

UNIVERSITY OF FORT HARE

TFN 111/111E

DEGREE EXAMINATIONS

June 2017

Time: 3 HOURS

Subject: BUSINESS MATHEMATICS

Marks: 100

This paper consists of 8 pages including cover page

Internal Examiners

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Instructions

1. This paper consists of 7 pages (including the cover page).
2. This paper is made up of 2 sections, Section **A** (Linear Programming and Differential Calculus) and Section **B** (Financial Mathematics).
3. Answer **ALL** Questions in both Sections.
4. Calculators may be used.
5. Graph paper will be provided.

SECTION A

Linear Programming and Differential Calculus [50 Marks]

Question 1 [14 Marks]

A farmer in Limpopo specializes in citrus fruit. He has 90 hectares of land and must use at least 20 hectares for grapefruit trees and 30 hectares for orange trees in order to have enough fruit to satisfy his orders for the season. He uses at least as much land for oranges as for grapefruit but not more than twice as much.

- a) Write down the constraints as linear inequalities (let x = orange hectares and y = grapefruit hectares). [5]
- b) Draw the inequalities on the graph paper provided (show all the necessary working). [5]
- c) If the profit on grapefruit is R300 per hectare and the profit on oranges is R400 per hectare, how should he plant his fruit trees in order to obtain maximum profit? [4]

Question 2 [11 Marks]

An orphanage is preparing a trip for 400 of its orphans. The company which is providing the transportation has 10 buses of 50 seats each and 8 buses of 40 seats, but only has 9 drivers available. The rental cost for a large bus is R900 and R700 for the small bus.

Let x = Large Buses

Let y = Small Buses

- a) Write down the constraints as linear inequalities. [4]
- b) Draw the inequalities on the graph paper provided (show/indicate the feasible region).

[4]

c) Write down the objective function.

[1]

d) Calculate how many buses of each type should be used for the trip for the least possible cost.

[2]

Question 3 [25 Marks]

a) Use the substitution method of the chain rule to differentiate the following:

$$y = \frac{4}{\left(\sqrt{5-x^3}\right)^3}$$

[4]

b) Differentiate the following with respect to x .

$$y = \frac{\ln(5x + 2)}{e^{(3x^2-2)}}$$

[4]

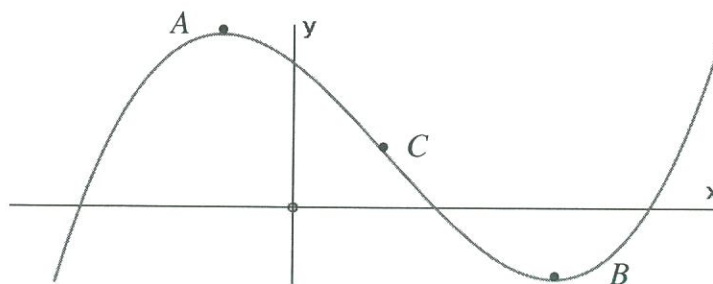
c) Find and simplify $\frac{dy}{dx}$ and hence find $\frac{d^2y}{dx^2}$ of the following function.

$$y = \ln\left(\frac{x}{x+1}\right)$$

[5]

d) The diagram below shows the sketch graph of $f(x) = x^3 + ax^2 - 11x + 30$.

A (-1; 36) and B are the turning points and C is an inflexion point of f



Determine the coordinates of C.

[4]

e) Consider the curve:

$$y = x^4 + 2x^3 - 2x - 2.$$

i. Show that

$$\frac{dy}{dx} = (4x - 2)(x + 1)^2.$$

Hence, find the coordinates for which y is stationary.

[5]

ii. Use the second derivative test to investigate the nature of the two stationary points identified above.

[3]

SECTION B
Financial Mathematics [50 Marks]

Question 4 [50 Marks]

- a) Mary took a loan of R1200 with simple interest for as many years as the rate of interest. If she paid R432 as interest at the end of the loan period, what was the rate of interest?
[5]
- b) If the simple interest on a sum of money at 4% per annum for 2 years is R400, find the compound interest on the same sum for the same period at the same rate.
[5]
- c) Cassidy wants to buy a TV and decides to buy one on a hire purchase agreement. The TV's cash price is R 5500. She will pay it off over 54 months at an interest rate of 21% p.a. An insurance premium of R 12,50 is added to every monthly payment. How much are her monthly payments?
[5]
- d) A box of chocolates costs R 55 today. How much did it cost 3 years ago if the average rate of inflation was 11% p.a.?
[4]
- e) A farmer buys a combine harvester at a cost of R1,6 million. The depreciation on the harvester is calculated at 18% p.a. on a reducing balance. Calculate the book value of the harvester at the end of 9 years.
[3]
- f) Convert an effective interest rate of 18% p.a. to a nominal interest rate per annum with weekly compoundings.
[3]
- g) Sizani wants to buy a computer, but right now he doesn't have enough money. A friend told Sizani that in 5 years the computer will cost R 9150. He decides to start saving money today at Durban United Bank. Sizani deposits R 5000 into a savings account with an interest rate of 7,95% p.a. compounded monthly. Then after 18 months the bank changes the interest rate to 6,95% p.a. compounded weekly. After another 6 months, the interest rate changes again to 7,92% p.a.

compounded two times per year. How much money will Sizani have in the account after 5 years, and will he then have enough money to buy the computer?

[6]

h) A farmer has just bought a new tractor for R800 000. He has decided to replace the tractor 5 years' time when its trade-in value will be R200 000. The replacement tractor is expected to increase by 8% per annum.

i. The farmer wants to replace his present tractor with a new one in five years' time. The farmer wants to pay cash for the tractor, after trading in his present tractor for R200 000, how much will he need to pay?

[4]

ii.

- One month after purchasing the present tractor, the farmer deposited x rands into an account that pays interest at a rate of 12% p.a. compounded monthly.
- He continued to deposit the same amount at the end of each month for a total of 60 months.
- At the end of 60 months he has exactly the amount that is needed to buy the new tractor, after he trades in his present tractor.

Calculate the value of x .

[6]

iii. Suppose that 12 months after the purchase of the present tractor and every 12 months thereafter, he withdraws R5 000 from his account, to pay for maintenance of the tractor. If he makes 5 such withdrawals, what will his new monthly deposit be?

[5]

i) A certain amount was invested on Jan 1, 2010 such that it generated a periodic payment of \$1,000 at the beginning of each month of the calendar year 2010. The interest rate on the investment was 13.2%. Calculate the original investment and the interest earned.

[4]

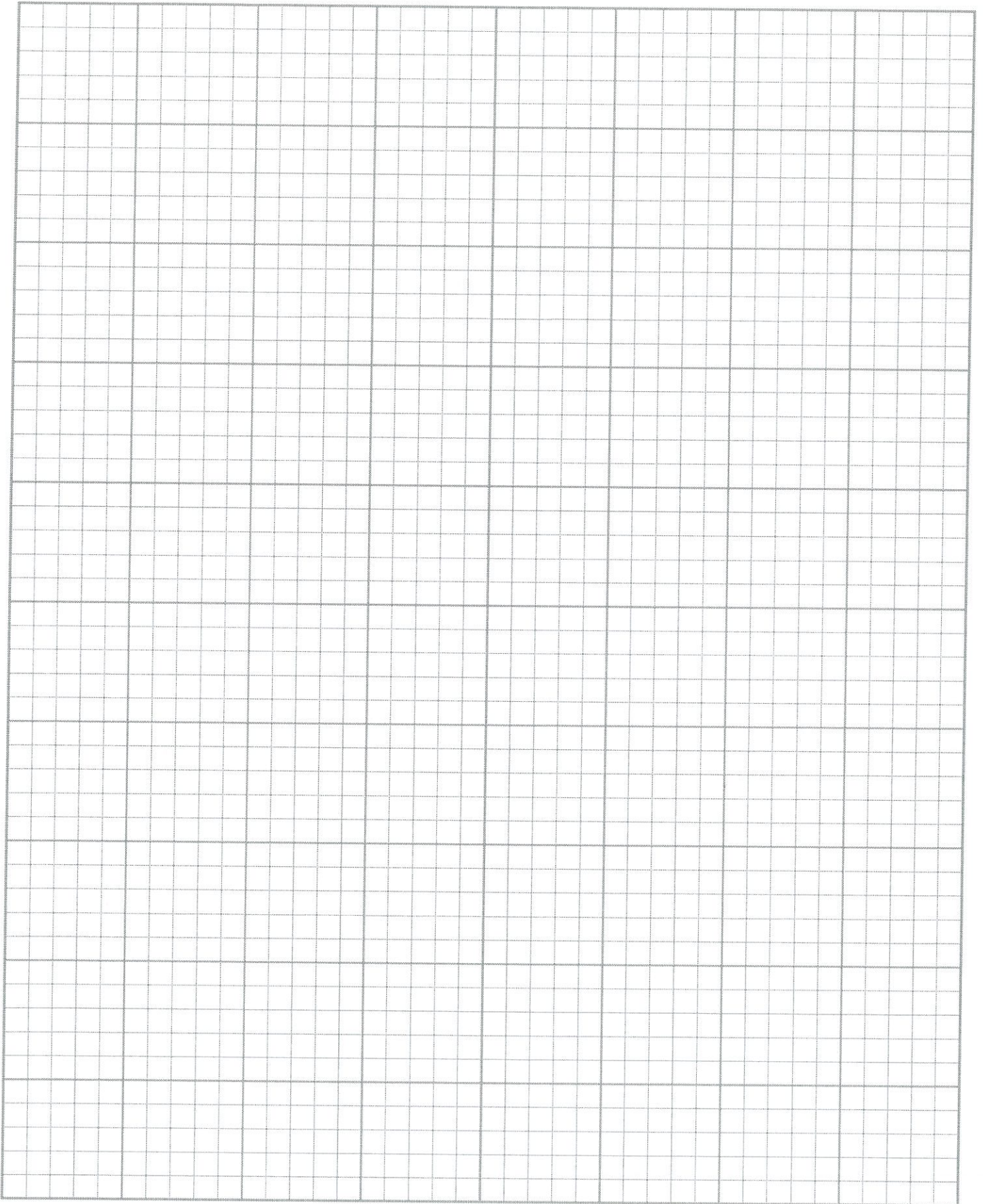
TOTAL: 100

FORMULA SHEET

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$	$\frac{d}{dx}[uv] = v \frac{du}{dx} + u \frac{dv}{dx}$
$\frac{d}{dx}(u/v) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$	
$F_v = P_v(1 + \frac{1}{m})^{mn} \times (1 + \frac{1}{m})^{mn} \times \dots$	$F_v = P_v(1 + \frac{1}{m})^{mn} + P_v(1 + \frac{1}{m})^{mn} + \dots$
$i_{eff} = (1 + \frac{i}{m})^m - 1$	$F_v = R \frac{(1 + i)^n - 1}{i}$
$F_v = R \frac{(1 + \frac{i}{m})^{mn} - 1}{i/m}$	$P_v = R \frac{1 - (1 + i)^{-n}}{i}$
$P_v = R \frac{1 - (1 + \frac{i}{m})^{-mn}}{i/m}$	$F_v = R \frac{((1 + i)^n)(i + 1)}{i}$
$F_v = R \frac{((1 + \frac{i}{m})^{mn} - 1)(1 + \frac{i}{m})}{i/m}$	$P_v = R \frac{(1 - (1 + \frac{i}{m})^{-mn})(1 + \frac{i}{m})}{i/m}$
$P_v = R \frac{(1 - (1 + i)^{-n})(1 + i)}{i}$	

student number

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