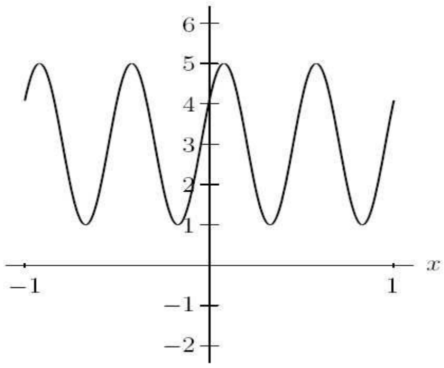
e) $f(-x+h)$

12. Which of the graphs (a-d) could represent an antiderivative or integral of the straight line shown in the figure below?





13. The amplitude and period of the function below are:



a) Amplitude $= 2$; Period $= 2$

b) Amplitude $= 2$; Period $= 3$

c) Amplitude $= 2$; Period $= \frac{1}{2}$

d) Amplitude $= 3$; Period $= 2$

e) Amplitude $= 3$; Period $= \frac{1}{2}$

14. In the figure below at $x = 0$, list the signs of the following in order:

- the function
- the first derivative
- the second derivative

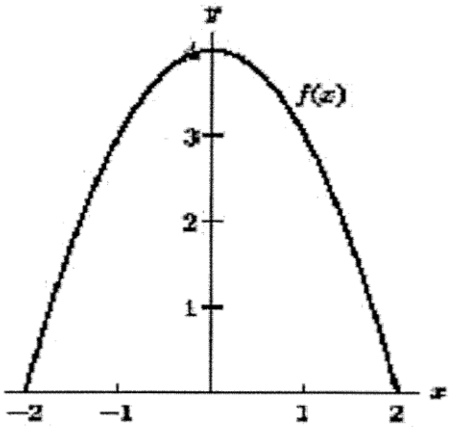
a) $+,\ 0, -$

b) $-,\ 0, +$

c) $-, 0, -$

d) $+, -, +$

e) $-, +, -$



15. Let $N = f(t)$ be the total number of books that Michael has read by age t, measured in years. Write down the units (years, books/year, etc.) that the answers to questions 15.1 and 15.2 are measured in:

15.1 $f^{-1}(50) = 8$

15.2 $f'(12) = 4$

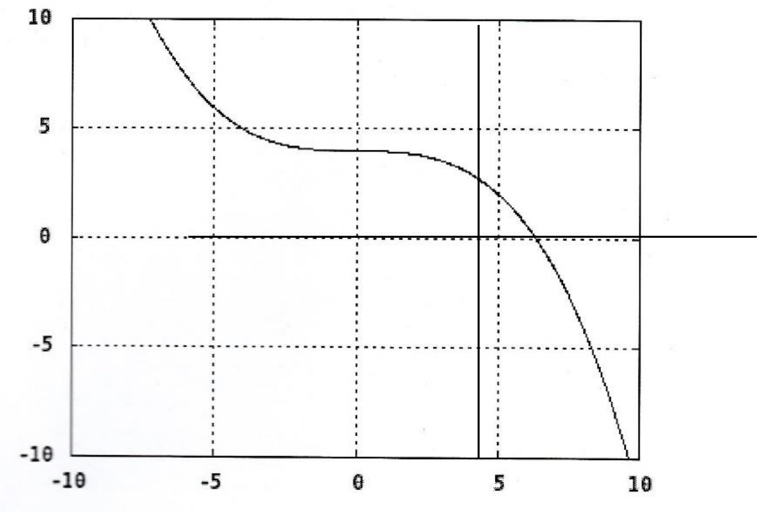
16. The graph of $y = f(x)$ is shown below. Rank $f^{-1}(5), f^{-1}(0), f(0)$ and $f(5)$ from smallest to largest:

a) $f^{-1}(5)$; $f(0)$; $f(5)$; $f^{-1}(0)$

b) $f(0)$; $f^{-1}(0)$; $f(5)$; $f^{-1}(5)$;

c) $f(0)$; $f(5)$; $f^{-1}(0)$; $f^{-1}(5)$;

d) $f^{-1}(5)$; $f(5)$; $f(0)$; $f^{-1}(0)$;



17. May not be published, according to confidentiality agreement with J. Epstein

18. Could this table represent a linear function?

x	1	2	4	8
y	12	14	16	18

(a) Yes, and I am very confident.

(b) Yes, but I am not very confident.

(c) No, but I am not very confident.

(d) No, and I am very confident.

Appendix C: Ethical clearance certificate from UFH

University of Fort Hare
Together in Excellence

ETHICAL CLEARANCE CERTIFICATE
REC-270710-028-RA Level 01

Certificate Reference Number:	MAM041SCOE01
Project title:	**Reform based approaches to the learning and teaching Calculus FOR Diploma studies in South Africa**
Nature of Project:	PhD
Principal Researcher:	Johanna (Hanlie) Coetzee
Supervisor:	Prof KJ Mammen
Co-supervisor:	

On behalf of the University of Fort Hare's Research Ethics Committee (UREC) I hereby give ethical approval in respect of the undertakings contained in the above-mentioned project and research instrument(s). Should any other instruments be used, these require separate authorization. The Researcher may therefore commence with the research as from the date of this certificate, using the reference number indicated above.

Please note that the UREC must be informed immediately of

- Any material change in the conditions or undertakings mentioned in the document
- Any material breaches of ethical undertakings or events that impact upon the ethical conduct of the research

The Principal Researcher must report to the UREC in the prescribed format, where applicable, annually, and at the end of the project, in respect of ethical compliance.

Special conditions: Research that includes children as per the official regulations of the act must take the following into account:

Note: The UREC is aware of the provisions of s71 of the National Health Act 61 of 2003 and that matters pertaining to obtaining the Minister's consent are under discussion and remain unresolved. Nonetheless, as was decided at a meeting between the National Health Research Ethics Committee and stakeholders on 6 June 2013, university ethics committees may continue to grant ethical clearance for research involving children without the Minister's consent, provided that the prescripts of the previous rules have been met. This certificate is granted in terms of this agreement.

The UREC retains the right to

- Withdraw or amend this Ethical Clearance Certificate if
    - Any unethical principal or practices are revealed or suspected
    - Relevant information has been withheld or misrepresented
    - Regulatory changes of whatsoever nature so require
    - The conditions contained in the Certificate have not been adhered to

- Request access to any information or data at any time during the course or after completion of the project.

- In addition to the need to comply with the highest level of ethical conduct principle investigators must report back annually as an evaluation and monitoring mechanism on the progress being made by the research. Such a report must be sent to the Dean of Research's office

The Ethics Committee wished you well in your research.

Yours sincerely

Professor Gideon de Wet
Dean of Research

18 July 2014

Appendix D: Consent from University where study was executed

WSU
Walter Sisulu University

DIVISION OF ACADEMIC AFFAIRS AND RESEARCH
DIRECTORATE OF RESEARCH DEVELOPMENT

Nelson Mandela Drive
Mthatha Campus
Private Bag X1
MTHATHA 5117
Tel: +27 47 502 2947/2647
Fax: +27 47 502 2185

Web Fax: 0866 541 093
E-mail: ecishe@wsu.ac.za
bandlec@gmail.com

Buffalo City
Potsdam Campus
EAST LONDON
Tel: + 43 708 5444
Fax: + 43 708 5458

05 August 2014

Mr. J. Coetzee
University of Fort Hare
South Africa

Dear Sir

Re: Request to conduct a Research study among Students at WSU

Permission is hereby granted provided that a copy of your completed study will be submitted to the Directorate of Research Development, WSU.

Regards

Dr. E.N. Cishe
Acting Director: Research Development

Walter Sisulu University

Appendix E: Cover letter and consent form

University of Fort Hare
Together in Excellence

The University of Fort Hare is conducting research regarding the learning and teaching of calculus. We are interested in finding out more about how to improve students' conceptual understanding of calculus. We are carrying out this research to help lecturers to improve their methodology.

Please understand that you are not being forced to take part in this study and the choice whether to participate or not is yours alone. If you choose not take part in answering these questions, you will not be affected in any way. If you agree to participate, you may pull out at any time and tell us that you don't want to continue with the tests. If you do this there will also be no penalties and you will NOT be prejudiced in ANY way. Confidentiality will be observed professionally.

We will not be recording your name anywhere on the questionnaire and no one will be able to link you to the answers you give. Only the researchers will have access to the unlinked information. The information will remain confidential and there will be no "come-backs" from the answers you give.

If possible, we would like return once we have completed our study to inform you of what the results are and discuss our findings and proposals around the research and what this means for you.

Principal Investigator Name: J. Coetzee
Signature J. Coetzee
Contact Details Cell: 0843944459

CONSENT

I hereby agree to participate in research regarding the learning and teaching of calculus. I understand that I am participating freely and without being forced in any way to do so.
I understand that this is a research project whose purpose is not necessarily to benefit me personally.

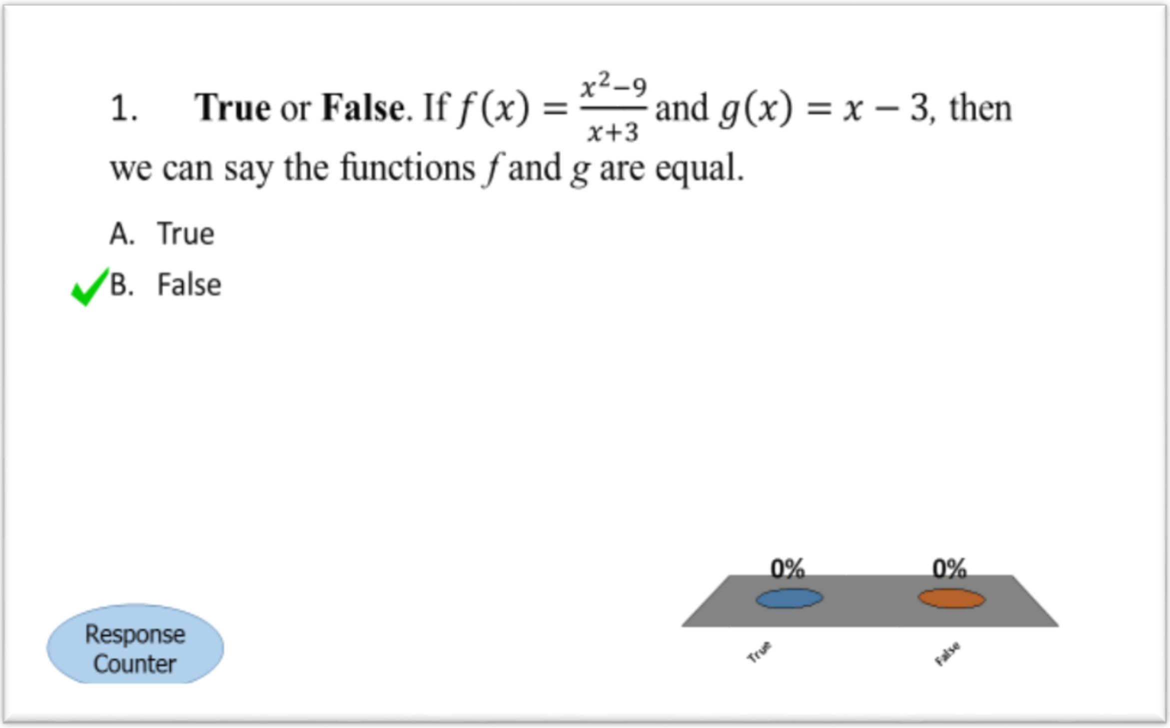
I have received the telephone number of a person to contact should I need to speak about any issues which may arise.

I understand that this consent form will not be linked to the questionnaire, and that my answers will remain confidential. I understand that if at all possible, feedback will be given to me on the results of the completed research.

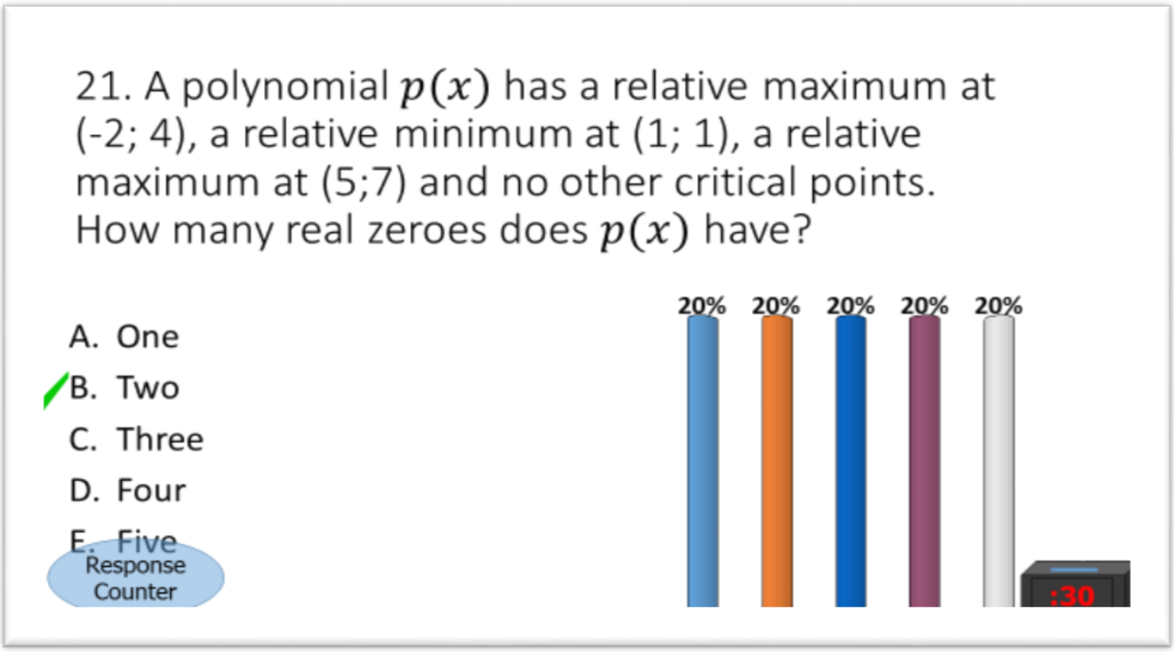
....
Signature of participant **Date:**......................

Document approved by UREC: 11 August 2011, V01

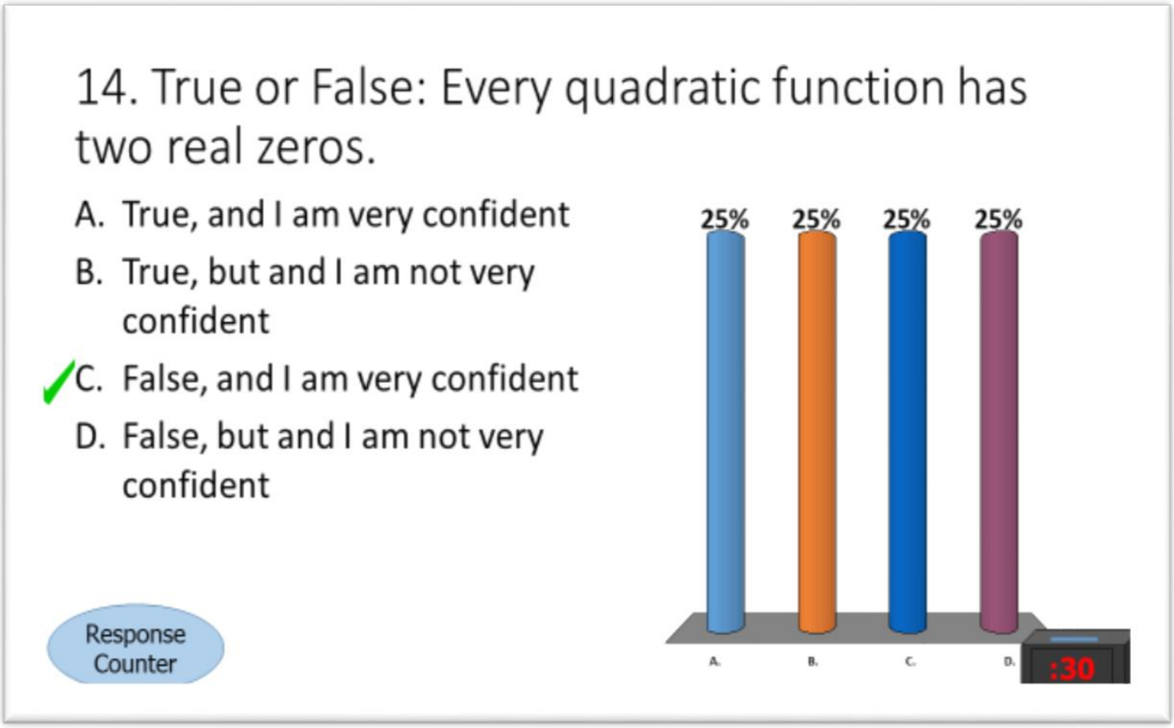
Appendix F: Examples of Questions captured in Turning Point



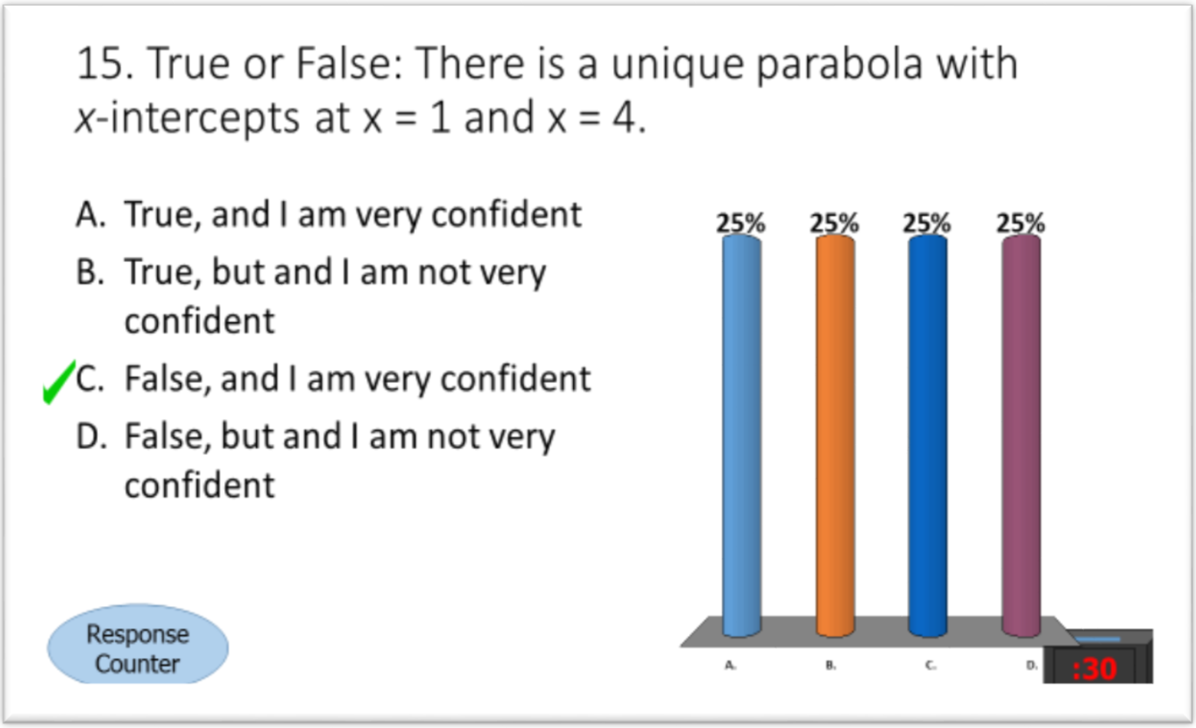
Source: Adapted from Q2, Limits, GoodQuestions (2005, p. 1)



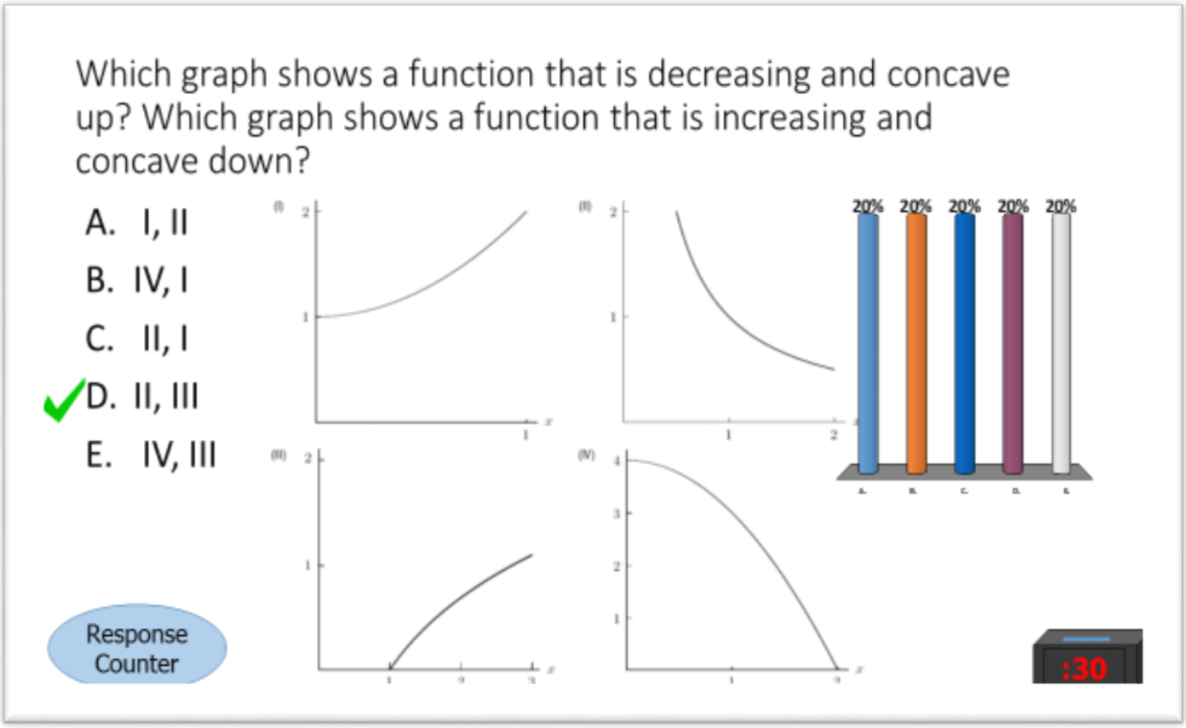
Source:Q28, Pre-calculus: Functions and Change, GoodQuestions (2005)



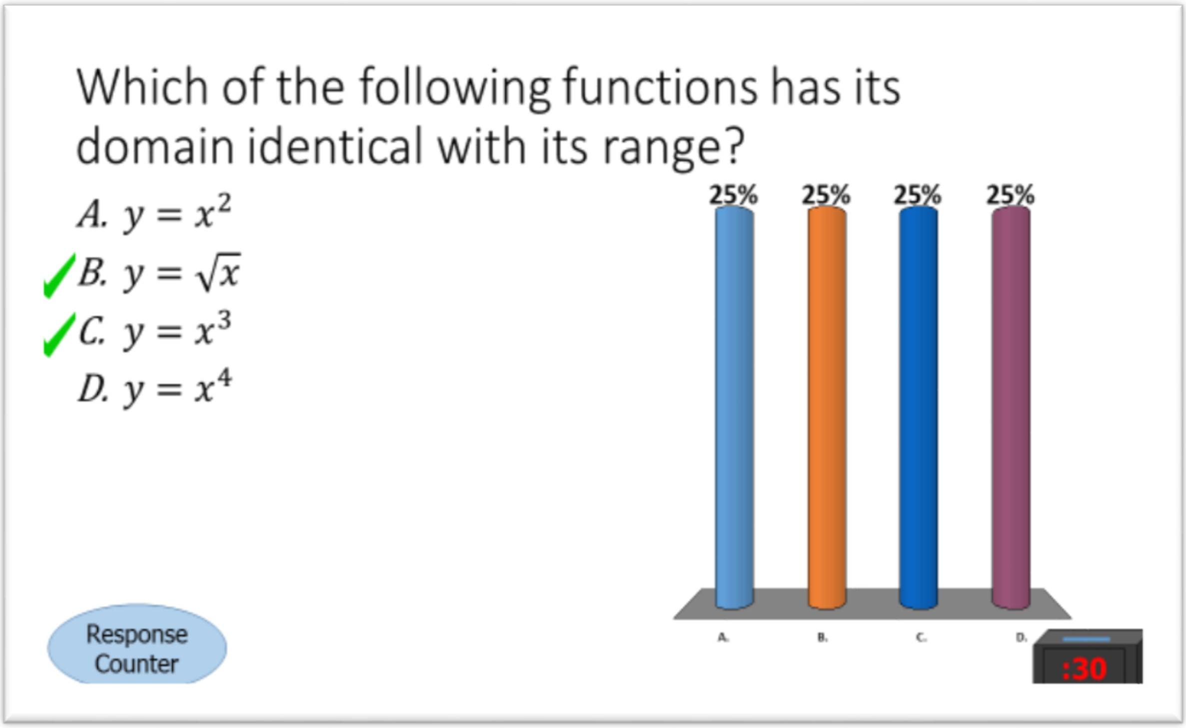
Source: Q26, Pre-calculus: Functions and Change, GoodQuestions (2005)



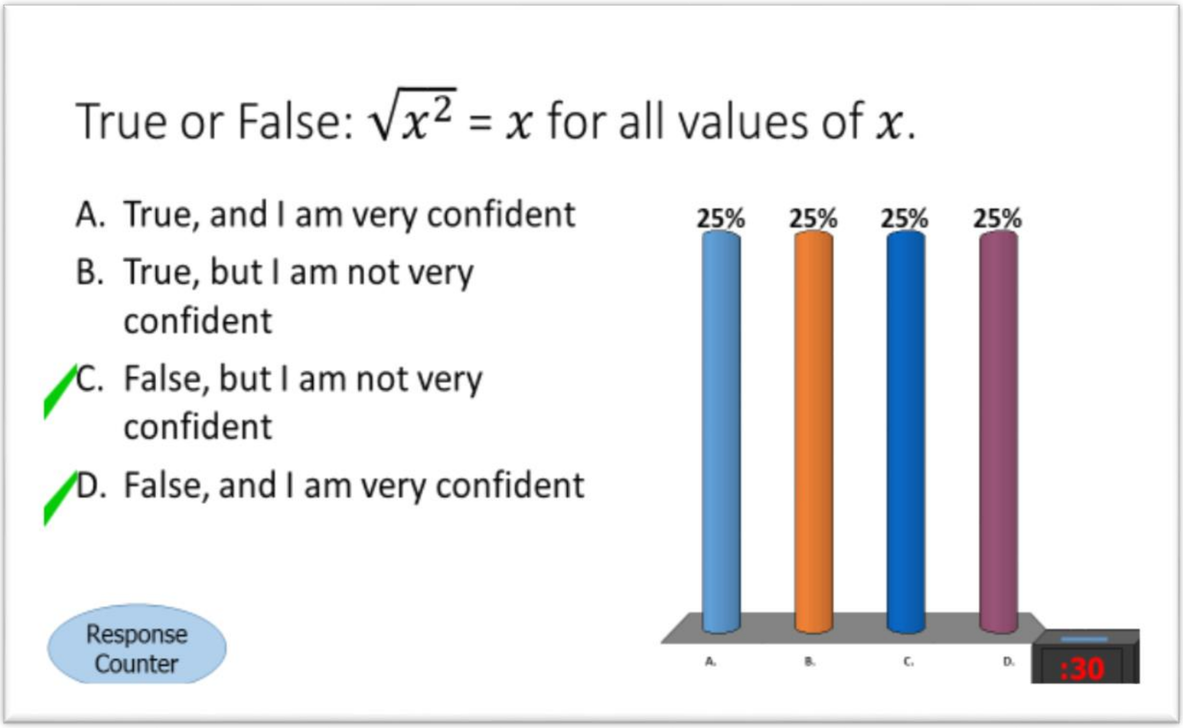
Source: Q27, Pre-calculus: Functions and Change, GoodQuestions (2005)



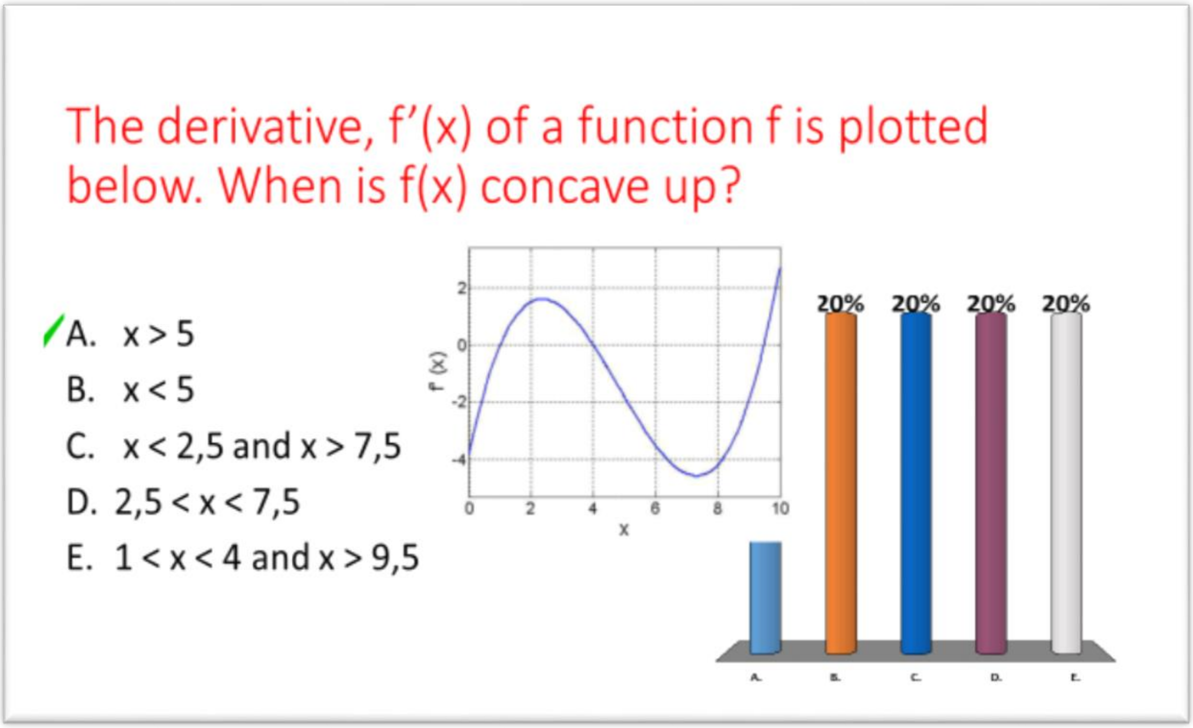
Source: Adapted from Q3, Calculus I: 2.1 How do we measure speed? GoodQuestions (2005)



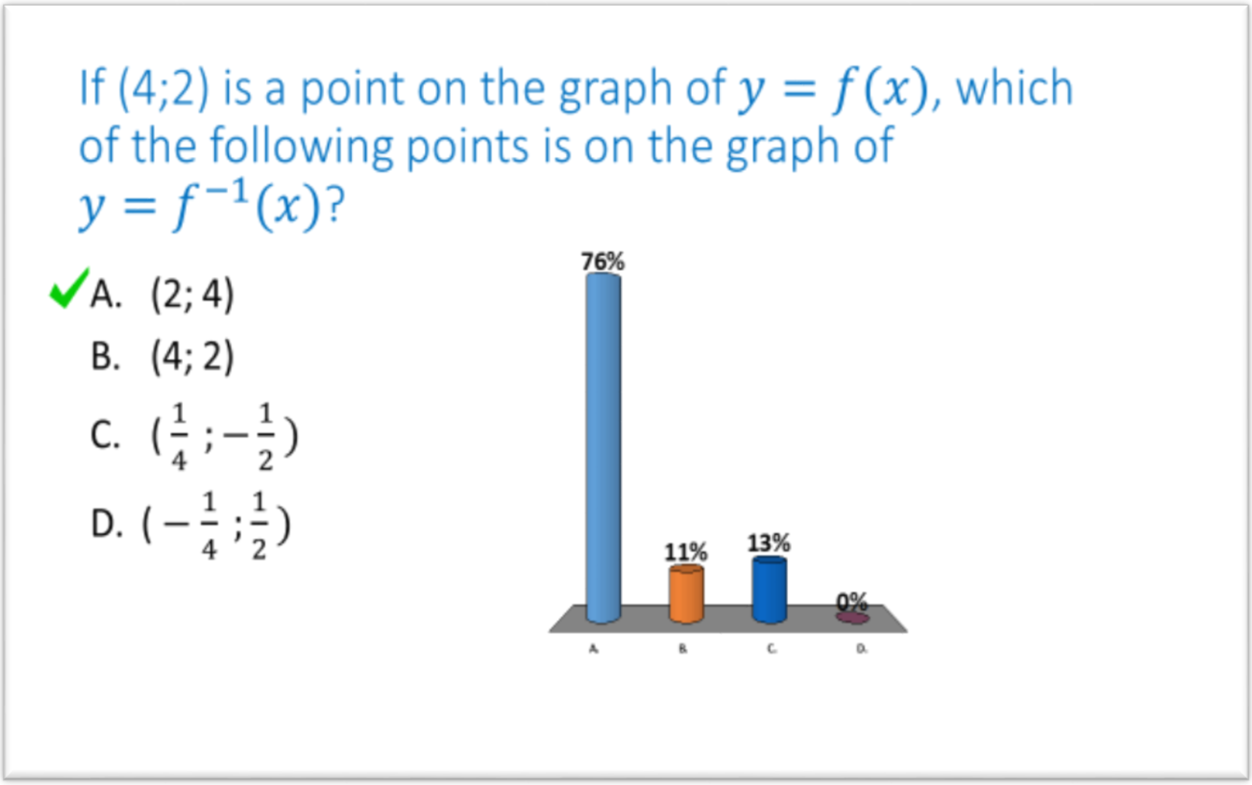
Source: Q 18, Pre-calculus: Functions and Change, GoodQuestions (2005)



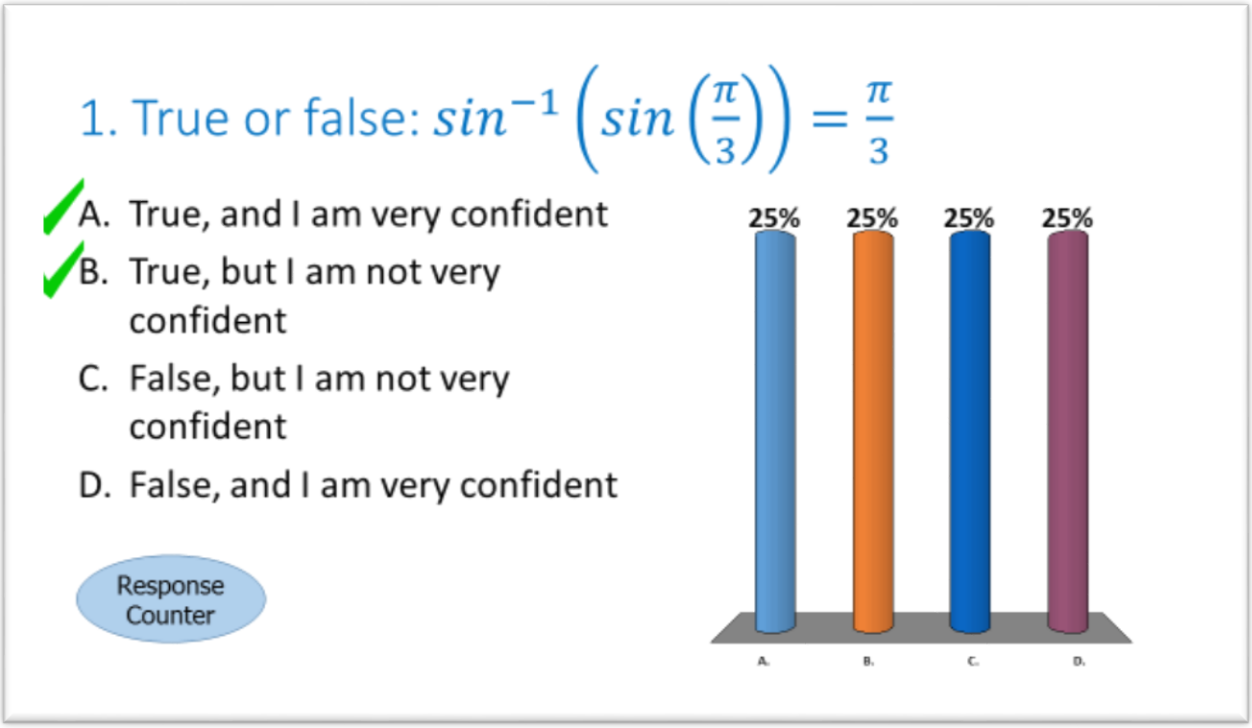
Source: Q 33, Pre-calculus: Functions and Change, GoodQuestions (2005)



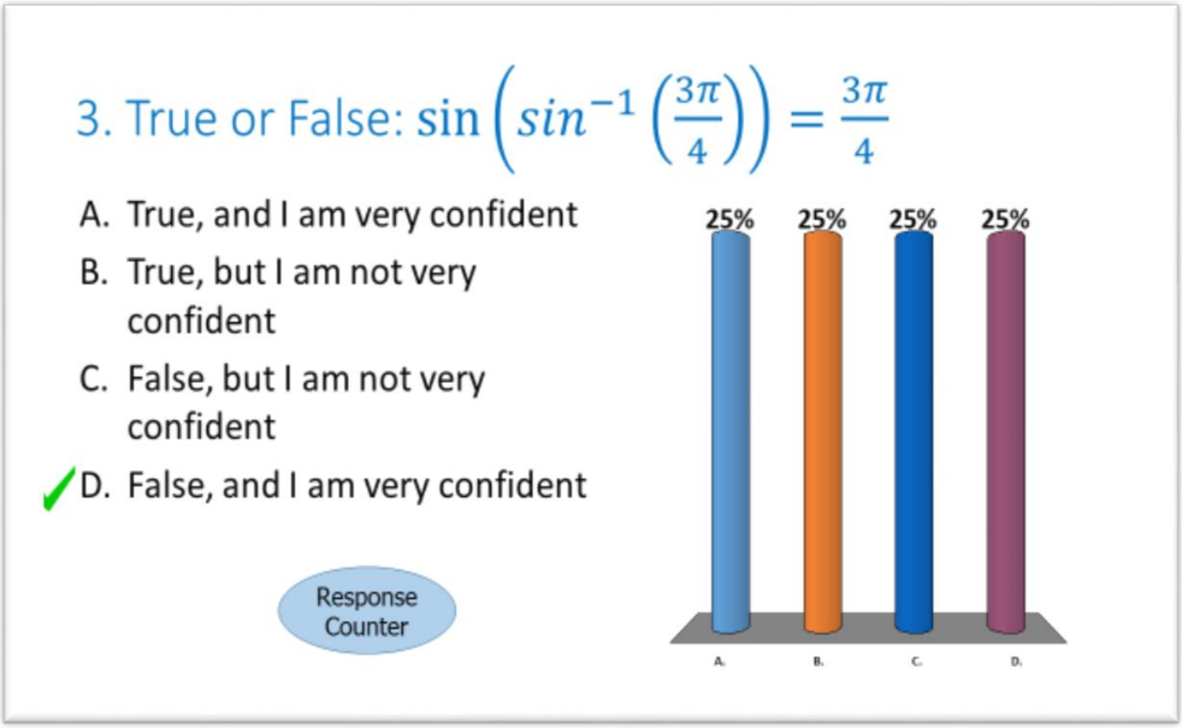
Source: Q9, Calculus II Section 6.1 Antiderivatives Graphically and Numerically, GoodQuestions (2005)



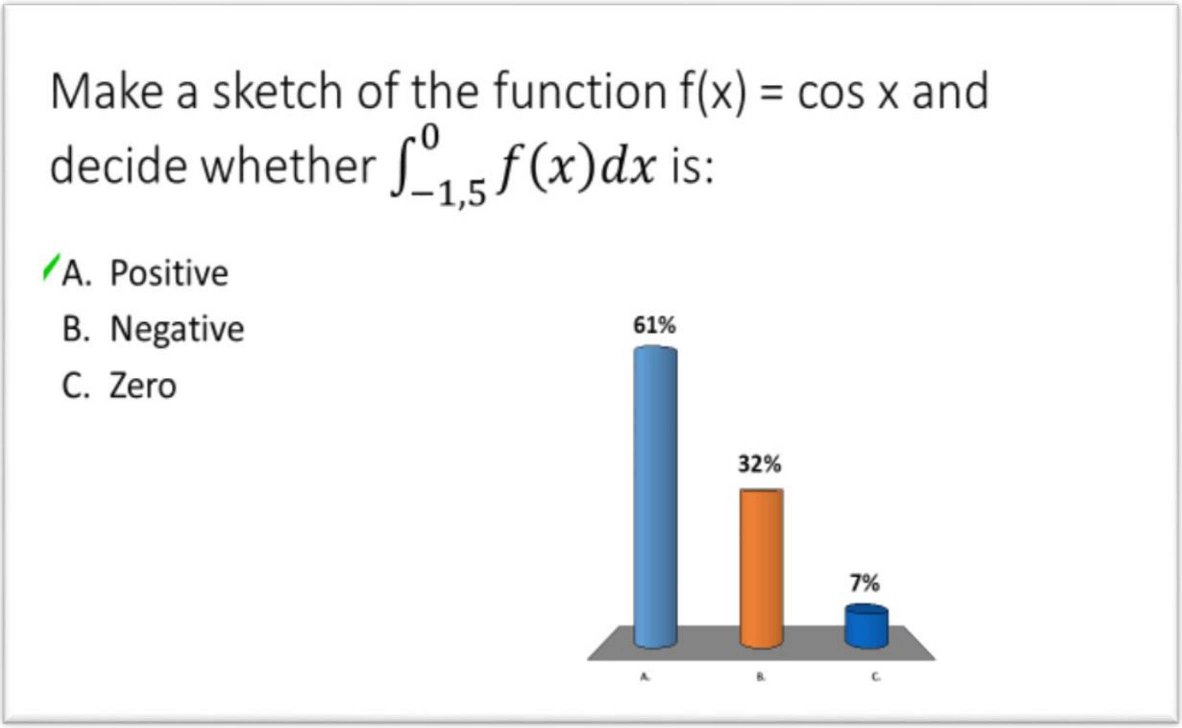
Source: Q10 Pre-calculus: New Functions From Old: Compositions, Inverses, and Transforms, GoodQuestions (2005)



Source: Q11 – Q13, Pre-calculus: Inverse Trigonometric Functions, GoodQuestions (2005)



Source: Q11 – Q13, Pre-calculus: Inverse Trigonometric Functions, GoodQuestions (2005)



Source: Classroom Voting Questions: Calculus II, Section 5.2, The Definite Integral, GoodQuestions (2005)

Appendix G: Certificate of Language Editing

3 Tranmere Road
Vincent
East London
5249
Tel. 27 43 726 6088 (H)
e-mail: anne3tran@gmail.com

J. Biggs

31 May 2017

TO WHOM IT MAY CONCERN

CERTIFICATE OF LANGUAGE EDITING

Title of thesis:	Reform-Based Approaches in the Learning and Teaching for Conceptual Understanding of Calculus for Diploma Studies at a South African University.
Student:	Coetzee, J.
Supervisor:	Mammen, K.J.
Degree:	Doctor of Philosophy (Education), Faculty of Education, University of Fort Hare

I confirm that the above paper was checked for language and the suggestions for improvement have been implemented.

J. N. Biggs

J. Biggs (B.A, HDE (Rhodes University))

South Africa