An Investigation into the level of understanding of two-dimensional shapes among learners at the end of the Intermediate Phase in a well-resourced former Model C school in the Eastern Cape : A case study.

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by

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Declaration

I, Caroline Harriet Selkirk, hereby declare that

(a) This dissertation is my own work.

(b) It has not been submitted for degree purposes at any University.

(c) The information derived from the published and unpublished work of others has been acknowledged in the text and a list of references is given.

Signature: CSl

Date: 31 March 2011

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Abstract

This study investigates the geometrical understanding according to the van Hiele theory, in terms of two-dimensional shapes, that learners have at the end of the Intermediate Phase in a well resourced former Model C school in the Eastern Cape. The research was situated in the Interpretivist paradigm and a qualitative research methodology was adopted. The research design was a case study and I used open and semi-structured interviews to collect the data. The participants were Grade 6 learners who had been purposively selected from the research school by using the results from an end-of-year Geometry assessment and consultation with the Grade 6 teachers. The research methodology, including ethical considerations and negotiating access to the research school, is discussed in Chapter 3.

The data collected as a result of the interviews is presented and discussed in chapter 4. Analysis of the data suggested that the learners had achieved different levels of geometrical understanding according to the van Hiele hierarchy. The data also indicated that learners in the Intermediate Phase at the research school have not had sufficient experiences with two-dimensional shapes to help them develop an understanding of the shapes. It was also found that the respondents struggled to find the geometrical language they needed to express their understanding of geometrical concepts. The conclusions and recommendations based on these research findings are in Chapter 5.

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Definitions

The following definitions have been used throughout this thesis.

The van Hiele Levels: Pierre van Hiele described five levels in his original theory (1986), naming them levels 0 to 4. Later, he revised the numbers of these levels (Van Hiele, 1999). The revised numbers of the levels, according to van Hiele, are used in this thesis, although some literature still refers to the original levels. I have written the revised levels first with van Hiele's original levels in brackets following. Van Hiele did not originally give the levels names. The names describing the first three levels are from van Hiele (1999:311) and the last two are from van der Walle (2004:310).

Level 1 (level 0) is the visual level.

Level 2 (level 1) is the descriptive level. Van der Walle (2004:309) calls Level 2 the level of analysis

Level 3 (level 2) is the level of informal deduction

Level 4 (level 3) is the level of deduction.

Level 5 (level 4) is the level of rigour.

Model C School: During the Apartheid era in South Africa, there were separate schools for the different race groups. Schools that were exclusively for white children, and thus better resourced than schools for children of other race groups, are now called Model C schools. These schools are state-controlled.

Manipulatives: In this study, this term describes physical objects that learners can handle like plastic shapes, cardboard cut-out shapes and construction sticks.

Learning Outcomes: There are eight **Learning Areas** (or subjects) that are taught during Grades R-9 in South African schools. Within each **Learning Area**, there are **Learning Outcomes** which have to be attained by the end of Grade 9. The **Learning Outcomes** describe developmental and intellectual skills and abilities that should be achieved by the learners as a result of their learning. Examples include the ability to evaluate information critically or to demonstrate an awareness of environmental issues.

Assessment Standards: These are defined for each grade and describe the depth and breadth of what learners should be able to know and do so that the Learning Outcomes can be achieved.

Abbreviations

In this document the following abbreviations have been used:

ACE	Advanced Certificate in Education
CDASSG	Cognitive Development and Achievement in Secondary School Geometry
ESL	English Second Language
FET	Further Education and Training
GET	General Education and Training
L.O. 3	Learning Outcome 3
MALATI	Mathematics Learning and Teaching Initiative
NCS	National Curriculum Statement
RNCS	Revised National Curriculum Statement
RUMEUS	Research Unit for Mathematics Education at the University of Stellenbosch
SACMEQ II	Southern African Consortium for Monitoring Educational Quality
TIMSS '99	Third International Mathematics and Science Study, 1999
VHGT	Van Hiele Geometry Test
WGT	Wu Geometry Test

Chapter 1 : Introduction

1.1 Background

The Revised National Curriculum Statement (RCNS) for Mathematics for Grades R to 9 (South Africa, 2002) states that there are five Learning Outcomes for Mathematics in the Intermediate Phase of schooling in South Africa. Learning Outcomes 3 and 4 deal directly with geometrical shapes and their measurement.

The Assessment Standards for Learning Outcome 3 state that, by the end of Grade 6, learners should to be able to describe and classify two-dimensional shapes and three–dimensional objects in terms of properties which include faces, vertices and edges, length of sides and angle size of corners. Learners need to be able to solve problems involving straight lines and triangles, use transformations and symmetry to investigate the properties of geometrical shapes and to be able to locate positions on coordinate systems (South Africa, 2002).

At the school where I teach, teachers in the Senior and F.E.T. phases have found that the performance of learners in tests of geometrical understanding is, on average, 20% lower than for Algebra assessments. At the beginning of this research, informal discussions with my colleagues revealed that we shared a common belief that learners appear to have memorized Geometry information in the Intermediate Phase. A possible consequence of this rote-learning is that most of these children do not develop a full understanding of the Geometry concepts, vocabulary and skills they should have by the end of Grade 6.

I work in an affluent, well-resourced former model C school. The learners at this school have access to appropriate Mathematics textbooks, support materials and a well-stocked library and Information Technology centre. The class sizes are not large, with an average of 28 to 30 learners per class. Our mathematics teachers are well-qualified, nurturing and competent. Our school provides excellent resources and infrastructure and offers a secure and supportive environment in which learners should thrive. However, despite all these advantages, we find that, year after year, learners do not develop a robust understanding of geometrical concepts.

This raises the question whether there is a disjunction between our Geometry methodologies in class and the conceptual development of the learners. To answer this question, it is necessary to examine what the developmental theorists have had to say about how children develop an understanding about space and shape in Geometry.

According to Jean Piaget (1896-1980), children aged from seven to twelve years are in the concrete operational stage of cognitive development. (Stone and Church,1979:392). This corresponds with the age of children in Grades 1 - 6 in South African schools (i.e. the Foundation Phase which is Grades 1 to 3 and the Intermediate Phase which is Grades 4 to 6). Farrell and Farmer, (1980) posed the question:

> "What does 'concrete' signify? Must the child manipulate physical models of the ideas to be learned? Usually but not necessarily! What is essential is that the experiences be real to the child and that those experiences reflect in as tangible a way as possible the concept or rule being developed"
>
> (Farrell and Farmer, 1980:59)

This seems to suggest that, during the Foundation and Intermediate Phases, learners require interaction with concrete items such as manipulatives, and tactile objects, in order to help them develop an understanding of geometrical concepts.

Piaget's ideas stimulated research by Pierre and Dina van Hiele, a husband and wife team in the Netherlands during the 1950's. The van Hiele's were interested in the development of geometrical thinking among children of school-going age. Their research was influenced by Pierre van Hiele's experiences when he taught at a Montessori school in 1951. Van Hiele wrote:

> "Because I had understood that the learning of facts could not be the purpose of teaching mathematics, I was convinced that development of insight ought to be the purpose"
>
> (van Hiele, 1986:4)

The area of Mathematics that van Hiele focused on was the teaching of Geometry to learners at school. Bennie (1999:64) stated that the van Hiele's "focused on levels of thinking in Geometry and the role of instruction in assisting pupils to move through the levels". Van Hiele claimed that, "an important part of the roots of my work can be found in the theories of Piaget" (van Hiele, 1986:5). However, he emphasized that:

> "The psychology of Piaget was one of development and not of learning. The problem of how to stimulate children to go from one level to the next was not his problem. It was mine. Piaget distinguished only two levels. In geometry it appears necessary to distinguish more."
>
> (van Hiele, 1986:5)

In Pierre van Hiele's doctoral thesis, completed in 1957, he postulated that the development of geometrical reasoning in children proceeds through five levels, which follow one another. Unlike Piaget's age and stage theory (1941) these levels are not age dependent but van Hiele suggested that a child needed to master one level of understanding before he could move to the next level (van Hiele, 1986).

Like Piaget, van Hiele suggested that children need to have the opportunity physically to interact with shapes in order to develop their geometrical understanding (van Hiele, 1999). Van Hiele stated:

> "The attainment of a new level cannot be effected by teaching, but still, by a suitable choice of exercises, the teacher can create a situation for the pupil favourable to the attainment of the higher level of thinking."
>
> (van Hiele 1955 cited in van Hiele, 1986:39)

Van Hiele's use of the phrase "suitable choice of exercises" above suggests that the teacher should provide learning opportunities in the classroom that will help his learners to develop an understanding of geometrical concepts.

In South Africa, the Geometry curriculum, as prescribed in the Mathematics Policy Document of the Revised National Curriculum Statement (South Africa, 2002), is sequential, each year laying the foundations for the next. This follows the Van Hiele theory of sequential levels of geometric understanding.

Van der Walle stated that:

> "The most important geometric agenda of the K-8 teacher is to provide experiences that move students from level 0 thinking to level 2 thinking by the end of the eighth grade. Not every teacher will be able to move children into the next level. However, all teachers should be aware that the experiences they provide are the single most important factor in moving children up this developmental ladder. Every teacher should be able to see some growth in geometrical thinking over the course of a year"
>
> (van der Walle, 2004:348)

However, it seems that this development in geometrical thinking is not the focus in many South African classrooms. Van Hiele warned that many teachers reduce Geometry to the memorization of definitions and structures. For example, alternate interior angles are recognized as part of a Z-form. This is illustrated in Figure 1.1 below.

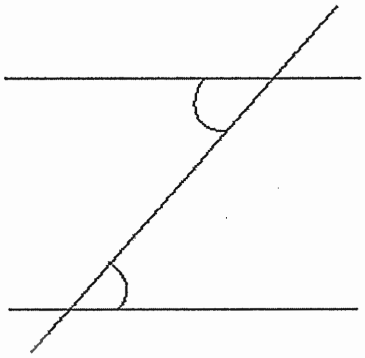


Figure 1.1 Alternate angles form a Z-shape

Van Hiele said:

> "This method can turn out to be harmful if the teacher, in his zeal for quick results, has these structures learned by the pupils. For by doing so he weakens the necessity for the pupil to come to a higher order of thinking..."
>
> (van Hiele, 1986:43)

This runs parallel to what my colleagues and I think is happening in our own school. It seems that children in our primary school are learning definitions such as "squares have four equal sides" and a rhombus is a "pushed over square". When they are shown a diagram of a rhombus and asked whether it is a square, they say, "Yes, it has just been pushed over". As a result, they have not had the opportunity to develop an understanding of the similarities and differences between squares and rhombi.

My initial overview of the literature revealed that much of the research that has been done on the level of geometrical thinking of learners in South Africa has been directed at previously disadvantaged schools. Feza and Webb (2005) researched the level of geometrical understanding of a small group of Xhosa first-language Grade 7 learners. This research took place in under-resourced rural, peri-urban and urban schools in the Eastern Cape. Feza and Webb discovered that none of the 30 children they studied had attained the requirements of the RNCS (South Africa, 2002) Assessment Standards in Learning Outcome 3. Ten learners were strictly at van Hiele's Level 1, fifteen learners were between Levels 1 and 2 and only five learners had attained Level 2. None of the learners could be categorized at van Hiele Level 3, the level required for Grade 7 Geometry. (Feza and Webb, 2005:40).

Kotze (2007) used van Hiele's research as the basis for investigating the geometrical understanding of Grade 10 learners and their teachers in under-resourced schools in South Africa. She found that major emphasis seemed to be given in classrooms to the mastery of basic skills in Geometry, such as calculating areas. She also found that the identification of geometric shapes in new positions was more problematic for the teachers than the learners. Kotze (2007) suggested that this would indicate that the teachers were still operating at Level 1 of the van Hiele hierarchy in some areas of Geometry.

Feza and Webb (2005) found that in the schools in which they conducted their research, many of the teachers were the products of "Bantu Education." They stated:

> "After the installation of the Nationalist government (and the subsequent adoption of apartheid as a national policy) in South Africa in 1948, a system of 'Bantu education' was introduced for black South Africans. Bantu education was for black people and was to be largely based in the Bantustans or homelands where natives would be prepared for life on reserves (Davies as cited in Feza and Webb, 2005:37), with different or inferior curricula – usually with no science or mathematics offerings (Hartshorne, as cited in Feza and Webb, 2005:37). The impact of the

> system can be seen even today and the children participating in this study all had teachers who are products of Bantu Education. These teachers were under-qualified to teach mathematics and were teaching in under-resourced schools mainly in rural areas of the Eastern Cape."
>
> (Feza and Webb, 2005:37)

Thus the teachers themselves had experienced little or no Mathematics instruction in their own schooling. The research suggested that because the teachers hold tenuous geometric conceptual frameworks themselves, the teachers perpetuate a lack of geometrical understanding among the primary school learners in their schools. Current research supports Feza and Webb's (2005) findings (Halat, 2008; Knight, 2006; Kotze, 2007; van der Sandt and Niewoudt, 2003).

Additional research done both overseas and in South Africa indicates that many teachers of Geometry at primary school have not progressed to the higher levels of geometrical understanding themselves. Knight (2006) found that the level of geometrical understanding of students registered to study education at the University of Maine was significantly lower than the level expected of the learners they were required to teach once they had qualified. She went on to assert that, if the teacher's level of understanding is only at van Hiele Level 1 or 2, the teacher will be unable to provide children with the requisite tasks and opportunities needed to develop appropriate grade and phase-specific spatial and geometrical concepts and understanding (Knight, 2006:10). Van der Sandt and Nieuwoudt (2003) conducted similar research among eighteen Grade 7 teachers and one hundred prospective teachers in South Africa. They also found that the level of geometric understanding of these teachers is not at the level required to promote successful teaching of Geometry.

Guay and McDaniel (1977) have suggested that there may be a link between spatial sense and general performance in Mathematics itself. Their research indicated that high achievers in Mathematics at primary school have greater spatial ability than low achievers. While Learning Outcomes 3 and 4 focus on the development of geometric and spatial competences, these competences and their attendant concepts, knowledge and skills may be transferable into the other Learning Outcomes of the Mathematics syllabus.

Many articles have been written about the poor performance of South African learners in Mathematics (including Geometry understanding) both at school and in tertiary institutions. These include analyses of how South African learners have performed in international tests of mathematical understanding such as SACMEQII (Southern African Consortium for Monitoring Educational Quality) and TIMSS'99 (Repeat of the Third International Mathematics and Science Study in 1999).

Data captured by the Southern African Consortium for Monitoring Educational Quality, (SACMEQII), indicated that South African learners have a low level of mathematical understanding. Kotze and Strauss (2007) stated that:

> "Data from grade 6 learners' scores for mathematics ... (show) ...
>
> - that the overall learner achievement was noticeably on the lower end of the acceptable limits on the SACMEQII benchmark.
> - the relationship between difficulty levels and learner competencies reveals a significant low level of general numeracy, understanding and skills. Test items focusing on higher order meta-cognitive skills were poorly answered."
>
> Kotze and Strauss (2007:29)

Thus Kotze and Strauss (2007) found that, not only did South African Grade 6 learners attain low scores for their mathematical skills, but that their problem-solving skills were not well developed either.

South Africa participated in the Repeat of the Third International Mathematics and Science Study in 1999 (TIMSS'99). Comparative results show that South African Grade

7, 8 and 12 learners performed very poorly in comparison to the other countries that participated in the study. The study included many African countries and other developing nations. Howie (2004) analysed the results from the Mathematics assessment in the TIMSS'99 and found that Grade 8 learners in the Eastern Cape achieved a mean score of 256 out of a possible score of 800. The international average was 487.

Howie (2004) also mentioned that, when comparing the results of those learners who spoke the language of the test, which is English, with the results of those who never speak the language of the test, the former group achieved on average 140 points higher than the latter group on the tests of mathematical proficiency. While this finding does not include a discussion on the possible use of different teaching and learning methodologies in different classrooms, it does nonetheless suggest that English speaking learners in former Model C Schools have an advantage over those learners who are not fluent in that medium of instruction. This could be a contributing factor, for some students at least, for poor performance in Geometry.

Poor performance in Mathematics at school and in matric leads to tension at tertiary level too. Studies by Usiskin (1982) and Senk (1989) from the University of Chicago have shown that students, who have not attained a van Hiele Level 3 of geometrical understanding by the end of primary school, are unlikely to achieve success in Geometry at secondary school and beyond as their level of understanding is too low. Augustine (2009), a Chemistry lecturer at the University of KwaZulu-Natal, claimed that 50% of his students lacked the basic Mathematics and Science skills to compete at university level, including the spatial skill of being able to read values from graphs. At the University of the Witwatersrand, mathematics and engineering lecturers claimed that their students were unable to visualize in 2- and 3-dimensional space (Huntley, 2009).

This preliminary review of the literature revealed that the problems my colleagues and I have experienced among our Geometry students are not unique but that geometrical understanding is problematic at tertiary level too. The question is : what is happening in Geometry lessons in the Intermediate Phase that is preventing learners from developing the level of geometrical understanding required to master the Grade 6 Geometry Assessment Standards?

1.2 Statement of the Research Problem

For many years, my experience as a Mathematics teacher in Grades 8 to 12 has been that the learners at the former Model C school where I teach perform consistently badly in Geometry assessments. By the time our learners have reached Grades 8 to 12, it is assumed, by their Mathematics teachers that they have reached a level of geometrical understanding that will enable them to cope with the demands of the Geometry curriculum. It would appear that this is generally not the case.

Our learners have had the opportunity to attend a well-resourced school which is able to provide them with textbooks and other support materials. They have well-qualified, nurturing teachers, who teach their learners in comfortable, well-resourced classrooms. The class sizes are on average 28 to 30 learners. The medium of instruction is English and many of the learners speak English as their home language. Many of the non-English mother tongue speakers speak English at home as a second language. The majority of the learners at our school have attended the same English medium school since Grade R and so it would be reasonable to assume that the learners are comfortable with speaking and hearing English. Despite all these advantages, it appears that many learners memorize Geometry definitions, with little understanding of what they have learned, have poor spatial sense and are unable to apply the knowledge they have.

1.3 Purpose of the Study

Learners in my school seem to have access to all they require to develop to the level of geometrical understanding required by the RNCS (South Africa, 2002) by the end of the Intermediate Phase and yet they are not attaining this level. The purpose of my research was to investigate the level of geometrical understanding of learners at the end of Grade 6 at a well-resourced school in the Eastern Cape, focusing particularly on their understanding of two-dimensional shapes.

1.4 Research Questions

My primary research question was:

- What was the level of geometrical understanding of the learners at the end of Grade 6 in a well-resourced school, with regard to two-dimensional shapes, where the Revised National Curriculum Statement (South Africa, 2002) is followed?

This was supported by the following secondary questions:

- How do the requirements of the Revised National Curriculum Statement (South Africa, 2002) align with the Van Hiele levels of geometrical understanding?

- How were children in the Intermediate Phase, and specifically in Grade 6, being taught Geometry at the school selected for the research?

- Were learners in the Intermediate Phase at the research school being given sufficient exposure to Geometry activities to promote the development of their geometrical understanding?

- What influence do activities such as jigsaw puzzles and tangrams have on the development of geometrical thinking in learners?

1.5 Research Objectives

1. My first objective was to determine the level of geometrical understanding learners had at the end of Grade 6 at a well-resourced, former Model C school, following the Revised National Curriculum Statement (South Africa, 2002). I focused on the learners' understanding of two-dimensional shapes. I did this to ascertain whether these learners had attained the level of geometrical understanding required for the study of Geometry in the Senior Phase (Grades 7 to 9).
2. I wanted to investigate how Geometry was taught in the Intermediate Phase at the above former model C school. I hoped that my findings might allow me to contribute to a more informed methodology for Geometry teaching in the Intermediate Phase.

1.6 Assumptions

The following assumptions were made:

1. That the geometrical understanding of children does develop according to the van Hiele theory. This theory states that children's geometrical understanding develops through 5 levels, from the basic recognition of shapes up to being able

to formulate formal proofs. The levels are sequential in that Level 1 must be mastered before Level 2 can be attained and so on. However, children may be in a transition between two levels. For example, if learners are in transition between the van Hiele Levels 1 and 2, where they still think in a Level 1 manner at times, they can be challenged by a "crisis of thinking" to move to Level 2, a higher level.

2. I also assumed that children would be able to remember the geometrical experiences they have had so that they will be able to recount them to me. This was important for me to establish whether prior experiences with shapes have affected the development of geometrical understanding among the learners interviewed.
3. That the teachers teaching at the former Model C research school in the case study were all well-qualified and committed to teaching Geometry to their learners.
4. That the learners at the former Model C school were proficient in the language of teaching and learning which is English.

1.7 Significance of the study

This research may help to shed light on the level of geometrical understanding of two-dimensional shapes for learners at the end of Grade 6 in a well-resourced, former Model C school in the Eastern Cape in South Africa. It may also influence the way Geometry is taught in the Intermediate Phase in my school.

1.8 Rationale of the study

A number of research studies have been carried out on the level of geometrical understanding among South African school children (Bennie, 1999; de Villiers and Njisane, 1987; Feza and Webb, 2005; Kotze, 2005). These studies indicate that the level of geometrical understanding among learners in the primary and secondary school is not what it should be. Some of these studies are briefly discussed below.

Research has been done on the level of geometrical thinking of learners in under-resourced schools in Grade 7 by Feza and Webb (2005). They discovered that these Grade 7 learners were not at the Van Hiele level required to master formal Geometry in higher grades. Kotze (2005) discovered, during her research with Grade 10 learners from rural and urban schools and their Geometry teachers, who were registered for in-service teaching certificates, that neither the teachers nor their learners were able to recognize shapes when they were rotated or placed in a different orientation. This is indicative of Level 1 thinking on the van Hiele hierarchy. De Villiers and Njisane (1987) found that Grade 10 learners in rural schools in Kwa Zulu Natal resorted to rote-learning definitions in Geometry as their level of geometrical reasoning was not sufficiently advanced to master Grade 10 Geometry concepts.

Bennie (1999), working at MALATI (Mathematics Learning and Teaching Initiative) project schools in the Western Cape, noted that the development of Geometry reasoning is sequential and that children need to understand concepts in the primary school before they can master formal Geometry in the higher grades. Bennie (1999) suggested that there was a need to investigate what is happening in Geometry classes in the Intermediate Phase so that we can discover how to help learners to improve their geometrical understanding.

Thus, research has been done in South Africa in rural, peri-urban and urban schools. Many of the learners in the research, however, come from under-resourced schools. The problems that we experience in teaching Geometry at my school, which is a well-resourced school, appear to be similar to the problems identified with learners from under-resourced schools. Therefore there is a need to investigate the level of geometric reasoning of the learners at the end of Grade 6 at well-resourced schools to try to understand why we experience the same problems. It may well be that learners at the end of the Intermediate Phase at the research school are not at the level of geometrical thought necessary to study Geometry in Grades 7 to 12. Knowing this may affect how teachers approach the teaching of Geometry in the Intermediate and Senior Phases at our school.

It may be possible that Geometry skills are able to be transferred to other areas of Mathematics too (van Hiele, 1986). As many authors have noted the concern over Mathematics results in South African schools, (Howie, 2004; Huntley, 2009; Kotze and Strauss, 2007) research into the development of geometrical understanding among South African Intermediate Phase learners may help to improve the overall performance of learners in Mathematics. This research may also assist in improving the teaching of Geometry in the Intermediate Phase in South African schools which may benefit learners when they study Geometry at secondary and tertiary level.

1.9 Delimitations of the study

This research has been carried out in one former Model C school in the Eastern Cape. A very small percentage of South African schools are former Model C schools. As a result, the findings may not be generalized to all schools in South Africa.

It is a case study and has provided qualitative data from fourteen purposively-selected Grade 6 respondents. This is not a large enough sample to be able to make generalizations about all Grade 6 learners but I hoped that the structured interviews and purposive sampling would allow me to investigate the geometrical reasoning and determine the van Hiele level of geometrical understanding of the Grade 6 learners at the research school.

Chapter 2 : Literature Review

2.1 Introduction

This chapter considers issues which affect Grade 6 pupils' understanding of two-dimensional shapes in Geometry in the Intermediate Phase of teaching and learning in a South African school.

In this literature review I will explore three theories, each of which explores the development of geometric reasoning in Mathematics. These theories will be discussed in terms of both international studies and local research. Also in this literature review, I will examine the roles of teachers as facilitators of development of geometrical thought in learners. The importance of language in the development of geometric thinking will be investigated. Finally, this chapter will introduce a number of assessment techniques which are used to test geometrical thinking.

2.2 The Mathematical Performance of South African Learners

Howie, (2004), Huntley, (2009), Kotze, (2007) and Roux (2005), among others, have documented the generally poor performance in Mathematics by South African learners. Howie (2004) analyzed the results obtained by South African school children who participated in the Repeat of the Third International Mathematics and Science Study in 1999 (TIMSS'99). She found that South African Grade 7, 8 and 12 learners performed very poorly in comparison to the other countries that participated in the study. Grade 8 learners in the Eastern Cape achieved a mean score of 256 out of a possible score of 800. The international average was 500 (Howie, 2004). The study included many African countries and other developing nations but the "South African learners performed significantly worse than those from all other participating countries in TIMMS-R including other developing countries." (Howie, 2004:159) Many of the South African learners were not English first language speakers and the test was in English. However, Howie (2004) noted that the learners in many of the developing countries were also not English first language speakers and yet their performance in the Mathematics test was better than the performance of the South African learners. Howie went on to say that:

> "This issue needs to be explored further as it appears from the data that the pupils from other developing countries do not seem to be equally disadvantaged by writing tests in their second or third language in mathematics or science. Important lessons may lie in the answers for South Africa."
>
> (Howie, 2004:156)

Thus Howie suggested that the reason for the poor performance of South African learners in Mathematics may not necessarily be a language issue alone and that there may be other factors involved which warrant further research.

Kotze and Strauss (2007), using the data obtained from the international research project undertaken by the Southern African Consortium for Monitoring Educational Quality (SACMEQ II), investigated the performance of Grade 6 learners in Mathematics in South Africa, They categorized the learners into 8 levels on the basis of their Mathematical skills and found that 7.8 % of Grade 6 learners in South Africa were only at the most basic level, the level where they could perform simple addition. This level is termed "beginning numeracy". Alarmingly, only 9.2 % of Grade 6 learners countrywide had achieved a competence level regarded as mathematically skilled, or better. Mathematically skilled learners are able to apply specific knowledge of mathematics in a range of problems involving number, measurement and geometrical shapes (Kotze and Strauss, 2007). An example used in the Grade 6 assessment, which was problematic for the majority of learners, required the knowledge of the properties of two-dimensional geometrical figures and their areas. According to the RNCS (South Africa, 2002), in the Learning Outcome "Space and Shape", all Grade 6 learners should be able to classify triangles on the basis of the length of the sides and the angle sizes. They should also know the similarities and differences between squares, rectangles and parallelograms. The Learning Outcome "Measurement" requires that Grade 6 learners should be able to calculate "perimeter using rulers or measuring tapes ... and area of polygons (using square grids) in order to develop rules for calculating the areas of squares and rectangles" (RNCS, 2002:14). The poor results achieved by South African Grade 6 learners in SACMEQ II are an indication of their poor performance in Geometry and other areas of Mathematics.

When Kotze and Strauss (2007) analyzed the data obtained from SACMEQ II, they found that only 2.1% of South African Grade 6 learners were competent at problem solving and only 1.3% of all the Grade 6 learners in South Africa were adept at abstract problem solving. Problem solving and abstract problem solving are necessary in Geometry but can also be developed during the study of Geometry. Van der Walle (2004) said this about the relationship between Geometry and problem solving:

> "Geometric explorations can develop problem solving skills. Spatial reasoning is an important form of problem solving and problem solving is one of the major reasons for studying mathematics."
>
> Van der Walle (2004:309)

Van der Walle (2004) and van Hiele (1986) suggested that the reasoning skills developed during the study of Geometry are able to be transferred to other areas of Mathematics. Katagiri (2004) argued below that mathematical reasoning skills are valuable life skills and are the most important component of children's education.

> "The most important ability that children need to gain at present and in the future, as society, science and technology advance dramatically, is not the ability to correctly and quickly execute predetermined tasks and commands, but rather the ability to determine for themselves what they should do, or what they should charge themselves with doing."
>
> (Katagiri, 2004:3)

He went on to say:

> "The most important ability that arithmetic and mathematics" (and here he includes Geometry) "courses need to cultivate in order to instill in students this ability to think and make judgments independently is mathematical thinking. This is why cultivation of this 'mathematical thinking' has been an objective of arithmetic and mathematics courses in Japan since the year 1950."
>
> (Katagiri, 2004:3)

Katagiri (2004) suggested that the mathematical thinking skills that are developed by learners at school will help them to think independently and to make decisions, both of which are important skills in adult life. It stands to reason that, if the development of mathematical (and geometrical) thinking is a critical component of children's education, then the past Mathematics results obtained by South African children on international tests should be of concern to teachers.

2.3 Geometrical Reasoning and Concept Development

2.3.1 Mathematical Thinking

According to van der Walle (2004), Mathematics is an organised field of knowledge, which focuses on patterns and relationships and has its own language, consisting of a precise mathematical vocabulary and symbols. Katagiri (2004) suggested that the development of mathematical thinking is the most important function of Mathematics courses. He stated:

> Mathematical thinking is even more important than knowledge and skill because it enables (one) to drive the necessary knowledge and skill.
>
> (Katagiri, 2004:5)

The skills and knowledge that learners might learn in a Geometry class include using a protractor to measure angles or learning that the angles inside a triangle add up to 180°. It is important for children to acquire these skills and knowledge but Katagiri (2004) also suggested that the Mathematical skills and knowledge that children acquire at school are not the sole purpose of teaching children Mathematics. The ability to make decisions in order to get meaningful solutions to Mathematical problems should be the focus of Mathematical teachers.

In Geometry, which is one branch of Mathematics, the focus should not be on rote learning of definitions but on developing an understanding of relationships between shapes and being able to make logical deductions about the properties of shapes. Hans Freuthenthal (as cited in de Villiers, 1995) had this to say about the practice of reducing Geometry teaching to the learning of definitions:

> Good geometry instruction can mean much – learning to organize a subject matter and learning what is organizing, learning to conceptualize and what is conceptualizing, learning to define and what is a definition. It means leading some pupils to understand why some organization, some concept, some definition is better than another. Traditional instruction is different. Rather than giving the child the opportunity to organize spatial experiences, the subject matter is offered as a preorganized structure
>
> (Freudenthal as cited in De Villiers, 1995:3)

From this we can deduce that in Freudenthal's opinion (De Villiers, 1995) at least, Geometry teachers should focus on assisting learners to develop an understanding of relationships between for example, geometric shapes and being able to classify and make logical deductions about the properties of these shapes.

Saul (2001) conducted a Mathematics workshop with Western Cape teachers in Stellenbosch and noted that these teachers were the products of Bantu Education, a system of education for black learners during the Apartheid era. He found that they had been taught to be more concerned with mastering the mechanics of Mathematics than developing insight into why they should use the methods they do. In his words:

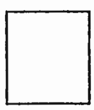
> "Rather, they seemed to look on mathematics as a body of knowledge one rehearsed and performed... And they could think: with varying degrees of success, they could solve the problems I set them. But somehow the notion that the solving of problems actually was the mathematics did not quite click."
>
> (Saul, 2001:3)

This may be the case with many teachers in Mathematics in general and in Geometry in particular. Saul suggested that it is the actual process of working through problems to arrive at feasible solutions that is the essence of mathematical thinking. His statement above reinforces De Villiers' view (1995) that Mathematics teachers need to teach for understanding rather than just focusing on definitions or skills development. When children are given the opportunity to think about how to solve problems, and given the opportunity to discuss their ideas with others, they learn how to think for themselves.

The process of learning how to think mathematically to solve problems should be part of children's education from their earliest years at school. Bell (2009) suggested that the early years of educating a child are the most important and that, "...any schooling beyond the age of twelve is remedial" (Bell, 2009:17). Children in the Foundation and Intermediate phases at schools in South Africa should have the opportunity to have as many experiences as possible with shapes in order to help them develop their ideas about shapes. By the end of Grade 6, the child who does not have a good understanding of what parallel lines are, or the similarities and differences between rectangles and parallelograms, is going to find that there is very little or no time in the higher grades to play with shapes to discover their properties. Learners may be forced to resort to learning definitions by heart in order to cope with the demands of the Geometry curriculum in Grades 7 to 12.

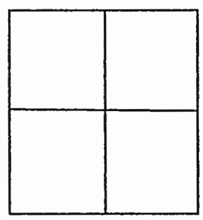
Geometry is a component of Mathematics which concentrates on Space and Shape (RNCS, South Africa, 2002). However, it is argued (Katagiri, 2004; van Hiele, 1986) that the deductive and inductive reasoning skills developed during the study of Geometry are able to be transferred to other areas of Mathematics. A simple example below, which was discussed by van Hiele (1986), demonstrates the validity of this argument.

There is a direct correlation between the area of a square and square numbers – the so-called perfect squares - such as 1, 4, 9, and 16. A square is a geometrical figure with all four sides equal in length. The area of a square is obtained by multiplying the length of the shape by the breadth. Thus, in a square with sides of length 1 unit, the area will be 1 X 1 =1. The area of a square with sides of length 2 units will be 2 X 2 = 4. If the sides have length 3 units, the area will be 3 X 3 = 9 and a square with sides of length 4 units will have an area of 4 X 4 = 16. The numbers obtained this way are called square numbers. This is illustrated in the diagram below:



Area = 1 X 1

= 1



Area = 2 X 2

= 4

Figure 2.1 Relationship between the area of a square and the so-called square numbers.

In algebra, children learn that 1 squared is 1, 2 squared is 4, 3 squared is 9 and 4 squared is 16. A learner who has a good understanding of squares and the calculation of their area in Geometry will be able to use that knowledge when he studies square numbers in Algebra.

From the above example, it can be seen that learners' experiences of two-dimensional Geometry can be reduced to the rote-learning of facts and the un-informed practice of skills: all-too-often, these learners do not know why they are being told to use particular facts, or how to apply new skills (De Villiers, 1995; Katagiri, 2004). This led Katagiri to recommend that teachers should strive to develop mathematical thinking which "... acts as the guiding force that elicits knowledge and skills" (Katagiri, 2004:11). His view was that a learner who can think mathematically is able to use the skills and knowledge that he has to solve new problems in Mathematics.

2.3.2 Geometrical Reasoning

In the Intermediate and Senior Phases of the General Education and Training Band (GET) of schooling in South Africa, all learners are required to study Mathematics. There are five Learning Outcomes for Mathematics in the GET band. Two of them, namely Learning Outcomes 3 and 4, deal with geometrical figures and their measurement (RNCS, South Africa, 2002). Learning Outcome 3 focuses on Space and Shape (Geometry) and Learning Outcome 4 addresses Measurement. In both Learning Outcomes, learners are required to work with two-dimensional shapes.

The Assessment Standards for Learning Outcome 3 state that, by the end of Grade 6, learners should to be able to describe and classify two-dimensional shapes in terms of properties including length of sides and angle size of corners. Learners need to be able to solve problems involving straight lines and triangles and to use transformations and symmetry to investigate the properties of geometrical shapes. Learning Outcome 4 requires learners to be able to calculate the perimeter and area of two-dimensional shapes and to be able to describe angles in terms of right, acute and obtuse angles. (RNCS, South Africa, 2002).

Learners in Geometry lessons, however, are required to acquire more than knowledge and skills. They also need to develop geometrical reasoning. The Revised National Curriculum Statement (RNCS, South Africa, 2002) identifies the type of reasoning patterns required for the learning and teaching of Mathematics. These include the ability to pose and solve problems, to investigate the properties of geometrical figures and reasoning and communication (Kotze: 2007).

Lauf (2004:23) suggested that, "Geometric reasoning is not only about proving results but it is also much wider" as it requires the formulation of a conjecture, the extensive empirical testing of the conjecture, using concrete examples and then finally, verifying, with an argument, called a proof, that the conjecture is true. An example would be the statement that the sum of the interior angles in a triangle is 180°. This claim can be explored by drawing numerous triangles and, by using a protractor, measuring the interior angles to confirm that their sum is, indeed, 180°. In order to be certain that this conclusion is true for every triangle, a deductive proof, requiring a logical chain of reasoning, would be required. A learner who is able to formulate or understand such a process of deductive thought (usually at high school) is considered to be capable of geometrical reasoning.

2.3.3 Concepts in Geometry

The Reader's Digest Universal Dictionary (1987:330) defines a concept as "a general idea or understanding, especially one derived from specific instances or occurrences, a thought or notion, especially one that is abstract or theoretical, a way of thinking about something". Farrell and Farmer (1980:77) described a concept as being "a classification of objects, object properties or events into a set by the process of abstraction". By way of example, long before children are given formal instruction in school about shapes such as triangles, they may have begun to classify triangles as having different properties from rectangles. They do this by playing with plastic shapes of triangles, observing pictures of triangles on posters and in books and discussing triangle shapes found in structures such as road signs. At the same time, these children could be doing the same type of informal investigations with rectangles. The literature indicates that it is vital for children to be exposed to as many opportunities as possible to interact with shapes in order to develop their spatial sense. (Duckworth, 1996; Mussen, Conger, Kagan and Huston, 1984; van der Walle, 2004; van Hiele, 1999)

Katagiri (2004:10) suggested that, "Concepts are made up of both connotative and denotative components." The denotative component is the symbol or name that is given to an object such as a Geometrical shape. The connotative component is the meaning that is implied in the name or symbol that goes beyond its literal meaning. Katagiri (2004) gave the following explanation to illustrate the connotative and denotative components of concepts in Geometry:

> Showing students a round top and telling them 'this is called a circle' is not enough when it comes to teaching the concept of the circle. Since the top will have properties such as material, size, a painted pattern and a method of use, the students will not yet ignore those aspects and may think of, for instance, a round wooden top as a circle. The other properties must be ignored. Instead, show students tops of various different sizes, and have them consider various other circular objects including cups, to elicit commonalities such as 'all of these shapes have the same length from one point (the central point) to the edge' "
>
> "Abstract thinking is used to clarify shared properties here. These abstracted properties are referred to as the concept's connotation."
>
> "The thinking method of concretization is used at this time to gather many different concrete examples, and to clarify the denotation or extension of the concept.
>
> (Katagiri, 2006:23)

From the above explanation, it is important for children who are developing an understanding of new Geometry concepts, specifically shapes, to be given many concrete examples of shapes to investigate. They also need opportunities to talk about their developing understanding of shapes with others. Wellington and Osborne (2001) said this about the need for children to talk about new concepts:

> "Pupils in the classroom are very much in the process of groping for new words and new meanings, endeavoring to construct new concepts and make sense of a new language. Confidence in its use, and the concepts that it represents, comes above all else, with practice."
>
> Wellington and Osborne (2001:123)

Although Wellington *et al.* (2001) were specifically writing about the development of concepts and new meanings for words in Science, there is a similar need for children to talk about new Geometry concepts in class, both to their teachers and among themselves. However, Geometry involves more than an understanding of concepts. It requires inductive and deductive reasoning skills. The next section explores three theories on how these reasoning skills develop in Geometry.

2.4 Theories on the development of geometrical understanding in children

A review of the literature regarding cognitive development in children revealed contrasting ideas about how a child's spatial or Geometric thinking skills develop. The review indicated that Piaget's Constructivist theory (Farrell and Farmer, 1980), van Hiele's theory of levels of geometrical understanding (van Hiele, 1984; van Hiele, 1986; van Hiele, 1999) and Gagne's instructional theory (Gagne, 1988) seem to be the main focus of how children acquire geometrical insight. Each theory is discussed below.

2.4.1 Piaget's Constructivist Theory

Jean Piaget (1896-1980), a developmental psychologist, became interested in the thinking that lay behind children's answers to questions, particularly their incorrect answers. Piaget's research with children led to the formulation of his theory of cognitive development during the 1920's and 1930's (Mussen, Conger, Kagan and Huston, 1984). A central component of Piaget's cognitive development theory is that all children move through a fixed sequence of successive stages as they mature (Farrell and Farmer, 1980). Piaget asserted that learners construct their own knowledge rather than being passive recipients of what is taught. Farrell and Farmer stated that:

> "Piaget holds that individual intelligence develops through the person's *interaction* with his or her environment. Piaget insists that knowledge is active. For him, to *know* an idea or object requires that the student manipulate it physically or mentally and thereby transform it."

(Farrell and Farmer, 1980:55)

The implication of Piaget's theory for Geometry teachers is that their learners need to be actively involved in their own learning.

According to Piaget (Stone and Church, 1979: Farrell and Farmer, 1980) there are qualitatively different intellectual abilities that develop in children in a fixed sequence of stages related to age. Piaget described the following four stages:

The sensori-motor stage occurs between birth and two years of age. During this stage, children use their five senses to explore their world.

The pre-operational stage (usually from two to seven years of age) is the stage when children develop the ability to represent objects with symbols. For example, they begin to associate numbers with the actual quantities the numbers represent, rather than just reciting the names of the numbers.

The concrete operational stage occurs roughly between the ages of 7 and 12 years of age. At this stage, children start to be able to classify and order objects. This is associated with a developing ability to reverse actions mentally.

The Formal operational stage starts at roughly 12-16 years of age. It is characterized by the ability to reason abstractly and to develop hypotheses and use deductive reasoning.

(Farrell and Farmer, 1980:48-64)

The age at which children move into each stage is not fixed (Farrell and Farmer, 1980; Stone and Church, 1979) and is meant to be a guideline approximating the earliest age for which characteristics of the stage begin to be demonstrated by 75% of the children tested. The oldest age in the range is the age at which the characteristics of that stage have been shown to be a stable intellectual characteristic of at least 75 % of the children tested (Farrell and Farmer, 1980). Piaget developed his theory while working with children in Paris and by observing his own children. However, research done among American children showed that they attained some of the characteristics of the formal operational stage of thought up to a year later than their European counterparts (Elkind, 1961). Bruner, Olver and Greenfield (Scribner and Cole, 1978) researched the link between levels of literacy and the attainment of Piaget's stage of formal operational thought among Woloi adolescents in Senegal in West Africa (Scribner *et al.*, 1978). Bruner *et al.* (Farrell and Farmer, 1980) found that, among the children they researched,

> "The sequence of stages remained the same but the first three stages were attained at much later ages and the formal operational stage characteristics were not found to be stabilized in the oldest sample tested (16 and 17 year olds)"

(Bruner *et al.* as cited in Farrell and Farmer, 1980:52).

One of the implications of the above research is that children of the same age may be in different stages of intellectual development. Farrell and Farmer (1980) suggest that the most useful age to serve as an indicator of a child's intellectual ability is the later age in the age range for each stage in Piaget's stage theory. This seems to indicate that the children in Grade 6 in South African schools would largely be in the concrete operational stage of cognitive development.

An important aspect of Piaget's theory of how concepts develop is equilibration (Stone and Church, 1979). Equilibration happens throughout the child's life and is the way that children increase their knowledge and understanding of the world around them. It involves two cognitive processes, the first of which is assimilation. When a child encounters an object for the first time, it is not viewed as something truly novel but as an example of an already known concept. For example, if a young child sees an Alsatian and is told that what he is seeing is a dog, he develops what is termed a "class of notions" of what a dog is. If he sees another type of dog like a Labrador, he is able to say that what he is seeing is a dog too. So he has developed a concept of a dog. Sometimes a small child will see a slightly different, hairy four-legged creature, a cat, and call that a dog too. He is using the mental process that Piaget calls "assimilation" to help him understand this slightly different object in his world. If someone is able to tell the child that what he is seeing is called a cat, he will develop a new category of animals in his mind, called cats. Piaget has called this mental process accommodation. The combination of the two mental processes results in an increase in the child's knowledge and understanding of the world around him (Stone and Church, 1979).

In the Geometry classroom, assimilation and accommodation need to work together to help the child develop geometrical understanding. For example, a learner, who has had the opportunity to develop the concept of what a square is, will think that a rectangle is also a square when he comes across one for the first time because of the cognitive process of assimilation. He will be uncomfortable with this conclusion though, as the rectangle will not fit completely into his concept of what a square is. By being given the opportunity to manipulate rectangular shapes physically, he will begin to realize that they are not squares, and will have to develop a new category of shapes called rectangles. This process of "accommodation" (according to Piaget), will result in the learner restoring his cognitive balance or equilibrium, hence equilibration.

"According to Piaget, the basic unit of knowledge is the schema, a mental picture or idea, ordinarily implicit (though it can be verbalized)..." (Stone and Church, 1979:41). A schema is an idea of what objects or a group of objects are like. Thus, in the example above, the learner might have developed a schema for a square. Equilibration produces changes in schemata. Once the learner has developed the new class of shapes called rectangles, it will help him to have a better understanding of what makes a square a square.

These schemata are a necessary end product of the study of Geometry in the Intermediate Phase at school. Children in the Intermediate Phase of schooling are required to "Recognize, visualize and name two-dimensional shapes" (RNCS, 2002). The majority of learners in the Intermediate Phase are at the stage of concrete operational thought. As mentioned earlier, two of the major descriptors of cognitive development of children at this stage are the abilities to classify and work with relationships where order makes a difference (Farrell and Farmer, 1980). If we consider the requirements of the RNCS (South Africa, 2002) for Grade 6 learners, we can see that there is an emphasis on the ability to classify shapes and to work with the relationships between similar shapes. Learning Outcome 3 sets out the following requirements for two-dimensional shapes. Learners must be able to:

- Describe and classify two-dimensional shapes in terms of the length of the sides and the angle size of corners.

- Use the vocabulary and properties of rotations, reflections and translations to describe the relationships between distinct two-dimensional shapes within patterns (including transformations and symmetry).
- Draw enlargements and reductions of two-dimensional shapes (at least quadrilaterals and triangles) using grid paper to compare their size and shape.
- Recognize and describe natural and cultural two-dimensional shapes and patterns in terms of geometric properties.

(RNCS, 2002)

Thus, it is during this stage of cognitive development that children start to look at, for example, triangular shapes and begin to classify them as isosceles, equilateral or scalene triangles on the basis of their differences, but know that they all belong to the set of triangles.

Piaget's data (Farrell and Farmer, 1980) suggests that the development of these cognitive abilities occurs gradually throughout the concrete operational stage as a result of interaction with sufficient experiences that require multiple classifications. Farrell and Farmer (1980:77) discuss the example of a teacher who might write the definition of a triangle on the board and draw next to it several different triangles. If the teacher were simply to drill his Grade 6 learners to recite the word triangle and the definition of a triangle, the authors suggest that the children would simply have learned the name of the concept and a verbal association describing the concept. The learners would not have developed a deeper understanding of what a triangle is, what its properties are, or how to distinguish a triangle from a figure containing many different shapes.

The implication of Farrell and Farmer's (1980) example is that Geometry teachers need to provide learners with opportunities to play with moulded-plastic shapes. They should give the learners opportunities to cut out triangular shapes in paper and card and encourage learners to measure the lengths of sides and sizes of angles. By being exposed to numerous examples, learners begin to differentiate between the different types of triangles, realizing that some triangles have all three sides of equal length (equilateral triangles), some have only two sides of equal length (isosceles triangles) and some have no sides equal in length (scalene and most right-angled triangles). These types of activities encourage the classification of triangles on the basis of the length of the three sides. Classification based on angles would follow naturally from this.

Piaget's research led to the development of a school of thought called Constructivism among developmental psychologists. Kilpatrick (as cited in Lerman, 1989:211) describes Constructivism thus,

> "Knowledge is actively constructed by the cognizing subject, not passively received from the environment. Coming to know is an adaptive process that organizes one's experiential world. It does not uncover an independent pre-existing world outside the mind of the knower."
>
> (Kilpatrick, 1987, cited in Lerman, 1989:211)

This view suggests that Constructivists would agree that, in Geometry, children need to be given opportunities to interact with various geometrical shapes from an early age to help them construct their own understanding of geometrical concepts. However, these opportunities need to be carefully structured by teachers so that their learners would benefit from their experiences with shapes.

Prawat (1992) warned that some teachers practice what he termed "Naïve Constructivism", where teachers think that their students are able to structure their own learning independently of any guidance from the teacher. He also suggested that some educationists think that merely giving learners activities to do results in learning taking place. He quoted a teacher as saying, "As long as children are active, then learning is going on" (Prawat, 1992:370). Dewey (Dewey as cited in Prawat, 1992) stressed that experiences in the classroom needed to be carefully chosen and structured in order that the learners' subject knowledge and understanding might develop. Thus, a Geometry teacher in the Intermediate Phase should select activities that will help learners to develop an understanding about the properties that classes of shapes have in common. Simply giving the children shapes to play with may not produce the understanding of classes of shapes required by the RNCS (2002).

Duckworth (1996), a Constructivist, who was strongly influenced by Piaget's theory, discussed a simple exercise a child in Grade 6 could do where a rectangle is divided into two triangles by cutting it along the diagonal as shown below:

Figure 2.2: Illustration of cutting a rectangle into two triangles by dividing it along the diagonal.

A Grade 6 child might be asked whether the area of the two triangles is the same as the area of the original rectangle. The child can confirm this by placing the two triangles back together to form the original rectangle. This exercise would show that the total area is independent of shape. Duckworth (1996) concluded that the understanding that the child who has done this exercise will have would be far deeper than someone who has never had the opportunity to do the exercise. It is this deeper understanding that forms part of Mathematical thinking. The implication for teaching children in the concrete operational stage, as children in Grade 6 are, is to provide learners with many opportunities to manipulate geometrical figures and to pose challenging exercises to encourage them to develop a deeper understanding of concepts such as area, perimeter, congruence and similarity..

Prawat (1992) held the view that the opinions that individual teachers have about teaching and learning influence their classroom practice. He suggested that teachers, who view children simply as recipients of information, tend to teach in a traditionalist manner in Geometry, showing their learners methods for doing calculations like the area of a triangle or a rectangle. Teachers who have this view of their learners tend to produce lessons in which, "Students play a relatively passive role" (Cohen as cited in Prawat, 1992:356) and that during such lessons, students are "accumulators of material who listen, read and perform prescribed exercises" (Cohen as cited in Prawat, 1992:356). This method of teaching may assist teachers in covering many aspects of Geometry rapidly in the classroom but the learners will probably have little understanding of the concepts that have been taught.

From the above brief discussion about Piaget's stage theory, it is apparent that Piaget's ideas could have had a profound influence on the manner in which Mathematics is taught by many teachers. Piaget's work also stimulated a great deal of research into how children learn. As Farrell and Farmer (1980) point out:

> "During the twenty years between the two world wars, Piaget's colleagues, along with psychologists in France, Russia and Great Britain, replicated and extended Piaget's research efforts with thousands of children from birth to ages 16 or 17."
>
> (Farrell and Farmer, 1980:49)

One of the educationists who were influenced by Piaget's theories was Pierre van Hiele, who said, "An important part of the roots of my work can be found in the theories of Piaget" (van Hiele, 1986:5). Van Hiele's theory is discussed in detail in the next section.

2.4.2 The Van Hiele Theory on the Development of Geometrical Understanding

Two Dutch educators, Pierre van Hiele and Dina van Hiele-Geldof, developed the van Hiele theory of geometrical thinking between 1955 and 1959. This theory "immediately attracted a lot of attention of the education authorities in the Soviet Union, but for nearly two decades, got little notice in this country (America)" (Hoffer, as cited in van der Walle, 2004:309). During the 1980's, the American education authorities began to realize that, by international standards, American children displayed relatively poor understanding of spatial relationships in Geometry. As a result, the Mathematics curriculum was revised and the van Hiele theory had a major influence on the development of the new Geometry curriculum in America. (van der Walle, 2004). Between 1990 and 1992, the National Education Authority recorded a significant improvement in the geometric reasoning of American children at grades 4, 8 and 12. The conclusion was that this improvement was due to an increased emphasis on Geometry and spatial skills at all grades. (Struchens and Blume as cited in van der Walle, 2004: 308)

Van Hiele (1986) stated that Piaget's ideas about how cognitive development occurs in children influenced him and his wife during the development of their theory of how children acquire geometrical understanding. However, there were important differences between the van Hiele theory and Piaget's stage theory. These differences are outlined below.

- The two theories differ on how children develop spatial sense. Piaget argued that the development of a learner's concept of space is part of the maturation process. A child in Piaget's pre-operational stage has no concept of conservation of volume but develops this understanding with exposure to practical experiences with different shaped containers filled with the same volume of liquid (Farrell and Farmer, 1980). Similarly, according to Piaget's Constructivist theory, children exposed to the correct choice of activities will develop spatial and geometrical understanding as they mature. The van Hiele model, on the other hand suggests different levels of geometrical thinking which depend on the correct experiences while learning and carefully selected instructional techniques. (van Hiele, 1986)

- The van Hiele stages are not age dependent as are the Piagetian stages of cognitive development (van Hiele, 1986).
- Piaget did not emphasize the importance of language in helping children to move from one level to the next, whereas in the van Hiele model, language is essential for the development of a child's geometrical understanding (van Hiele, 1986).

Van Hiele (1986) claimed that, by reaching a higher level of geometrical thinking, the potential of a child will have been increased and so it is unlikely that a learner will fall back to a lower level, even though subject matter could be forgotten (van Hiele, 1955 cited in Van Hiele, 1986). Piaget stated that a child in the formal operational stage of thinking:

> "... has the capacity to use formal operations but is not compelled to do so. He or she may revert to any of the earlier modes of thinking ... for earlier stages are not eradicated but integrated into later stages."
>
> (Farrell and Farmer, 1980:61)

This would imply that a child in the formal operational stage, who is presented with a new problem to solve, may revert to using concrete examples to help him decide on a solution to the problem.

The van Hiele theory has generated much research and has influenced the development of Geometry curricula in countries such as Russia, the Netherlands and America (De Villiers and Njisane, 1987; Feza and Webb, 2005; Kotze, 2007; van der Walle, 2004; Way, 2007, University of Chicago, 2009). Although some questions have been raised with regard to certain aspects of the theory and the original five levels (0 to 4) were adapted and renamed (1 to 5) in 1981 (van Hiele, 1986), contemporary views support the van Hiele model (Feza and Webb, 2005; Kotze, 2007; Roux, 2005; van der Walle, 2004; Way, 2007; Wu and Ma, 2005.)

Some research has indicated that not all people use a single van Hiele level of reasoning all the time. While interviewing Grade 9 learners for MALATI (Mathematics and Teaching Initiative at Stellenbosch) Bennie (1999) found that the learners interviewed did not operate at a single van Hiele level of reasoning but seemed to use several levels at the same time. Clements and Battista (2001) challenged the van Hiele notion that learners' geometrical understanding develops in a simple linear manner.

> "Conceptualizing geometric growth as being strictly visual, then strictly descriptive/analytic, then logical and so on is neither accurate nor optimal for educational theory or practice"
>
> (Clements and Battista, 2001:129)

This seems to contradict van Hieles' (1986) claim that, once a child has reached a higher level of geometrical thinking, it is unlikely that a learner will fall back to a lower level, Other researchers appear to have obtained results that support Clements and Battista's (2001) view.

Kotze (2007) found in her research into the geometrical reasoning of 290 grade 10 learners and 29 Grade 10 teachers that there was not a gradual development of geometrical understanding from Levels 1 to 4. She found that learners operated at the visual level (Level 1) for certain tasks and at Level 4 (the deductive level) for other tasks. Bennie (1999) found that the Grade 9 learners she interviewed were operating at Level 2 at times and at other times were operating at a "pre-recognition" level, a level preceding Level 1, the visual level. Bennie suggested that the learners she interviewed had memorized what van Hiele called "structures" (1986:42). In Grade 8 and 9 Geometry, children can learn the structure that alternate angles form a Z shape. This type of structure can assist a learner to perform simple calculations but, because it avoids what van Hiele calls a "crisis in thinking" (1986:43), it does not help him to progress to a higher level of thinking. It might be that the apparent oscillation between levels is simply the result of children memorizing structures that enable them to perform simple tasks at the next level of geometrical thinking, although they have not yet progressed into that stage. When learners are making the transition from one level to the next, they will exhibit the type of thinking of each of the levels in different situations.

The van Hiele theory postulated that there were five levels in the development of children's understanding of spatial concepts in geometry. (van Hiele, 1986). Originally, the levels were named Level 0 to 4 by van Hiele in 1959. However, van Hiele later revised the names for the levels calling the lowest level, **Level 1** and the subsequent levels **2, 3, 4** and **5** (van Hiele 1986). For clarity, van Hiele's later nomenclature, Level 1 to 5 is used throughout this thesis.

Clements and Battista (1990:354) suggest another level of geometric thinking, which precedes van Hiele's Level 1. They call this level "pre-recognition". At this level, children attend to only one visual aspect of a shape. For example, children at this stage might be able to separate triangles and quadrilaterals but not be able distinguish different types of triangle or quadrilateral. These children would focus only on the number of sides of the shapes and not on any other features that the shapes might have in common.

The van Hiele levels are sequential in that the '*products of thought*' at each level become the '*objects of thought*' for the subsequent level (van der Walle, 2004). For example, the products of thought at the end of the first level, **Level 1**, are classes or groupings of shapes that appear to be alike. These groups of shapes become the objects of thought at **Level 2** (the second level) where the child begins to think about the properties of shapes.

In order to progress through the levels, children have to have access to activities which enable them to explore spatial concepts. Children, at a particular level, need continuous opportunities to interact with activities designed to help them consolidate ideas at that level. At the same time, they need opportunities to explore and talk about geometric activities from the next level of development. This gives them the best opportunity to advance to the next level. (van der Walle, 2004: 347-348).

2.4.2.1 The Van Hiele Levels

Van Hiele (1986) described five levels of geometrical thinking, of which only the first three are relevant for learners in the Intermediate Phase of schooling, which is the focus of this study (Feza and Webb, 2005; van Hiele, 1999; Way, 2007).

Level 1 – The level of Visualisation

When van Hiele first described the levels in 1957, he omitted this level. It was only a few years later that he realized he needed to include it (van Hiele, 1986) and so it was called Level 0 until 1981. At that time, it assumed its new title, **Level 1**. At this level, learners recognize a shape by using global, visual characteristics of the shape, rather than by distinguishing parts. It is non–verbal thinking, similar to the way in which some children recognize single words before they know the individual letters. A child at this level might say “it is a square because it looks like one” or “it looks like a box” (van Hiele, 1999). At this stage, appearance is everything and, if for example a square is rotated through 45°, then it is no longer regarded as a square because it looks different (van der Walle, 2004).

Level 2 - The Descriptive level

This is the level at which “...conceptions are pointed out by their mathematical names and are put in formal connections” (van Baalen as cited in van Hiele, 1986) At this level, learners cannot understand any reasoning involving properties of figures (van der Walle, 2004). Learners in this stage are able to identify and describe the component parts and properties of shapes. At this stage, learners need to develop the appropriate language to describe the properties of shapes (van Hiele, 1986). Learners are able to focus on a class of shapes and think about what makes a rectangle, a rectangle. If a square is rotated through 45°, it is still regarded as a square because it has the properties of a square. Ideas about shapes can be generalized to all shapes that fit that class. For example, an equilateral triangle is simply perceived to be a triangle as it has three sides. However, the relationship between three equal sides and three equal angles is not yet realised (van der Walle, 2004; Way, 2007).

Level 3 - The level of Informal Deduction

According to van Hiele (1986), this level is reached when learners are able to operate with known relations of shapes with which they are familiar (van Hiele, 1986). Learners at this level are also able to apply the properties of the shapes. Van Hiele (1999) calls this the level of 'Informal Deduction'. At this level, learners are able to apply what they already know to explain the relationships between shapes. They start to develop an ability to engage in "if – then" reasoning which enables them to explain why all squares must also be rectangles (van Hiele, 1999). These learners can reason that if all squares have their opposite sides equal and all four angles equal, they must all be rectangles, as rectangles also have opposite sides equal and all four angles equal 90°. However, squares must be a special type of rectangle as all the sides in a square are equal, not just the opposite ones.

Level 4 – The level of Deduction

This is the level where the structure of a system with axioms, definitions, postulates and theorems begins to have meaning for learners (van Hiele, 1986). In order to be said to be functioning at this level, learners need to demonstrate an ability to prove geometrical properties through a series of deductive arguments. An example of this would be the ability to prove the diagonals of a rectangle bisect each other with a series of deductive arguments rather than just looking at the diagonals in a few rectangles and measuring their lengths. This sort of reasoning is beyond the requirements of the RNCS syllabus (South Africa, 2002) for learners in the Intermediate Phase of school and would be suited to the Euclidean Geometry component of Mathematics Paper 3 (NCS, South Africa, 2007) examined at the end of Grade 12.

Level 5 - The level of Rigour

This is the highest level of the van Hiele hierarchy. It is connected with the possibility of comparing, transposing and operating with the relations between shapes (van Hiele, 1986) and there is an appreciation of the distinctions and relationships between different axiomatic systems. Geometric studies at this level require learners to solve problems that require extensions of known theorems and indirect proofs. According to van Hiele (1986), it is also the level a learner should have attained to study Mathematics at tertiary institutions.

However, van Hiele warned, "You see that we did not try to describe levels higher than the fourth (fifth)" (van Hiele, 1986:47). Van Hiele saw no point in trying to search for higher levels of geometrical understanding and asked, "And what are we to do with such levels?" (van Hiele, 1986:47). He was concerned that too high a value might be placed on the attainment of levels that were higher than level 5 and that research into these higher levels might occur. Van Hiele said:

> "Some people are now testing students to see if they have attained the fifth or higher levels. I think this is only of theoretical value. I think that for years and years, very bad instruction has been given in mathematics. But that was because of neglect of levels 2, 3 and 4 – on the fifth and higher levels the mischief has already done its work. So I am unhappy if, on the ground

> of my levels of thinking, investigations are being made to establish the existence of fifth or higher levels. This is a method to realize one's theoretical lusts, but I would much prefer that a beginning be made on the improvement of education with the aid of the levels of thinking."
>
> (Van Hiele, 1986:47)

Thus van Hiele emphasized that the purpose of the van Hieles' research into the levels of geometrical thinking was to improve classroom practice. Van Hiele had been motivated to research geometrical understanding because of his experience as a Mathematics teacher, teaching Geometry to children at school. In his words, "There were parts of the subject matter that I could explain and explain, and still the pupils would not understand." (van Hiele, 1986:39) Van Hiele wanted to help Geometry teachers to develop geometrical thinking in their learners. The van Hiele theory includes more than the description of the type of thinking that occurs in each level, it also describes how teachers can help their learners to progress from one level of geometrical thinking to the next.

2.4.2.2 Progress through the Van Hiele levels

Van Hiele (1984) described how progress from one level to the next would occur. In the words of Pierre van Hiele, in his doctoral dissertation:

> "The maturation that leads to a higher level happens in a special way. Several stages can be revealed in it. (This process must be considered above all a process of apprenticeship and not just as a ripening of a biological sort)"
>
> (van Hiele, 1984:246-247)

Van Hiele emphasized that the role of the Geometry teacher is to provide the type of instruction that will facilitate his learners' progress to the next level of geometrical thinking. Children do not automatically acquire this understanding as they mature but require a particular sequence of instruction to help them develop to a higher level. The van Hieles' termed the different stages in this process "phases". In the words of van

Hiele, there are five "phases which in the process of apprenticeship lead to a higher level of thought." (van Hiele, 1984:247). The phases were described in the doctoral dissertation of Dina van Hiele-Geldof (1984) and are briefly summarized below:

> **Information.** This is the first phase during which the learner is given examples of a particular shape, for example, to manipulate so that he can gather information about the shape. The teacher's role during this phase is to establish what the learners know about the shape.
>
> **Guided /directed orientation**. The learner explores the object by measuring it or folding it down the axis of symmetry for example. The teacher directs the activities so that the learner may develop what van Hiele-Geldof describes as "more conscious perception in a geometric sense" (van Hiele-Geldof, 1984:218). This means that the learner begins to identify properties of the shape.
>
> **Explanation/ explication.** The learner learns the language necessary to express what he has found out about the shape.
>
> **Free orientation**. The learner is given more complex tasks to help him find out as much as he can about the object he is investigating. He may begin to see that a square is a rhombus because a square has all the properties of a rhombus. The learner begins to recognize shapes as a result of their properties.
>
> **Integration.** The learner summarizes what he has learned about the shape and is able to operate with the shape as a totality of properties.
>
> (Dina van Hiele-Geldof, 1984)

Before a Geometry teacher can begin the above sequence of phases, it is important for him to know which level his learners have reached. Van Hiele-Geldof (1984) stressed that teachers need to start instruction at the child's level. Van Hiele-Geldof said:

> It is wrong for a teacher to present subject matter as a completely finished entity, to point out what paths to follow or to explain methods which the

children can develop for themselves. Such instruction is devoid of the attractiveness of finding things out for oneself. It also lacks the satisfaction that accompanies discovery.

(Van Hiele-Geldof, 1984:48)

Typically, at the beginning of each new grade, teachers should perform some type of diagnostic assessment to establish the level of geometrical reasoning of their learners. As van der Walle (2004:311) said "A developmental approach to instruction demands that we listen to children and begin where we find them." Lack of diagnosis of the level of geometrical understanding of learners may result in learners memorizing definitions without any real understanding of geometric concepts (De Villiers, 1995). As a consequence, these learners may subsequently be unable to solve geometrical problems and may not progress to the higher van Hiele levels. This may eventually lead to failure in Geometry at the higher grades as a good understanding of geometric concepts is required to cope with making deductions about relationships between figures (van der Walle, 2004). In summary, the progression of a learner from one level of geometrical understanding to the next is largely dependent on the type of instruction provided by the teacher.

2.4.3 Gagne's Theory of Instruction

To conclude the discussion on different theories on how children acquire geometrical understanding, I will consider Gagne's Instructional theory below. Gagne's theory has been used in a variety of instructional situations. It has been used by the armed forces for military training (Gagne, 1962) and by teachers in the classroom (Gagne, 1988). Although, Gagne's theory does not deal specifically with the development of geometric concepts, it has been applied by Geometry teachers, particularly those who use computers to facilitate their lessons (Gagne, 1988).

During the 1960's, Robert Gagne, an educational psychologist, (1916-2002), proposed an alternative to Piaget's theory (Farrell and Farmer, 1980). He did his research with human subjects in real classroom situations and classified all human learning into nine major types, which are hierarchical. Farrell and Farmer (1980), have the following to say about Gagne's hierarchy:

> "The hierarchy ... contains categories ranging from simple to complex, with each successive layer depending upon and subsuming those directly under it."
>
> (Farrell and Farmer 1980:131)

Thus Gagne's theory suggests that there are lower order skills which have to be mastered before skills that are higher up the hierarchy can be attempted. Gagne's original instructional theory was modified and adapted for Instructional Technology (Gagne 1988). This involves the use of computers to help children acquire concepts in areas such as Geometry.

A brief summary of each of Gagne's "Instructional Events" (Gagne, 1988) is given in Table 2.1 with a description of how it could be used in computer-aided instruction on the concept of equilateral triangles, for example. This sequence of instruction could also be used in a Geometry lesson without the use of a computer. The important feature of the sequence of instructional events is that the lower order skills should be practised before more complex skills are introduced to the learners. Gagne (1988) stated that, before any instruction begins, the "prerequisite skills must be retrieved so that they are in the forefront of memory, in other words, prominently attended to in working memory" (Gagne, 1988:114).

Type	Description	Action
1	Gain attention	The learner is shown a variety of computer generated triangles (or physical models of triangles in an ordinary classroom).
2	Identify the objectives	The question is posed: "What is an equilateral triangle?"
3	Recall prior learning	The definition of a triangle is reviewed.
4	Present structures	A definition of an equilateral triangle is given.
5	Guided learning	The learner is shown examples of how to create equilateral triangles on the computer. This is termed "organizing and elaborating the content" by Gagne (1988:115). In an ordinary classroom, the learners could be shown how to construct an equilateral triangle using a protractor and ruler.
6	Elicit performance	The learner is asked to create five different examples of equilateral triangles on the computer (or to construct them on paper using a protractor and ruler).
7	Provide feedback	The examples the learner has produced are checked on the computer as correct or incorrect (or the teacher could check them manually).
8	Assess performance	The computer (or teacher) provides scores and remediation if necessary.
9	Enhance retention/transfer	The learner is shown pictures of different shapes and asked to identify the equilateral triangles. (Gagne, 1988)

Table 2.1 The different types of instructional events in Gagne's theory (modified from Farrell and Farmer, 1980: 131-138)

An important component of Gagne's Instructional theory is what he termed "skill automaticity" (Gagne, 1988:123). He suggested that intellectual skills that are practised many times start to be performed automatically and require little conscious attention. This frees up the attention for other tasks that require problem-solving. He claimed:

> "The *principal factor* affecting the development of higher level thinking in learners is the release of attention by the automization of basic skills."
>
> (Gagne, 1988:123)

Gagne suggested that Geometry teachers need to give their learners the opportunity to practise Geometry skills repeatedly so that they become automatic. In Grade 6, these skills might be constructing angles of a particular size or line segments of a particular length and then using these skills to construct a square or a triangle. Gagne claimed that the mastery of these skills would free up the attention of the children to develop higher-level thinking in Geometry.

2.5 Research on the Development of Geometrical Thinking

I have decided to focus on the van Hiele theory for the remainder of this literature review as it has been the theory which has influenced the development of Geometry curricula in countries such as Russia, the Netherlands and America (De Villiers and Njisane, 1987; Feza and Webb, 2005; Kotze, 2007; van der Walle, 2004; Way, 2007, University of Chicago, 2009).

The van Hiele theory has generated a great deal of interest in the education world and stimulated numerous research studies. In this section, I will look at the studies done in the international arena and then will focus on South African projects which have been influenced by the van Hiele theory. The research studies that have been done, using the van Hiele theory as a framework, fall into two categories depending on the subjects of their research. The international studies I discuss involve:

- Learners at primary school.
- Student teachers.

The South African studies involve:

- Learners at primary school.
- Teachers and student teachers.

2.5.1 International Research

2.5.1.1 Learners at primary school

Usiskin (1982), working at the University of Chicago, researched the level of geometrical understanding (according to the van Hiele hierarchy) that learners had attained by the end of their primary schooling. He found that many of these learners had not attained a level of understanding (Level 3) that would enable them to perform the deductive reasoning required for secondary school Geometry.

This finding was replicated by Halat (2007), who tested the level of geometrical understanding of 273 Grade 6 learners at two government middle schools in North Florida, USA. He used the Van Hiele Geometry Test developed by Usiskin (1982) in the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project at the University of Chicago. Halat found that none of the students in the study had progressed beyond Level 2 (the descriptive level) on the van Hiele hierarchy. Most students were at Level 1 and some had not yet attained Level 1 and were at the level termed "pre-recognition" by Clements and Battista (Halat, 2007:354). This is the level where children attend to only one visual aspect of a shape such as the number of sides. They would not be able to distinguish between a square and a rectangle, for example, classifying them both as four-sided shapes (Halat, 2007).

In the research study described above, Halat (2007) wanted to find out whether there would be an improvement in geometrical understanding among Grade 6 learners taught Geometry at a school that used a "reform-based" curriculum designed on the van Hiele theory. As a control group, he chose Grade 6 learners who had been taught Geometry using conventional methods at another school in Florida. After five weeks he re-tested the children to determine whether the type of instruction they had received had resulted in an improvement in the level of understanding of the children in both groups. He found that the level of geometrical understanding had improved in both the control and the experimental groups. He noted that the implementation of the van Hiele theory-based materials did have a positive impact on the Geometry understanding of the Grade 6 children but it was not as marked as he had expected. He did suggest that five weeks was not long enough for the van Hiele-based materials to have an effect. When Dina van Hiele (1984) did her research with primary school children, using van Hiele-based instruction, she worked for a six month semester. At the end of this period of time, she noted a marked improvement in the level of understanding of her subjects (van Hiele-Geldof, 1984).

During another study, a case study involving six 12- to 13- year olds in Form 1 at a school in Singapore (this is the age of the learners in Grade 6 in South African schools), Meng (2009) found that one of the learners was operating at van Hiele's Level 1, the visual level, two were at Level 2 and the rest were at Level 2, the level of informal deduction according to the van Hiele hierarchy. Meng was researching the effect of using instruction that was modelled on the van Hiele "phase-based" instruction (van Hiele, 1984) to help children improve their level of geometrical reasoning. (This instructional technique has been discussed in section 2.4.2.2). She found that, "after phase-based instruction with Geometer's Sketchpad, a computer program, the van Hiele levels (of all the children) either increased or remained the same" (Meng, 2009:89) She concluded that in her sample, the combination of well-designed instructional activities coupled with appropriate guidance from the teacher resulted in all six of the learners either progressing to Level 2 or remaining at Level 2, if they had started at Level 2.

Some researchers have been interested in the effect of different instructional programs on helping primary school children to progress to higher levels of geometrical understanding (Bennie, 1999; Davis and Hyun, 2005; Halat, 2007). It was van Hiele himself who said;

> "The transition from one level to the following is not a natural process; it takes place under the influence of a teaching-learning program."
>
> (van Hiele, 1986:50)

Van Hiele (1984) discussed the importance of the five phases such a teaching-learning program should have in order for it to help children attain the next level of understanding. These five phases have been discussed in Section 2.4.2.2.

Davis and Hyun (2005) researched the effect of a well-structured learning and teaching program on the development of higher levels of spatial and geometrical understanding in children. They tested the spatial and mapping skills of 18 culturally and linguistically diverse 5- to 6- year olds at a school in Ohio, USA. These children were exposed daily for a year to directed instruction using technology, classroom discussions and opportunities to practise their spatial skills with an Art teacher. The researchers found that the geometrical complexity of the maps that the children drew showed a steady improvement throughout the year of the study. The researchers concluded:

> "With respect to the hierarchical levels of thinking proposed by Piaget and van Hiele, this study found that children can construct geometric meaning for topological, projective and Euclidean space in an interactive and synthetic way when given Constructivist opportunities for collaborative engagement, discourse and reflection."
>
> (Davis and Hyun, 2005:91)

Thus, Davis and Hyun found that children who were involved in practical activities involving space and shape and who were given opportunities to discuss their ideas with each other were able to improve their spatial and mapping skills. This research seems to confirm what van Hiele says about the importance of a well-structured learning program in helping children to develop geometrical and spatial understanding.

Wu and Ma (2005) tested 5 581 primary school children in Taiwan to determine their understanding of Geometry concepts. The researchers used an instrument (Wu's Geometry Test) that was based on the van Hiele level descriptors and the Van Hiele Geometry Test developed by Usiskin (1982) in the Cognitive Development and Achievement in Secondary School Geometry Project at the University of Chicago. The focus of the test was the concepts: circle, triangle and quadrilateral. They found that the majority of the primary school children in the study could identify shapes such as circles and triangles. However, many of the children (almost 50% of them) experienced problems identifying extremely obtuse shapes such as triangles and quadrilaterals that contain a large obtuse angle. Examples of obtuse figures are illustrated below.

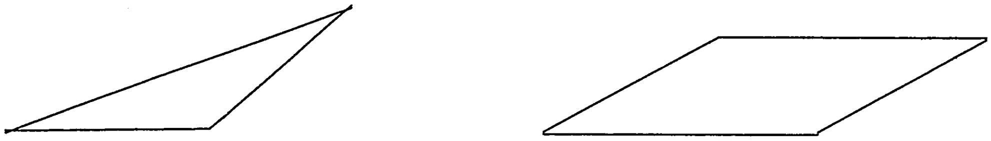


Figure 2.3 Obtuse Shapes.

Wu and Ma (2005) concluded that children are very rarely exposed to these types of figures in their daily lives and in textbooks. The implication of this research finding is that children need to be exposed to many different examples of shapes to enable them to develop their spatial sense and, hence, geometrical understanding. If children are learning about triangles, they need to be given examples of obtuse- and right-angled triangles in addition to the acute-angled triangles that can be seen in many structures in the man-made and natural environment.

The research done on primary school children internationally suggests that many primary school learners are at van Hiele Levels 1 and 2 (Halat, 2007; Meng, 2009;Wu and Ma, 2005). The research also suggests that the level of geometrical understanding of primary school learners can progress to higher levels with carefully designed instruction over a period of time. The van Hiele theory emphasized the important role of the teacher in providing the instruction needed for the children to progress through the levels. This has generated research into the level of geometrical understanding of people training to be teachers. Two of these studies are discussed below.

2.5.1.2 Studies involving student teachers

Knight (2006) found that the level of geometrical understanding of students registered to study education at the University of Maine was significantly lower than the level expected of the learners they were required to teach once they had qualified. She found that merely exposing the education students to a Mathematics course for a semester did not improve their geometrical reasoning skills. However, when student teachers were exposed to a semester Mathematics course in conjunction with an education course designed to help them with their geometrical reasoning, a significant number of them progressed to the next level of geometrical thought.

Halat (2008) did a case study researching the level of geometric understanding among 125 primary school teachers in training at a university in Turkey. He found that 7.2% of the student teachers were at the pre-recognition level as described by Clements and Battista (2001). 11.2% of the student teachers were at Level 1 on the van Hiele hierarchy while 36% were at Level 2 and 44% were at Level 3. He concluded that the majority of the student teachers were at a level where they would be able to teach Geometry adequately to primary school learners. However, it is worth noting that almost 20% of the student teachers in Halat's (2008) study were at a level of understanding lower than Level 3, the level learners should attain to study Geometry at secondary school (Senk, 1989; Usiskin, 1982). Both Knight (2006) and Halat (2008) concluded that they would recommend that all student Mathematics teachers should attend Geometry courses during their studies to help them develop geometrical understanding.

2.5.2 Research in South Africa.

When South Africa participated in the Repeat of the Third International Mathematics and Science Study in 1999 (TIMSS'99), South African learners performed very poorly in comparison to the other countries that participated in the study, including African countries and developing nations (Howie, 2004). Howie (2004) analyzed the results of the Mathematics component of the TIMSS'99 and found that South African Grade 8 learners achieved a mean score of 275 out of a possible score of 800. The international average was 487. In the Eastern Cape, the province where my case study was done, the mean score was even lower at 256.

Kotze and Strauss (2007) analyzed the Mathematics results of South African Grade 6 learners who had participated in an international research project undertaken by the Southern Africa Consortium for Monitoring Educational Quality, known as SACMEQII. They made the following conclusions:

- There was "a significantly low level of general numeracy, understanding and skills" (p 30) among the Grade 6 learners.
- There was a high correlation between the achievement of the learners and the economic status of their families.

- Lower achievement levels were found among learners from rural schools and higher scores came from children at urban schools.
- “The overall learner achievement was noticeably on the lower end of the acceptable limits on the SACMEQ benchmark” (Kotze and Strauss, 2007:29).

These research findings indicate that South African learners do not achieve Mathematics results that are on a par with the results obtained by children of the same age in other countries. It is also a cause for concern that children from families of lower socio-economic status, and in rural areas, achieve significantly lower results than children from wealthier homes in urban areas (Howie, 2004; Kotze and Strauss, 2007). Geometry is one of the areas of Mathematics that causes concern (Kotze, 2007). These findings have prompted South African researchers to investigate the level of geometrical understanding of learners in rural schools. Some of these studies are discussed in the following section.

2.5.2.1 Learners at primary school

The RNCS (South Africa, 2002:80) states that learners should be “able to describe and represent the characteristics and relationships between two-dimensional shapes in a variety of orientations” by the end of Grade 7. This corresponds to van Hiele’s Level 3. There is agreement among some researchers, both in South Africa and internationally, that learners need to have achieved van Hiele’s Level 3 by the end of Grade 7 but many learners do not attain this level (Feza and Webb, 2005; Knight, 2006; Kotze, 2007; van der Walle, 2004).

Research done by De Villiers and Njisane (1987) showed that only 45% of the Grade 12 learners taking Mathematics in KwaZulu Natal at that time had mastered van Hiele’s Level 2 and that most children who were in Grade 8 were at van Hiele Levels 1 and 2.

They suggested that these levels of understanding could be due to the manner in which Geometry is taught at school. De Villiers (1995) discussed the possibility that Mathematics teachers might be encouraging rote learning of definitions in Geometry instead of insisting that learners participate in constructing their own knowledge and understanding. As discussed earlier in this thesis, Piaget and van Hiele suggested that learners need opportunities to interact meaningfully with materials designed to help them develop their spatial sense and geometrical thinking skills. Learners lacking such opportunities in primary school may be forced to resort to memorization to cope with formal Geometry later on in their schooling.

Blandford (1908 as cited in De Villiers, 1995) had this to say about the learning of definitions in Geometry:

> "To me it appears a radically vicious method, certainly in Geometry, if not in other subjects, to supply a child with ready made definitions, to be subsequently memorized after being more or less carefully explained. To do this is surely to throw away deliberately one of the most valuable agents of intellectual discipline. The evolving of a workable definition by the child's own activity stimulated by appropriate questions is both interesting and highly educational."
>
> (Blandford as cited in De Villiers, 1995:38)

This insight by Blandford was made before the development of Piaget's Constructivist theory or van Hiele's theory. However, he highlighted the importance of learners being actively involved in their own learning. He also seemed to suggest that, by encouraging learners to develop their own ideas and definitions, their intellectual development might be stimulated.

Feza and Webb (2005) interviewed thirty English second language Grade 7 learners from historically disadvantaged schools and found that none of the thirty learners had attained the requirements of the RNCS (South Africa, 2002) Assessment Standards in

Learning Outcome 3 or could be categorized as Van Hiele Level 3, the level required to cope with Grade 7 Geometry (Usiskin, 1982). In fact, the majority of the learners in the study (25 out of the original 30) had not yet attained Level 2, the descriptive level.

In another study, Kotze (2007) analyzed the responses of 29 Mathematics teachers and 290 Grade 10 learners in historically disadvantaged schools on a Geometry test designed to assess their levels of geometrical understanding. She focused on spatial visualization and used problems based on the Geometry content of the RNCS (South Africa, 2002). She found that "space and shape were problem areas for the learners and the teachers" (Kotze, 2007:32). Kotze found that both groups experienced problems in activities involving three-dimensional figures, particularly when the surface area and volume of these figures had to be calculated.

Further, Kotze (2007) stated that:

> "It appeared that major emphasis is given in classrooms to the mastery of basic skills: calculation accuracy seemed to be preferred to the application of procedures. "
>
> (Kotze, 2007:33)

It would seem from Kotze's findings that many teachers concentrate on teaching skills such as the calculation of the area of a triangle rather than helping learners to develop their ideas about space and shape.

The learners in Kotze's (2007) study were unable to solve problems when shapes were rotated or viewed from another angle. However, Kotze (2007) noticed that the teachers in her study had greater difficulty identifying shapes in new positions (shapes having been rotated through 45 degrees or reflected about a line) than their learners did. She concluded that, "This tendency underlined the traditional emphasis on rote learning among teachers" (Kotze, 2007:33).

An interesting observation that Kotze (2007) made was that her respondents performed better on Level 4 (deduction) than on Level 3 (informal deduction) and better on Level 2 (shape analysis) than Level 1 (visual characteristics). This seems to contradict the van Hiele theory of a progression through the levels. However, a study of two of the questions used in Kotze's measuring instrument indicates that, in a question designed to test for understanding at van Hiele's Level 1, learners had to draw all the symmetrical lines in a figure that had been provided. Another question designed to test for van Hiele's Level 2 understanding asked learners to state whether the statement "every parallelogram is a rhombus" (Kotze, 2007:28) is true or false. A learner who could answer the second question but failed to draw in all the symmetry lines in the first question might appear not to have attained Level 1 on van Hiele's hierarchy despite seeming to have attained Level 2.

This anomaly could be explained if one considers that the learner who has rote-learned the properties of quadrilaterals would be able to answer the question used to assess Level 2 understanding and thus appear to have attained this level. However, this learner might not be able to rote-learn how to draw the symmetrical lines in a shape which they may or may not have seen before. Such an activity requires geometrical understanding. Therefore this learner might appear to have attained Level 2 but not Level 1. So, it could be that children, who have rote-learned properties of quadrilaterals (for example), may appear to have attained a higher level of geometrical understanding, simply because they can answer certain questions that require recall rather than understanding. Van Hiele (1986) discussed the possibility of this occurring, saying that:

> "... the teacher may reduce subject matter to such a form that the thinking activities needed by the pupil remain limited to a lower level...now the teacher knows he will have to continue remodelling the subject matter at a lower level for his pupils; he knows that for the time being he will have to expect little ingenuity."
>
> (Van Hiele, 1986:43)

To illustrate what he meant by saying that teachers remodel subject matter in Geometry to a lower level, van Hiele used the example of the technique, used by many teachers of Geometry, of introducing structures when teaching learners about the angles formed by parallel lines that have been cut by a transversal. Alternate angles can be recognized by the Z-shape they make and corresponding angles make an F- shape.

Figure 2.4 Alternate angles form a Z-shape and corresponding angles form an F shape.

Van Hiele claimed that this structure may help learners to solve problems but has not helped them to progress to a higher level of thinking. In fact, van Hiele claimed that this type of teaching, which may produce rapid results for the teacher, may be harmful for learners as it prevents them from experiencing the "crisis in thinking" (van Hiele, 1986:43) necessary for them to progress to the next level of geometrical thought.

In the light of her findings, Kotze (2007) asked the question "Is a 'feel' for space and shape a prerequisite for mastering geometry?" The response from van Hiele might be:

> "Still there are teachers who scarcely or ever meet the above mentioned difficulties. How can we account for this? There are two methods to solve these problems in such a way that, at least in the first year, you will never meet them. The first method is to explain theorems at school and afterward have them learned by heart at home. You can also demand that the pupils be able to give proofs with the figures in different positions than they have seen at school. The pupil not yet at the required level can help himself by simply learning by heart the order of the steps in the proof. It is very difficult to determine if the pupil understands the proof, for he knows exactly which

> properties of the geometric figures he must use. If he is not at the [third] level, he does not know why he has to use them, but he is not asked why. And he also knows what relations he must use. If he has not attained the level, he does not know why, but he is not asked why. The teacher, after proposing a new problem, will state that the pupil has a lack of ingenuity; he is not aware that the pupil is not at the level at which he would be able to make use of such ingenuity"
>
> (van Hiele, 1986:43)

This suggests that learners who have learned Geometry theory by heart may be able to reproduce theorems and definitions on demand but, because they lack an understanding of the geometrical thinking behind the theory, they are unable to apply the knowledge that they have to new situations. This may be what Kotze (2007) noticed about the teachers and learners in her study who were able to answer the questions that required recall of material that had been learned but lacked understanding of concepts like symmetry.

2.5.2.2 Studies involving teachers and student teachers

Van Der Sandt and Hercules (2003) investigated the geometrical understanding of 18 Grade 7 teachers and 100 prospective teachers and found that both the in-service teachers and the teachers in training had not acquired the level of geometrical understanding necessary to teach Geometry to Grade 7 learners. According to the van Hiele theory, teachers need to provide instruction for their learners at a higher level than the level their learners are at so that their learners can experience what is termed a "crisis in thinking" (van Hiele, 1986:43). Van Hiele mentions that, "The crisis in thinking ... is exactly what is necessary for the development of thinking by the pupil" (van Hiele, 1986:43). Thus, the teachers in van der Sandt *et al.*'s (2003) research group may be unable to provide instruction at the level required to help their learners progress to the next level of geometrical understanding on the van Hiele hierarchy.

Vygotsky's (1962) theory of conceptual development emphasized the importance of teachers providing instruction within their students' zone of proximal development, a process called scaffolding, which would enable the students to master concepts at a higher level. Knight (2006) suggested that teachers who have not reached a higher level of geometrical thinking than their learners will be unable to provide opportunities for scaffolding to occur, thus impeding the progress of their learners.

The research done with South African learners and teachers seems to indicate that many teachers still place undue emphasis on rote learning in Geometry. This prevents their learners from being able to experiment with Geometry activities and discuss their ideas in class. The research of Piaget and van Hiele indicates that children need opportunities to engage with activities that stimulate their thinking about shapes and spatial concepts to increase their geometrical understanding. The importance of language in helping children to progress through the van Hiele levels is discussed in the next section.

2.6 The importance of Language in Geometry

Van Hiele (1986) emphasized that teachers should use mathematical language in the classroom that can be understood by the children they are teaching. Van Hiele stated that:

> A teacher, beginning the teaching of geometry, should address himself to the pupils in a language they understand; he must not use the language of the [second] level, because the pupils have not yet attained that level.
>
> (Van Hiele, 1986:45)

The implication from the above quote is that teachers should assess what level of geometrical understanding their learners have reached before they start to teach Geometry to their learners. The teachers should then use the geometrical vocabulary that the learners will understand. Van Hiele (1986) claimed that, if a teacher uses the appropriate language during the teaching of Geometry, "His learners will try to understand him and they will also succeed" (van Hiele, 1986:45). These words of van Hiele imply that, if teachers use geometrical terms with which their learners are not familiar, their learners will not be able to understand the concepts being discussed. This may prevent learners from progressing to higher levels of geometrical understanding.

Usiskin, (1996) who was instrumental in developing the van Hiele Geometry test, which is used to assess the level of geometrical thinking of children and adults, claimed that Mathematics is:

- A written language.
- A spoken language and the spoken language is important for the understanding of mathematical concepts.
- A symbolic (pictorial) language. In Geometry there are many shapes to be recognized and their properties explored.

(Usiskin, 1996:231-242)

This suggests that the "language" to which van Hiele referred above is not just mathematical vocabulary. The teacher who is teaching Geometry to children who are at Level 1 on the van Hiele hierarchy should be aware of the different types of language involved in the teaching of triangles, for example.

- There is the way in which we describe a triangle called ABC in mathematical symbols (i.e. $\triangle$ ABC),
- The spoken word "triangle" represents the concept of a shape with three sides,
- There are differently shaped triangles, equilateral, isosceles, right angled and scalene to name a few. Learners need to spend time recognizing the different triangles and exploring their properties.

Van Hiele was not the only educationist concerned with the role that language plays in the development of children's understanding of Geometry. The work of some other researchers in this field is considered below.

Rubenstein and Thompson (2002) have noted that there is a mathematical vocabulary that needs to be learned by children studying Mathematics. Many of the terms used in Mathematics could be confused with their everyday meanings. When a term such as "face" is used in connection with the sides of a right prism such as a cube, for example, learners may be confused because they are familiar with the everyday meaning of face. There are other words such as "odd', "plane", "round" and "square" which have double meanings. Some words like parallelogram and rhombus are used only in Mathematical English and so the learners may not have come across these words before. Rubenstein and Thompson (2002) concluded that there is a need for Mathematics teachers to ensure that they spend time in the classroom helping learners to develop their Mathematical vocabulary.

Wellington and Osborne (2001), who researched children's understanding of the language of Science, had this to say:

> "Many of the words of science are complete strangers to pupils. Often students can answer questions in science without truly comprehending any of them."
>
> (Wellington and Osborne, 2001:19)

Although this quotation refers to the language used in Science, it applies equally to the language used in Mathematics. As Rubenstein and Thompson (2002) noted, Mathematics also has its own vocabulary and children may answer questions in Geometry too, without comprehending the questions or the words they are using in the answers. This may be illustrated by using the following passage from a children's book:

> "Twas brillig and the slithy toves
>
> Did gyre and gimble in the wabe;
>
> All mimsy were the borogroves,
>
> And the mome raths outgrabe...
>
> Somehow it seems to fill my head with ideas – only I don't know what they are!"
>
> (Lewis Carroll, Through the Looking Glass, cited in Wellington and Osborne, 2001:19)

The language used in the above quote is nonsense and, yet, Wellington and Osborne suggest that readers who understand the syntax of the English language would be able to answer the following questions about the passage without understanding what "slithy toves", "borogroves" or "mome raths" are:

Question	Answer
What activity did the slithy toves get up to?	Gyring and gambling.
Where did they do this and when?	In the wabe, at brillig.
How were the borogroves at this time?	Mimsy
What was the reaction of the mome raths?"	They outgrabe

(Wellington and Osborne, 2001:19)

In the same way some of the words and phrases used in Geometry classrooms may not be understood by learners. According to van Hiele (1986), this may happen if the teacher uses vocabulary from a higher level of geometrical understanding than his learners have reached. Learners may be rote learning geometrical definitions and terms they do not fully comprehend, answering questions in Geometry assessments at school and getting them marked correct simply because they recognize cue words in the questions, not because they understand the concepts.

Feza and Webb (2005) suggested that poor language competency in English is a barrier to the attainment of the higher van Hiele levels of geometric thinking. When one considers that many South African learners are not English mother-tongue speakers but are learning Geometry through the medium of English, one realizes that mathematical vocabulary may present them with a major challenge in trying to develop their geometrical understanding. According to Ramirez (cited in Nel 2006:150), it takes more than six years to develop proficiency in English when English is the second language of learning. This has profound implications for many non-English mother-tongue speakers in classrooms in the Foundation and Intermediate Phases in former Model C schools. These learners have to learn Mathematics in a language in which they are not yet proficient and they have to master the language of Mathematics itself. Research by Roux (2005) indicated that these learners are at high risk of not attaining the van Hiele Levels 1 to 3.

Roux (2005) studied the relationship between proficiency in the language of instruction and the level of geometrical understanding reached by children. She found that learners who were proficient in the medium of instruction did significantly better on the visual (Level 1), the descriptive (Level 2) and the informal deduction (Level 3) levels than those who were less proficient. She discovered that two aspects of language proficiency namely, reading comprehension and vocabulary, were particularly strong predictors for geometrical thinking on these first three levels. Roux (2005) concluded that it is essential for teachers to pay attention both to the general use of language and to the acquisition of the geometric terminology in the classroom. Roux (2005) also recommended that there be opportunities in the classroom for learners to communicate geometrical concepts to each other during group work. This is one way that learners can verbalize their thoughts to help them consolidate concepts in Geometry.

Unfortunately, in most classrooms, there is little opportunity for learners to communicate with each other in groups. Much of the talk in classrooms has the "sequence which is known as initiation-response-feedback" (Wellington and Osborne, 2001:106). The teacher asks a question, the learner responds and the teacher evaluates the response. Cobb, Wood and Yackel, (1993) suggested that there are problems associated with this pattern in schools. The learners become fearful of answering incorrectly and so they give up trying to respond, the teacher often allows too little time for a learner to formulate a response and the majority of the talk in the classroom becomes "teacher-talk". Research at Kings College, London showed that,

> "In the observation of 39 lessons, less than 5% of the time was given to group discussions and less than 2 per cent of the teacher-pupil interactions were genuine discussions with an exchange of differing views."
>
> (Wellington and Osborne, 2001: 106)

This research suggests that learners lack the opportunities in class that they need to verbalize their thinking about new concepts or geometrical problems they have been given to do.

From the discussion above, it seems that, for learners to develop an understanding of geometric concepts, attention needs to be given to the language used in the Geometry classroom. This is particularly important with children whose mother tongue is not the language of instruction.

2.7 Measuring instruments developed to test geometrical thinking

The van Hiele theory has stimulated a great deal of research and a variety of instruments have been developed in different countries to determine the level of geometrical understanding of children and adults.

Usiskin (1982) developed the Van Hiele Geometry Tests (VHGT) at the University of Chicago in a project termed CDASSG (The Cognitive Development and Achievement in Secondary School Geometry Project). The VHGT have been used in a number of research projects to test the van Hiele level of geometrical understanding of learners, student teachers and teachers (Halat, 2007; Halat 2008; Knight, 2006; Roux, 2005). One of the tests is a multiple choice test with five questions covering each level in the van Hiele hierarchy. If a student answers four or five of the five questions for a level correctly, then the student is considered to have reached that level. The tests are the property of the University of Chicago and written permission has to be requested from Professor Usiskin if a researcher wants to use one of the tests. Each copy of the test must have "Copyright© 1980 by the University of Chicago" printed onto it and a copy of any report written involving results using the instruments must be sent to the university (University of Chicago, 2009).

Pusey (2003) described a research study at the University of Stellenbosch, in which E. C. Smith (1987) gave a modified version of the CDASSG test to secondary school learners as well as a Geometry test designed by the RUMEUS (Research Unit for Mathematics Education at the University of Stellenbosch) group at Stellenbosch University. He found that the RUMEUS test outperformed the CDASSG test for the placement of learners in the van Hiele levels. However, he noted that the CDASSG test was shorter and the multiple-choice format made it much more convenient to administer and less time-consuming than the RUMEUS test (Pusey, 2003).

Wu and Ma (2005) developed the WGT (Wu's Geometry Test) in Taiwan as there were no suitable instruments in Chinese. They based the test on the van Hiele level descriptors and the sample responses identified by Fuys, Geddes, Lovett and Tischler (1988). Wu and Ma (2005) designed the test specifically for use in Elementary schools where the learners would be at van Hiele Levels 1 to 3. The WGT contains 25 multiple-choice questions to test Level 1, 20 for Level 2 and 25 for Level 3. The test focuses on triangles, quadrilaterals and circles and was used to test 5 581 primary school children in 23 cities in Taiwan in 2005 (Wu *et al.*, 2005).

In Kotze's (2007) case study on the level of geometrical understanding of Grade 10 teachers and their learners in South Africa, she designed her own instrument, comprising questions involving space and shape for Grade 10 learners. She piloted the questions for the instrument on groups of ACE students enrolled at the University of the Free State over three years. As she was able to work on the instrument over three years, she was able to modify it to develop good content validity. The questions were based on L.O. 3 of the RNCS (South Africa, 2002) and focused on areas of Grade 10 Geometry that are usually "problematic" (Kotze, 2007: 27). The instrument contained questions on the properties and classification of shapes, symmetry and calculations based on shapes such as area calculations. Kotze (2007) was able to assess the individual components of the test separately to ascertain how the learners performed in each area of Geometry. She then coded the questions according to the van Hiele level they tested (Kotze, 2005: 28), which enabled her to decide which van Hiele level the learners had attained.

2.8 Conclusion

This literature review has attempted to cover some of the theories about how children acquire geometrical understanding. These are Piaget's Constructivist theory (Farrell and Farmer, 1980), van Hiele's theory about the acquisition of levels of geometrical understanding (van Hiele, 1986) and Gagne's Instructional Theory (Gagne, 1988).

Although the three theories discussed have been developed by different researchers and are distinct from each other, there are common threads that run through all three.

They all emphasize a hierarchy through which learners move during their cognitive development. In Piaget's theory, the sequence is age dependent but learners have to have experience with objects in order for them to use the mental processes of assimilation and accommodation to increase their understanding and knowledge (Stone and Church, 1979). Piaget's theory influenced the development of van Hiele's theory of progression through five levels of geometrical understanding (van Hiele, 1986). In van Hiele's theory, development of understanding in Geometry can be seen as a continuous process with the teacher playing an important role (van der Walle, 2004). Van Hiele emphasized the need for learners to be challenged by carefully structured learning experiences to move from one level of geometrical understanding to the next (van Hiele, 1984, van Hiele, 1986). The teacher needs to provide instruction in Geometry at the level of understanding that the learners have attained using "level-appropriate" language in the classroom to assist the learners in attaining the next level of understanding. In Gagne's Instructional theory, a child needs to practise lower-order skills so that the performance of these skills becomes automatic and can be done without thinking. According to Gagne, this would free the mind for higher-order problem solving activities. Some Geometry teachers structure their lessons based on Gagne's theory (Farrell and Farmer, 1980).

Current research supports the van Hiele theory of levels of geometrical understanding (Feza and Webb, 2005; Halat, 2007; Knight, 2006; Roux, 2005; van der Walle, 2004) and so I used the van Hiele levels as the basis for my research into the geometrical thinking of Grade 6 learners. During this review, I discussed some of the instruments that have been developed to test geometrical understanding based on the van Hiele levels (Kotze, 2007; Usiskin, 1982; Wu and Ma, 2005). As I wanted to investigate the geometrical understanding of Grade 6 learners at a well-resourced school, I used these instruments to help me formulate my interview questions. This will be discussed in the next chapter (Chapter 3) on my research methodology.

Chapter 3 : Research Methodology

3. Introduction

This chapter aims to outline the research methods that were chosen for the study, the rationale behind choosing them and the limitations of the research design. The purpose of this research was to investigate the level of geometrical understanding, in terms of two-dimensional shapes, of Grade 6 learners at a former Model C school.

The research followed a case study design with purposive sampling being used to select fourteen Grade 6 respondents. To increase the validity of the research, a pilot interview was done. The fourteen respondents were interviewed using semi-structured interviews to explore their understanding of geometrical concepts. Data analysis included triangulation of hand-written notes taken during interviews with the audio recordings of the interviews and the researcher's own observations of the learners in class. The initial analysis was done during the data collection. More detailed analysis of the interviews was done later to identify trends. The chapter concludes with a discussion of the ethical considerations and the limitations of the research.

3.1 Research Orientation

3.1.1 Interpretive paradigm

The research is situated in the interpretive paradigm. Interpretivism is based on the view that:

> "Humans can only understand the world as it appears to them – not as it 'really' is. Interpretivist researchers are therefore interested in the *meaning* that people make of phenomena".
>
> (Janse van Rensburg, 2001:16)

Interpretivists believe that individuals construct their own meanings about events and that it is these meanings that are worth researching (Janse van Rensburg, 2001) Terreblanche and Durrheim (1999:6) describe the interpretive approach as one that, "believes that the reality to be studied consists of people's subjective experiences of the external world." This view holds that people interpret objects and events differently. Neumann (1997) would have agreed, describing the interpretive approach as:

> "The systematic analysis of socially meaningful action through the direct detailed observation of people in natural settings in order to arrive at understandings and interpretations of how people create and maintain their social worlds."
>
> (Neumann, 1997: 68)

Thus interpretivists are interested in what people think about their life experiences and how they make sense of the world around them. Each person is a unique individual and will construct his own understanding of the events he experiences. This is similar to what happens in Geometry lessons.

During a Geometry lesson, learners may be shown examples of two-dimensional shapes and told about their properties. Each learner will construct his own understanding of what those shapes are according to the way in which he or she perceives the shapes or their properties (van der Walle, 2004). De Villiers (1995) argued eloquently that Geometry is more than the mere learning of definitions of the properties of shapes. He quoted the Palestinian philosopher and poet, Khalil Gibran in The Prophet:

"The astronomer may speak to you of his understanding of space, but he cannot give you his understanding."

(Gibran, cited in De Villiers 1995:3)

In the same way, each learner in a Geometry class, learning about triangles, for example, will construct his own understanding of what a triangle is. Each child's understanding may be different from the understanding that the teacher has about triangles even though the teacher may have attempted to share his understanding with his learners.

Interpretive research involves interaction between the researcher and the respondent, which would lead the researcher to use data collection methods such as interviews. Janse van Rensberg (2001:16) suggested that interprevist researchers "look for rich, detailed information of a qualitative nature through in-depth interviews (and) observations".

Neuman (1997) stated that interpretive researchers believe that the answers that people give to research questions depend on the social situation they find themselves in. In many of the studies I have read on the assessment of van Hiele levels of the geometrical understanding of respondents, the instruments that have been used are mainly multiple choice tests (Knight, 2006, Kotze, 2007; Usiskin, 1982; Wu, 2004). I decided that, owing to the age of the children involved in my research (11-12 year olds), such a method of data collection would be unsuitable. The test setting might be threatening for them and an answer on a multiple choice test would not tell me how the respondent was thinking. I was interested in finding out the meanings that Grade 6 learners ascribe to geometrical concepts such as parallel lines and two-dimensional shapes. The interaction between each learner and me was important to enable me to probe answers or to reformulate questions if they were not understood.

3.1.2 Qualitative research

In quantitative research, the researcher gathers specific information on many cases but in a case study, the researcher gathers a large amount of information about a few cases and becomes immersed in the case (Neuman, 1997). Much of the research into the van Hiele levels of geometrical thinking of children and adults has been quantitative (Halat, 2007; Halat, 2008; Knight, 2006; Wu and Ma, 2005). These researchers have administered an instrument, containing questions designed to establish the level of geometrical reasoning of the respondents. This has been followed by statistical analysis of the data leading to conclusions about the number of subjects who have attained van Hiele's Level 1 or Level 2 and so on.

I decided that I was more interested in discovering how children were thinking about geometrical concepts and the way in which learners create meaning for themselves in Geometry. Quantitative data is "... in the form of numbers from precise measurement" (Neuman, 1997:329) whereas qualitative data "... are in the form of words from documents, observations, transcripts" (Neuman, 1997:329).

Qualitative research describes and analyzes the individual's beliefs, thoughts and perceptions (McMillan and Schumacher, 2006). Neuman (1997) says that:

> Qualitative research relies largely on the interpretive approach to social science... A qualitative researcher focuses on subjective meanings, definitions, metaphors, symbols and descriptions of specific cases.
>
> (Neuman, 1997:329)

I therefore considered that the qualitative approach was the better one to choose for this investigation. I was interested in the individual meanings and descriptions that Grade 6 learners have about two-dimensional shapes. It seemed that a qualitative and interpretive approach would enable me to discover what their ideas about these shapes were.

3.2 Research design

3.2.1 Case study

The research took the form of a descriptive case study. A case study is a specific instance that is frequently designed to illustrate a more general principle (Nisbet and Watt, cited in Cohen, Manion and Morrison, 2001:181). An instance is a bounded system like a child, a class, a school or a community, which is closely scrutinized. A case study is able to provide an in-depth description of a particular instance and is used when there are a limited number of cases, usually less than 50. (Mouton, 2001). The specific instance in this case is the fourteen Grade 6 learners at the school chosen for the study.

In a case study, the researcher, "might gather a large amount of information on one or a few cases, go into greater depth and get more details on the cases being examined" (Neuman, 1997:331). McMillan and Schumacher (2006) suggested that, in a case study, a single phenomenon is studied in depth with a limited number of respondents. The phenomenon I was interested in was the level of geometrical thought of learners at the end of Grade 6, which is the end of the Intermediate Phase of schooling in South African schools.

A case study involves systematically gathering enough information about a group to enable the researcher to, "...understand effectively how it operates" (Berg, 2001: 225). I wanted to gain insight into how fourteen Grade 6 learners at a well resourced school were thinking about geometrical concepts. Simply doing a test to ascertain the level of thought that each learner may have reached would not have given me the information I was seeking. In a test one has to accept an answer and mark it right or wrong. That does not help the marker to understand why the child gave the wrong answer and what he was thinking when he chose that answer. Case studies are able to give insight into situations in a far deeper way than numerical analysis can (Cohen *et al.*, 2001) and so a case study design seemed the most appropriate.

The data gathered in a descriptive case study is much more detailed (Neuman, 1997: 331). One of the advantages of case studies is that they produce extremely rich, detailed and in-depth information (Berg, 2001: 225). Supporting this view, Mouton (2001) mentions that, in a case study, the researcher is able to establish a rapport with the research subjects and that this may help respondents give more detailed information than they might have done in a more formal setting.

I was able to establish a good relationship with the learners I interviewed at the school. In addition, I was given access to assessments that the fourteen learners had completed in Geometry and interviews with teachers about the fourteen respondents. This all contributed to building a thick description of the case. A thick description, in the words of the philosopher, Gilbert Ryle (cited by Henning, van Rensberg and Smit, 2004:85), is "filled with discussion and analysis, rich in explanation and argument".

Another advantage of case studies is that they have high construct validity (Mouton, 2001), the construct in this study being the geometrical understanding of the Grade 6 learners. In the context of this research with fourteen Grade 6 respondents, the results from a single test could give an incorrect assessment of the learners' level of geometrical understanding. Thus, I thought that the probing nature of the qualitative questions and answers used in this descriptive case study would give a more accurate indication of the understanding that the individual learners may have achieved.

Adelman (cited in Cohen *et al.*, 2001: 184) claimed that case studies are a "step to action. They begin in a world of action and contribute to it." He went on to state that the insights that case studies uncover may be used in educational policy making or in personal development of staff and that case studies present research in a far more publicly-accessible format than most other research reports.

Case studies do, however, have certain weaknesses. They are susceptible to researcher bias, particularly in the interview situation (Mouton, 2001). The results obtained from a case study cannot be generalized. The collection of data, in this case, interviews, and the analysis of the data (i.e. the transcription of the interviews and the search for trends) is very time consuming. Cohen *et al.* (2001) stressed that case studies need to demonstrate both reliability and validity. Reliability refers to how reproducible the results of a study are (Neuman, 1997); validity refers to whether the research measures what it is supposed to.

When a researcher collects data in the field, by doing interviews, for, example, validity refers to the confidence that can be placed in the data and the analysis of that data by the researcher. Neuman (1997) stated that, "When an indicator is valid, it is valid for a particular purpose and definition" (Neumann, 1997:138). Cohen *et al.* (2001) argued that the selection of cases in a case study is extremely important because researchers who do case studies focus on the "significant few rather than the insignificant many" (Cohen *et al.*, 2001:184).

According to Yin (cited in Berg, 2001: 230), descriptive case studies require that the investigator present a descriptive theory, which establishes the overall framework for the investigator to follow throughout the study. Building from this, Berg (2001:230) suggested that descriptive case studies require the formation and identification of a viable theoretical orientation before enunciating research questions. The framework I used was the van Hiele theory of successive levels of geometrical thinking. As van Hiele (1986) claimed that he had been influenced by Piaget's Constructivist theory, my initial research question was formulated in relation to van Hiele's theory, mindful of the importance that both he and Piaget had placed on children constructing their own understanding of geometrical concepts.

3.2.2 Description of the case

The boundaries of this case study included the fact that Grade 6 male and female learners attending a former Model C school in East London were interviewed. The learners had different levels of achievement in a school-based Geometry assessment, they came from different backgrounds and cultures and they had been taught by two different Mathematics teachers. One school was chosen for the study. I was able to obtain detailed information about the school's Mathematics Academy and to interview the teacher who runs the Academy. I was also able to attend some Grade 6 Geometry lessons, which helped me to gain insight into how Geometry was taught in Grade 6 at this school.

3.3 Research methods

3.3.1 Research Instruments: semi-structured interview schedule

Open and flexible semi-structured interviews were used in the study, which is the technique that Feza and Webb (2005) used in their research into the geometrical thinking of Grade 7 learners in rural and peri-urban schools. Their rationale for using this technique was that it helped to clarify what learners understood about geometrical concepts and it enabled the researcher to explore in greater detail the explanations given by the respondents.

A questionnaire was designed to be used in the interviews. The principles of good questionnaire design were followed for this research (Neuman, 1997:233-236) and care was taken to ensure that:

- The questions were clearly worded and unambiguous, so that the learners would have no difficulty in understanding them.
- There were no double-barrelled questions i.e. two or more questions joined together that might cause confusion.
- Leading questions were avoided.
- There were no questions that contained double negatives.
- The questions were phrased using the terms that Grade 6 learners would use in Geometry.

Some of the questions selected for the interviews were closed and had only one possible answer. For example, the learners had to name certain figures from the pictures or the shapes they were shown. Many of the questions were open-ended questions, giving the learners the opportunity to formulate their own answers. Neuman (1997) claimed that mixing open and closed questions offers a change of pace during the interview and helps the researcher to establish a rapport with the respondents. He also stated that interjecting periodic probes helps to reveal the respondents' reasoning. He did mention that this makes the time taken for the interview longer and requires better trained interviewers. I conducted all the interviews, which meant there was no need to train interviewers. I am a qualified Mathematics teacher with twenty years experience in working with learners at school and hoped that I might be able to use appropriate probes to clarify and investigate the learners' responses to interview questions. Neuman (1997) warned that open-ended questions and probes into respondents' answers are more difficult to quantify. However, because my research did not involve any statistical analysis of responses and my intention was to obtain rich descriptions of individual learners' geometrical thinking, I was willing to include such questions.

The rationale behind the design of the questionnaire (Appendix D) used in the interviews with the Grade 6 learners was to try to include questions that would test whether they had attained Levels 1, 2 or 3 on the van Hiele scale of geometrical thinking. To assess this, there was a progression in the level of complexity of the questions which were asked. This method is similar to that used by Piaget in his research with children of different ages. He was interested in, "...what lay behind children's answers, particularly their incorrect answers" (Farrell and Farmer, 1980:49). I was interested in the incorrect answers given to the interview questions by the Grade 6 learners and wanted to probe the thinking behind those incorrect answers.

Some of the questions involved the use of K'NEX® rods and connectors to build models of two-dimensional shapes. According to Gagne's Instructional Theory, there is a hierarchy of skills and the higher order skills depend on the attainment of the skills lower down in the hierarchy (Farrell and Farmer, 1980). The ability to build a model of a triangle for example is a performance skill which would be a higher order skill on the Gagnean hierarchy as it requires that children should know what the properties of a triangle are.

The University of Chicago (2009) has developed a comprehensive test (the VHGT or Van Hiele Geometry Test) to ascertain the level of geometrical understanding of a subject. This test is available on the internet and can be downloaded for use by any researcher although written permission from the University is required. I contacted Professor Usiskin of the University of Chicago by e-mail (Z. Usiskin, personal communication, June 2, 2009) requesting permission to use the VHGT. He warned me that the test was not suitable for use with primary school children as it tested for levels beyond what they could have attained and the subject matter was too advanced. I obtained a copy of the test from the University of Chicago website and found that some of the questions could be modified to be used in an interview with learners at this level. Four of the questions I used are based on the simpler questions from the VHGT developed by the University of Chicago. I contacted Professor Usiskin again to request permission to use the questions and he responded giving me permission to use the questions. The questions I used from the VHGT involved diagrams of different two–dimensional shapes such as triangles, squares and rectangles, with questions like "which of these are triangles?" The questions tested whether learners were able to recognize shapes of a similar type which would mean that they had possibly attained van Hiele's Level 1, the visual level.

Neuman (1997) stated that a pre-test or pilot test improves the reliability of the research. I conducted a pilot test of the structured interview at the end of November 2009 with my daughter who was in Grade 7 at the time. The interview lasted 30 minutes and, on the basis of that interview, the interview schedule was modified. Neuman (1997) claimed that the principle of using pilot tests extends to replicating the measures that other researchers have used. Some of the questions used in the interview schedule were based on questions that Feza and Webb (2005) used in their semi-structured interviews. The questions and activities they used are supported by the literature (Fuys, Geddes, Lovett and Tischler, 1998; Murray, 1997; Roux, 2005). As a result of the pilot interview, more answer codes were added to questions 1 and 2 and question 7 was added to the schedule (see Appendix D). However, the pilot test verified that the instrument did enable me to investigate the geometrical understanding of children of this age. It also enabled me to have a clear idea of how long the interviews would take.

Henning, van Rensberg and Smit (2004) warned that the interview schedule (the instrument) may be biased by the interviewer's view of the world. To overcome this, they recommended that the interview not be analyzed for content only but that it should include some aspect of discourse analysis to highlight possible hidden meanings that were created during the interview. Every effort was made, while formulating the interview questions, to keep them as objective as possible. Most of the questions required naming of shapes or the division of shapes into groups based on similar properties. This provided very little opportunity for researcher bias to influence the answers owing to the nature of this study. However, each interview was digitally recorded and carefully analyzed to try to limit any researcher bias.

I designed the interview schedule to incorporate activities that would indicate which van Hiele level each learner had reached. The responses to the initial questions were probed more deeply to investigate how the learner was thinking similar to the method followed by Feza and Webb (2005). There were at least three activities included in the schedule that were designed to test each level up to van Hiele's Level 3. This was done to confirm what level a learner had attained.

To test whether learners had attained van Hiele's Level 1, the learners interviewed were shown a picture of a house surrounded by bushes, and asked to identify any shapes they could see in the picture (Question 1). They were asked to identify plastic shapes (Question 2) and to name them. They were also given K'NEX® and asked to make a triangle and a square (Questions 5 and 6). Feza and Webb (2005) used a similar technique in their research into the level of geometrical understanding of Grade 7 learners. They used manipulatives such as construction sticks, shapes and a picture of a convent in their interviews and matched the responses of the Grade 7 learners in their study with the appropriate van Hiele level of geometrical reasoning.

Van Hiele's Level 2 is known as the descriptive (van Hiele, 1999) or analysis level (van der Walle, 2004). At this level, learners can identify and describe properties of shapes. For example, a square is different from all other quadrilaterals because of its four sides of equal length, the opposite parallel sides, four 90° angles and its symmetry. Questions chosen for the interview to test this level included Question 3 where the learner was asked to describe a square over the telephone to a friend without using the word 'square' and Question 4 where a plastic square shape was rotated and they were asked whether it was still a square.

Van Hiele's Level 3, the level of informal deduction, is where learners start to order the properties of shapes logically. They begin to see relationships among groups of shapes (van der Walle, 2004). The questions towards the end of my interview schedule tested whether this level of geometrical reasoning had been attained. One of the final questions of my interview (Question 11.5) was about the properties of isosceles triangles. I established first whether the learner knew what an isosceles triangle was and then asked the learner to choose the property that was true of every such triangle. This is a similar question to one found on the original VHGT (Van Hiele Geometry Test), but, whereas the original question appeared as a written multiple choice question, the flexible format of the interview allowed time for discussion of the different options in the question to ensure the learners understood what was being asked and to explore their understanding of isosceles and equilateral triangles.

The design of the interview schedule has been informed by current research into good questionnaire design based on the work done by others (Bennie, 1999; Feza and Webb, 2005; Roux, 2005; Usiskin, 1982) into the level of geometrical thinking of respondents.

interviews were started before noon as the researcher did not want learners to be fatigued for the interviews

The technique of purposive sampling as described in Maxwell (1996) was used to select the learners for the interviews. Maxwell goes on to state that “random sampling is a poor method to use for a small sample due to the high likelihood of chance variation” (1996: 71). The method of purposive sampling is advocated by Neuman (1997) when a researcher wants to gain an in-depth understanding of particular individuals rather than generalize to a larger population. The population of Grade 6 learners in a school such as the one studied, is a heterogeneous one, consisting of male and female learners from a variety of cultural backgrounds. In accordance with “... the method of maximum variation sampling, a subset of purposive sampling” (Maxwell, 1996:71), the intention was to choose four learners who performed well on the scale of geometrical achievement, four who performed at a satisfactory level and four whose level was below the satisfactory level.

I visited the research school in the first week of November 2009 to observe the learners in two different Grade 6 classes during Geometry lessons. At the school that was selected for the study, each Grade 6 class has a class teacher who teaches them all the learning areas in the RNCS (South Africa, 2002) and stays with them all day. These teachers know their learners and their abilities very well. Henning *et al.* (2004:85) noted that, “Data that is collected and noted without involving the whole or larger picture of meaning is very bland and thin”. For this reason, I wanted to gather as much background information about the learners as possible. I asked the teachers for information about the selected learners and asked the learners questions to learn about their backgrounds and their prior experiences with shapes.

After watching the lessons, and consulting with the class teacher of each class, I selected twelve learners to interview. The learners were chosen on the basis of their performance in the end of term Geometry assessment (Appendices H and I) and their class teachers' evaluation of their mathematical ability. Care was taken to ensure that, among the learners selected there were male and female learners from different cultural backgrounds. Although I originally intended to interview twelve learners, this number grew to fourteen learners. Two additional learners were added to the original number for the following reasons:

- During the study, I attended the annual school prize-giving and noted that the learner who received the prize for coming first in Mathematics in Grade 6 had not been included among the original selection. I was interested in the level of geometrical thinking of a learner who had achieved the highest marks in Mathematics in the grade and so this learner was included after the others had been selected.
- Permission letters were sent to all the parents of the learners selected for the study. One learner could not return the letter as his parents were away and so another learner was selected to replace him. When the first learner's parents returned, he brought the signed permission letter back to school and was very keen to be interviewed too. So I included him among the respondents. This brought the total number of learners interviewed up to fourteen instead of the original twelve.

Some of the learners who were selected for the study attended the Mathematics Academy at the school so I visited the academy to observe these learners during an Academy Geometry lesson. Learners who attend the Mathematics Academy at the school have been selected on the basis of their mathematical ability, using an assessment at the start of each academic year. Those who perform well on this assessment are invited to attend the Academy provided they pay an additional fee. It is an elite group, consisting of up to ten learners from the same class, who meet once a week with a qualified Mathematics teacher to do advanced Mathematics, beyond the curriculum for their age. The rest of the class remains with their class teacher to consolidate the Mathematics activities they have been doing during the week. I noted whether each learner interviewed attended the Mathematics Academy or not. This information assisted with the compiling of a “thick” description of each learner (Henning *et al.*, 2004:85).

3.3.3 Data collection

I interviewed the learners individually at a table in a small hall near the Grade 6 classrooms. It was a familiar, non-threatening environment to reduce any anxiety the learners may have felt at being interviewed. At the start of each interview, questions were asked about the learner’s home language, extra-mural activities and their experience with various toys and games designed to develop spatial perception. These questions helped to relax the learners and assisted in compiling a “thick description” of the learners (Henning *et al.*, 2004:85)

I had an interview schedule for each interview, which set out all the questions that would be asked (Appendix D). The interviews were taped and I made notes onto the interview schedule about anything I observed during the interview like body language or what the learner did with some of the shapes or pictures (this would not be picked up in the tape recording). Neuman (1997) recommended that field researchers keep detailed notes about their interviews. He said that:

> Personal notes serve three functions: They provide an outlet for the researcher and a way to cope with stress; they are a source of data about personal reactions; they give him or her a way to evaluate direct observation or inference notes when the notes are later reread.
>
> (Neuman, 1997:366)

For this reason, after each interview, I added any extra observations I had made during the interview onto the written interview schedule. I hoped that these notes would help me to analyze the data collected from the interviews more accurately at a later stage.

Flick (2006:284) claimed that using machines for recording interviews aids in making the documentation of data independent of the perspectives of the interviewer. He did warn, however, that the presence of recording machines may influence those being interviewed. At the start of each interview, the learners were shown the tape recorder and asked whether the interview could be taped. None of the learners objected to the interview being taped. The tape recorder used was a small one, the size of an average cell phone and so it was not intrusive. The interview questions for the assessment were relatively impersonal and the learners did not appear to be affected by the presence of the digital tape recorder.

During the interviews, I used manipulatives (Osborne and Gilbert, cited in Feza and Webb, 2005), such as K'NEX® and plastic shapes. Feza and Webb (2005) used pictures, construction sticks and cardboard shapes during their interviews with Grade 7 learners. The use of manipulatives enabled me to explore geometric ideas such as sorting shapes into groups based on similar properties more easily and to ascertain the level of the learners' thinking about two dimensional shapes in Geometry.

Feza and Webb (2005) noted that some of the Grade 7 learners they interviewed became very fatigued during the interviews and so they had to keep the interviews to half an hour in length and bring the same learner back two or three times to complete the interview. With this in mind, I was careful not to allow my respondents to become fatigued. Each interview lasted between thirty and forty-two minutes and the learners appeared to concentrate well during the interviews.

3.3.4 Analysis

The unit of analysis for this research was the level of geometrical thinking of the fourteen Grade 6 respondents. One of the criticisms of qualitative research is the validity of the results. To counteract this possibility, I used a triangulation technique to check that the data was valid. Triangulation is a technique used in geographical surveying so that by using three different points or angles, the true position can be found. In qualitative research, the term is used to build up a complete picture by "interpreting and sourcing in various ways". (Henning *et al.,* 2004:103). With that analogy in mind, Berg (2001:4) said that, "the different methods we use in research are called different lines of sight." By combining several lines of sight, researchers obtain a better; more substantive picture of reality. Berg (2001) claimed that, in qualitative research, we should use three data-gathering techniques (triangulation) to investigate the same phenomenon.

To this end, I asked the teachers of the learners for background information on the learners and the copies of the assessments that the learners wrote at the end of the year for the Geometry module (Appendix I). During the individual semi–structured interviews I made notes on a written schedule, detailing what the learners did with the manipulatives as well as their responses to the questions. (Appendix E) The interviews were taped so that they could be transcribed in detail to capture any information not written down.

Another important reason for the tape recordings was that most of the questions had open-ended answers and I needed to probe some of the answers to discover what the learners' understanding of certain geometrical ideas was. Neuman (1997:257) warned that, during an interview, if the researcher is summarizing the longer responses to questions onto a recording sheet, owing to time pressure, much information can be lost. Being able to play the interview back from a recording enabled me to record the exact words spoken by the learners in response to the questions. The benefit of transcribing each interview, verbatim, was it allowed me to re-evaluate the exact meaning of the responses to the questions.

Each interview was transcribed verbatim and the data from the interviews was summarized into tables to identify trends among the learners. The learner responses to individual questions were analysed to identify the Van Hiele Level each learner was operating at for the shape or activity in the question. Interesting trends and anomalies were noted and extracts from the interviews were used to illustrate the discussion. On the basis of the learner responses and their observations about shapes during the interviews, I was able to categorise their thinking as Level 1, 2 or 3 on the van Hiele hierarchy.

3.4 Limitations

The principle limitation of a case study is that the results obtained cannot be generalized. It is thus not possible to apply the results obtained in this study to the entire population of Grade 6 learners in former Model C schools in the country. Case studies are also susceptible to researcher bias, particularly in the interview situation. (Mouton, 2001:150). The focus of this research was two-dimensional shapes and so the questions were objective. However, in an interview, there is a danger that the interviewer may ask leading questions (Neuman, 1997). Notwithstanding these limitations, the study benefits from a single interviewer conducting the investigation and from the comprehensive notes and recordings that were made of the semi-structured interviews.

3.5 Ethical considerations

I wrote to the Chief Director, Strategic Planning at the Department of Education, Eastern Cape on 11 October 2009 to request permission to interview learners at the school selected for the case study (Appendix A). The letter outlined what the purpose of the research was and how the research in the school would be conducted.

I then contacted the headmaster of the selected school and visited him to obtain his consent to the research being done at his school. At the meeting, I explained the purpose of the research and assured him that both the anonymity of the learners involved and the school would be preserved. He agreed to the research being carried out at his school but referred me to the Grade Head for Grade 6 at his school so that she could give me permission to work with the Grade 6 learners and teachers. This was done and permission was granted. I was put in contact with the Learning Area Head of Mathematics in the Grade 6 area because the Grade Head was not comfortable with her Geometry class being visited or her learners being selected. The Learning Area Head, Mrs C, was very co-operative and interested in the research. She introduced me to another Grade 6 class teacher, Mr L who was also prepared to participate in the research.

Once the learners to be interviewed had been selected, the teachers (Mrs C and Mr L) gave the learners a letter that I had prepared for their parents requesting permission to interview their children (Appendix B) The parents were assured that the anonymity of their children would be preserved and that the findings would be kept confidential. All the parents of the children selected gave permission and one example of the consent letters can be found in Appendix C.

3.6 Conclusions

The research was qualitative. To this end, open and semi-structured interviews were used to interview learners from a well-resourced former Model C school in a case study design. The learners were selected using the technique of purposive sampling so that the interesting cases could be investigated. Background information on the selected learners was obtained from their class teachers and the teacher in charge of the Mathematics Academy, where appropriate. These assisted in developing a rich, detailed description of each learner's geometrical understanding. The direct contact research took place during November and December 2009 after the Grade 6 end-of-year exams had been completed at the school. The transcription of the interviews was completed by May 2010 and the analysis of the information occurred thereafter. The analysis of the information is presented in the following chapter.

Chapter 4 : Data Presentation, Analysis and Interpretation

4.1 Introduction

The purpose of this chapter is to present the data from the interviews with the fourteen Grade 6 learners selected for the study. First, the chapter provides a discussion of the school chosen for the case study and information about the Mathematics and other academies at the school. I compare and contrast the requirements of the Revised National Curriculum Statement (2002) in the light of the material presented in the Geometry module done in Grade 6 at this school. Thereafter I will present background information on each learner which will be summarized into a table for easy reference. Part of my research design required me to construct a semi-structured informal questionnaire and the learner responses to these questions are presented as summarized, easy-to-reference tables. This information is supported by an in-depth discussion on the trends that have been identified from the learner responses.

4.2 School profile

The school chosen for the case study is a former Model C primary school situated in a fairly affluent middle-class suburb in East London. There are approximately thirty learners per class and five classes per grade from Grades 1 to 7. The classrooms in the Intermediate Phase are well-equipped, with each classroom having its own computer for the use of the learners. Each classroom also has an interactive whiteboard which enables the teachers to access information from the Internet to supplement the material covered during lessons. The teachers have attended training sessions on the use of the interactive whiteboards and the school has staff members, who have been trained in the use of Information Technology, at the school to assist teachers who need extra help with the interactive whiteboards.

There are three Grade R classes that are housed in the same set of buildings as the Foundation Phase classrooms. There is an interesting feature at the Foundation Phase at the school. The Foundation Phase teachers found that, in Grade 1, there is a difference between the academic achievement and physical development of children who turn seven during the last few months of the academic year and those who turn seven earlier in the year. To accommodate these younger children, the school has introduced a smaller class in Grade 1 with a teacher who moves at a different pace from the other Grade 1 teachers. This teacher is able to spend more individual time with each learner to help the learners achieve the best they possibly can during their first year at school. I highlight this aspect to illustrate that the research school tries innovative ways to help their learners academically. The Foundation Phase has its own campus, separate from the rest of the school. There is also a Pre-Primary school which educates three to five year old children and is a feeder school for the Grade R classes at school A. The medium of instruction is English.

To foster the talent of those learners who show special aptitude in the areas of Mathematics, Art, Music, Computers and Cricket, the Intermediate Phase has a number of Academies for which the children are selected. I will discuss some of these Academies in detail in Section 4.2.2 of this chapter.

4.2.1 Gaining access to the school

I met with the headmaster of the school to ask him for permission to work among the Grade 6 learners. He was very positive and said that he would allow me to do research at his school. However, he wanted me to contact the Grade 6 head, Mrs S to ask her for permission. He said that it was not his management style to take a decision like this on behalf of his staff and wanted her to have the opportunity to be involved in deciding whether to allow me to work in her grade.

I contacted Mrs S and she agreed to allow me to do my research in the Grade 6 department of the school. I explained the purpose of my research and requested that I be allowed to observe a few Geometry lessons. Mrs S explained that she was uncomfortable with my visiting her class to watch a Geometry lesson, as were some of the other Grade 6 teachers. She referred me to the Grade 6 teacher in charge of Mathematics, Mrs C. Two of the teachers, Mrs C and Mr L were prepared for me to observe their classes during a Geometry lesson.

I met with Mrs C at the start of the fourth term and she was helpful and co-operative. She explained to me that, at the research school, they do not do any Geometry in Grades 4 and 5 but that the Geometry for the Intermediate Phase is covered during the last term of Grade 6. I have subsequently discovered that this is not entirely accurate as the children who attend the Mathematics Academy are exposed to shapes during Grades 4 and 5. However, for the majority of the learners, it is the case that they do not do any Geometry in Grades 4 and 5. I think this is unfortunate as all learners would benefit from the opportunity to work with shapes in class in Grades 4 and 5, and many are unable to do so as they do not attend the Mathematics Academy.

Mrs C gave me a copy of the Geometry module that they were about to start during the course of the following week (See Appendix G). She told me that, as the learners had done no Geometry in Grades 4 and 5, Grade 6 Geometry started with a discussion on shapes. They then did a module covering constructions and theory about the names of shapes and calculation of areas.

She also gave me a copy of the test (see Appendix H) which is used at the end of the year to assess the children's Geometry knowledge. Mrs C offered to let me have the results of this test so that I could use the results to help me build a clearer picture of how the learners perform in a Geometry assessment. She was also prepared to give me background information about the learners selected for the interviews, which could help me to gain insight about the learners. Mrs C suggested that I do the interviews during the week 30 November – 4 December 2009, doing two or three interviews per day. By that time, the learners would have completed all their work for the year and end-of-year assessments. She felt that this would be the least disruptive time for the learners to miss class in order to be interviewed.

I was given permission to use a small school hall near the Grade 6 classrooms for the interviews.

4.2.2 The Academies

The research school has four specialized academic Academies in the Intermediate Phase. These are:

- Art
- Computers
- Mathematics
- Music

In order to attend one or more of these Academies, learners need to demonstrate an aptitude in that particular area. At the start of each academic year, all the children in each grade write a selection test for entry to the various Academies. Ten children per class are selected on the basis of their results for the tests and are invited to join the Academies. There is an extra tuition fee to cover the cost of attending an Academy but children whose parents cannot pay the fee are unable to attend the Academy.

Unfortunately, a consequence of this practice is that some children, who have an aptitude for Mathematics or Art, for example, are excluded from being able to benefit from the Academies. Some of the less-able children are chosen to fill the classes in the Academy, simply because their parents are able to pay the fee. When I discussed this practice with the head of the Mathematics Academy, he disagreed with it strongly. He expressed disappointment that some of the more able mathematicians in Grades 4 to 6 were unable to attend the classes in the Mathematics Academy owing to financial constraints. He said that occasionally this practice has resulted in less able learners, whose parents can pay the additional fee, attending the Mathematics Academy where the lessons were too advanced for them.

4.2.2.1 The Mathematics Academy

The children who have been selected, and who have paid the extra fee, attend the Mathematics Academy once a week during their normal Mathematics period. This frees the home-class teacher to do remedial Mathematics exercises with children left in the classroom. As there would be ten fewer children in the classroom, the children that remain are able to have more individual attention, which benefits them.

Those children who go to the Mathematics Academy may have an extension lesson on the work they are busy covering in class which includes more challenging examples. Alternatively, they might also explore new areas of Mathematics which are not covered in class, but may be part of the following year's curriculum.

The Mathematics teacher in the Mathematics Academy is a qualified Mathematics teacher, with many years experience in teaching Mathematics. He chooses challenging problems to encourage the learners to develop their problem-solving skills and he also spends time practising Mathematics Olympiad questions with the Grades 4 to 7 learners in the Academy to help them when they participate in these Olympiads. The research school participates in all the major Mathematics Olympiads during the year and the Grade 4 to 7 learners always perform very well, regularly achieving bronze, silver, gold and platinum awards.

4.2.2.2 The Art Academy

Learners who attend the Art Academy go to one lesson a week while their classmates have class Art. At the Academy they are encouraged to try different media such as oil paints and charcoal. Some of the work is selected to be displayed at the annual Primary Schools Art Exhibition. Many of the children achieve awards for their Art.

4.2.2.3 The Computer Academy

At the Computer Academy, the learners are shown how to use different software packages and make their own presentations, do research using the Internet or learn how to use a spreadsheet effectively.

4.2.2.4 The Music Academy

One of the requirements of the Music Academy is that learners must own a musical instrument such as a flute or clarinet. They attend individual Music lessons on a weekly basis to learn how to play their chosen instrument. They are encouraged to enter the annual Music Eisteddfod and to perform in front of the school at Assemblies and other functions.

4.3 A comparison between the Revised National Curriculum Statement, the Geometry module taught in Grade 6 at the research school and the Van Hiele levels

4.3.1 Revised National Curriculum Statement : Learning Outcome 3

There are five Learning Outcomes for Mathematics in the Intermediate Phase. Of these five, Learning Outcome 3 deals with Space and Shape, which is Geometry. Table 4.1 below presents the Assessment Standards that need to be attained by learners in the three grades in the Intermediate Phase according to the RNCS (South Africa, 2002). This full comparison is presented so that I can compare the Assessment Standards of the RNCS (South Africa, 2002) with what the Grade 6 learners actually do in Geometry at the school chosen for the case study.

L.O. 3 : SPACE AND SHAPE (GEOMETRY)

The learner will be able to describe and represent characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations and positions (RNCS, 2002:48-51).

Table 4.1 Assessment Standards for Learning Outcome 3 of the RNCS (South Africa, 2002) for the Intermediate Phase (Grade 4-6).

Grade 4	Grade 5	Grade 6
We know this when the learner; Recognizes, visualizes and names two-dimensional and three-dimensional objects in the environment, including • Rectangular prisms, spheres, cylinders and other objects; • Prisms and pyramids; • Circles and rectangles; • Polygons in terms of the number of sides up to eight-sided figures	We know this when the learner; Recognizes, visualizes and names two-dimensional and three-dimensional objects in natural and cultural forms and geometric settings including those previously dealt with and focusing on: • Similarities and differences between cubes and rectangular prisms; • Similarities and differences between squares and rectangles	We know this when the learner; Recognizes, visualizes and names two-dimensional and three-dimensional objects in natural and cultural forms and geometric settings including those previously dealt with and focusing on: • Similarities and differences between tetrahedrons and other pyramids; • Similarities and differences between rectangles and parallelograms
Describes, sorts and compares two-dimensional shapes and three-dimensional objects from the environment according to geometrical properties including: • Shapes of faces • Number of sides • Flat and curved surfaces, straight and curved sides	Describes, sorts and compares two-dimensional shapes and three-dimensional objects from the environment and from drawings or pictures according to properties including: • Number and/or shape of faces • Number and/or length of sides	Describes, and classifies two-dimensional shapes and three-dimensional objects in terms of properties including: • Faces, vertices and edges; • length of sides; • angle size of corners
Investigates and compares (alone or as a member of a group or team) two-dimensional shapes and three-dimensional objects studied in this grade according to properties listed above by: • Three-dimensional models using cut-out polygons (supplied) • Drawing shapes on grid paper.	Investigates and compares (alone or as a member of a group or team) two-dimensional shapes and three-dimensional objects studied in this grade according to properties listed above by: • making models of geometric objects using polygons they have cut out; • cutting open models or geometric objects (e.g. boxes) to trace their nets; • Drawing shapes on grid paper.	Investigates and compares (alone or as a member of a group or team) two-dimensional shapes and three-dimensional objects studied in this grade according to properties listed above by: • Making three-dimensional models using ; • Drinking straws to make a skeleton, • Nets provided by the teacher; • Drawing shapes on grid paper; • Using a pair of compasses to draw circles, patterns in circles and patterns with circles.

Grade 4	Grade 5	Grade 6
Recognizes and describes lines of symmetry in two-dimensional shapes, including those in nature and its cultural art forms.	Recognizes, describes and performs rotations (turns), reflections (flips) and translations (slides) using geometric figures and solids	Uses the vocabulary and properties of rotations, reflections and translations to describe the relationships between distinct two-dimensional shapes and three-dimensional objects within patterns (including transformations and symmetry).
Makes two-dimensional shapes, three dimensional objects and patterns from geometric objects and shapes (e.g. tangrams) with a focus on tiling (tessellation) and line symmetry.	Makes two-dimensional shapes, three dimensional objects and patterns from geometric shapes and describes these in terms of: • Tessellations; • Line and rotational symmetry; • Movement including rotations, reflections and translations	Draws enlargements and reductions of two-dimensional shapes (at least quadrilaterals and triangles) using grid paper to compare their size and shape
Recognizes and describes natural and cultural two-dimensional shapes, three-dimensional objects and patterns in terms of geometric properties.	Recognizes and describes natural and cultural two-dimensional shapes, three-dimensional objects and patterns in terms of geometric properties.	Recognizes and describes natural and cultural two-dimensional shapes, three-dimensional objects and patterns in terms of geometric properties.
Describes changes in the view of an object held in different positions	Describes and sketches views of a simple three-dimensional object in different positions	Draws and interprets sketches of simple three-dimensional objects from different positions (perspectives)
Locates position on a coded (labelled) grid including • Maps from given instructions • Column and row	Locates position on a coded (labelled) grid including maps and traces a path between positions following verbal and written instructions	Locates position on a coded grid, describes how to move between positions on the grid, and recognizes maps as grids.

Table 4.1(cont) Assessment Standards for Learning Outcome 3 of the RNCS (South Africa, 2002) for the Intermediate Phase (Grade 4-6).

In terms of two-dimensional shapes, the focus of this research, learners at the end of Grade 6 need to be able to:

- Recognize and name quadrilaterals such as squares, rectangles and parallelograms.
- Classify shapes such as triangles and quadrilaterals on the basis of the length of their sides and the size of the interior angles.
- Discuss the similarities and differences in the properties of rectangles, squares and parallelograms.
- Describe relationships between triangles and quadrilaterals using symmetry and transformations such as rotations and translations.

4.3.2 Grade 6 Geometry Module at the Research School

Earlier in this chapter, I reported that the majority of the learners at the research school receive no Geometry tuition in Grades 4 or 5. However, during the fourth term in Grade 6, all of the learners receive a printed module (Appendix G) covering the following aspects of Geometry:

- Definitions of the terms
  - point, "segment ", ray, line,
  - parallel and perpendicular lines, diagonal lines, horizontal and vertical lines,
  - circumference, diameter, radius,
  - angle, vertex, right angle, acute, obtuse, straight and reflex angles,
  - quadrilateral,
  - triangle, equilateral, right-angled triangle, hypotenuse, isosceles triangle,
  - Pictures of a square, rectangle, parallelogram and rhombus.
- Instructions on how to:
  - Bisect a given line segment.
  - Bisect a given angle.
  - Bisect a circle.
  - Construct a rectangle.
  - Construct a square.
  - Construct a perpendicular line on a horizontal line segment.
  - Construct a triangle, given the length of all three sides.
  - Construct a triangle given the length of two sides and the included angle (this definition is not given, simply the length of the two sides and the size of the angle between them).
  - Construct an isosceles triangle given the length of the three sides. The two angles on the "base" are the same.
  - Use a protractor.

- o Construct an equilateral triangle.

- Definition about the sum of the angles inside a triangle adding to 180°. Four examples of different triangles showing this concept.
- A worksheet of constructions to practise which included a few questions of the type “what type of triangle have you drawn?”
- Two worksheets on measuring angles.

Based on the above, the focus of the Geometry module appears to be on the learning of definitions and the acquisition of selected geometrical skills. These skills include the construction of triangles, squares and rectangles and the calculation of the third angle in a triangle if the size of the other two angles is given. The material in the module appears to follow Gagne’s Instructional Theory (Gagne, 1988), in which he suggests that Geometry teachers encourage their learners to practice geometric skills repeatedly until they become automatic. The geometric skills that are listed below are included in the Grade 6 Geometry module:

- Measuring angles using a protractor.
- Using a pair of compasses to do constructions such as bisecting an angle.
- Using a ruler, protractor and pair of compasses to construct polygons.
- Calculating the size of the third interior angle of a triangle, when the other two are known.

It is a simple task to assess whether a child has acquired the above skills or learned the definitions. The end-of-year Geometry assessment, which is discussed below, is designed to do just that.

4.3.3 End of Year Geometry Assessment

The end of year Grade 6 Geometry assessment (Appendix H) is divided into two parts:

Part One: Terminology

This part consists of twenty questions, giving definitions and asking for the name.

Examples: "A triangle that has all three sides equal is called a?"

"How many end points has a segment got?"

There are no diagrams on the question paper.

Part Two: "Problem Solving"

This part is also out of 20 marks.

- Question 1 involves measuring two angles using a protractor.
- Question 2 shows two diagrams, one a triangle and the other two adjacent angles on a straight line. The learners are asked to compute the size of an angle in each diagram. The first question requires an understanding that the angles in a triangle add up to 180°. The second question requires that a child understand that the angles on a straight line are supplementary. However, there is no statement that the figure is a straight line. The learners have to assume that it is a straight line, which is not good practice.
- Question 3 asks the child to draw an angle of a particular size using a protractor and to bisect it.
- Question 4 involves the construction of a circle with a given radius. The learner has to calculate the size of the diameter.
- Question 5 requires the learner to construct a square, to draw in the diagonals and to measure them.
- Questions 6 and 7 ask the learner to construct triangles, given the measurement of the three sides to construct an isosceles triangle and two angles and a side to construct an acute-angled scalene triangle.

Based on the above considerations, it is evident that the emphasis of the Geometry assessment at the end of Grade 6 was based on recall of definitions, and the ability to do constructions, and a few simple calculations.

I included the results the learners obtained for Part 2 of the Geometry assessment in Table 4.1 which summarizes background information about the fourteen learners who were interviewed during this study. This table appears in Section 4.4. The results that the learners achieved for Part 2 have been converted to percentages for easy reference. Part 1 contained memory recall questions and so I did not think that the results for this part of the assessment were relevant in a study on geometrical understanding.

As the next section compares the RNCS (South Africa, 2002), the Geometry module and the van Hiele Levels, I have included a brief summary of the first three van Hiele Levels here:

- Level 1 (the visual level) is where learners recognize a shape by its appearance (van Hiele, 1999).
- Level 2 (the descriptive level) is where figures are "bearers of their properties" (van Hiele, 1999:311).
- Level 3 (informal deductive level) is where properties become logically ordered and one property follows from another property (van Hiele, 1999).

To differentiate between the use of the term 'level' and the van Hiele levels, the specific van Hiele levels are referred to as "Level 1", 'Level 2" and so on in the analysis.

4.3.4 Comparison between the RNCS, the Geometry module and the Van Hiele levels

If one compares the Assessment Standards required by the RNCS (South Africa, 2002) for Space and Shape, the Geometry module taught at the research school and the Van Hiele levels of geometrical thinking, one finds that:

- Provision is made in the RNCS (South Africa, 2002) for the gradual development of concepts such as the properties of two-dimensional shapes from grades 4 to 6. Grade 4 learners are required to, "recognize, visualize and name two-dimensional shapes... in their environment" (RNCS, 2002:48) but, during Grade 5, learners are required to, "recognize, visualize and name two-dimensional shapes...focusing on similarities and differences between squares and rectangles" (RNCS, 2002:48). This gradual development of concepts does not happen during the ordinary Mathematics classes at the research school because all the Geometry is left to the last term in Grade 6. The learners who attend the Mathematics Academy, however, do get the opportunity to work with geometrical shapes from Grade 4.

- By the end of Grade 5, learners should be able to "describe, sort and compare two-dimensional shapes...from drawings or pictures according to properties such as number and/ or length of sides" (RNCS, 2002:48). This would link with van Hiele Level two. According to van der Walle (2004), at this level, learners should be able to sort a collection of shapes into groups with common properties. As most of the learners at the research school do not do any work with shapes during Grades 4 and 5, many of them may have not attained this Assessment Standard by the end of Grade 5.

- The RNCS (South Africa, 2002) suggests that, by the end of Grade 6, learners should be able to describe and classify two-dimensional shapes in terms of angle size of corners and length of sides. This corresponds with Level 2 on the van Hiele hierarchy of geometrical thinking. Van Hiele (1999) stated that at Level 2 learners should be able to describe the properties of a rectangle, that is, that it has four sides, the opposite sides are parallel, the opposite sides are the same length, it has four right angles and the diagonals are the same length. Irrelevant features such as size or orientation should become insignificant. The activities for exploring the properties of shapes in the Grade 6 Geometry module are limited to following instructions to construct quadrilaterals and triangles (Appendix G). The construction of a two-dimensional shape such as a square, for example, may help a child to discover that all four sides are equal in length or that the size of each interior angle within a square is ninety degrees. However, research suggests that children need to have the opportunity to investigate many different squares to develop their ideas about squares (van der Walle, 2004).

- Although the RNCS (South Africa, 2002) requires that the learners learn the language of transformations and be able to perform translations, rotations and reflections of figures, this was not covered at all in the module. According to the RNCS (South Africa, 2002), learners in Grade 5 are required to be able to make two-dimensional shapes and describe them in terms of "tessellations, line and rotational symmetry and movement including rotations, reflections and translations" (RNCS, 2002:51). Some of the learners who were interviewed during the research were unable to identify a square once it had been rotated through 45°. They were unable to see that the square had simply been transformed by using a rotation but it was still a square. Learners given sufficient practice in manipulating shapes or drawing them on grid paper will be able to understand that a square or rectangle retains its properties even though it may have been moved somewhere else, "flipped" over or rotated. Transformations are new additions to the Grade 6 curriculum and it is possible that it is threatening for the teachers to cover topics with which they are not familiar.

- The RNCS (South Africa, 2002) encourages the development of geometric vocabulary and the practical investigation of three-dimensional objects from Grade 4 onwards. Learners should be able to describe properties such as number and length of sides of two-dimensional figures. Learners should be able to describe properties such as number of faces, vertices and edges of three-dimensional figures (RNCS, 2002:49). This concept development is necessary for the learners to be able to visualize three-dimensional objects in order to calculate their surface area and volume in the higher grades in the school.

Many authors have emphasized the importance of helping learners acquire the correct vocabulary of geometric terms in order to help learners express their ideas about shapes (Feza and Webb, 2005; Roux, 2005; Van Hiele, 1986). The Geometry module that is taught in the research school contains no material on three-dimensional objects. I checked with the teacher-in-charge of Technology in Grade 6 and discovered that they have not covered three-dimensional objects in Technology either. Although the main focus of this research is the concept development of two-dimensional shapes, the RNCS (South Africa, 2002) places two- and three- dimensional shapes together in each Assessment Standard for each Grade. Three-dimensional shapes are composed of two-dimensional faces, for example, a cube has six square faces. Geometry teachers should expose their learners to two-dimensional shapes and three-dimensional objects in order to meet the requirements of the RNCS (South Africa, 2002).

- The RNCS (South Africa, 2002) states that Intermediate Phase learners should investigate and compare (on their own or in groups) two-dimensional shapes by drawing shapes on grid paper or making three dimensional models (RNCS, 2002:51). The Geometry module (Appendix G) contains none of the above-mentioned activities. Van Hiele (1999) recommended that learners should have multiple opportunities to manipulate and explore shapes. Van der Walle (2004) suggested that learners in the visual level (Level 1), which is possibly the level that many learners in Grades 4 to 6 are, should be sorting, identifying and describing different shapes using many physical models that the learners can manipulate.

Learners should also be given the opportunity to build, make and draw shapes and to take three-dimensional objects apart and put them back together again. Van Hiele (1999) recommended that learners be given shapes like the ones in traditional tangram sets. They can be encouraged to arrange them to make other shapes and to investigate questions like "which shapes can be made from two other shapes?" The RNCS (South Africa, 2002) makes mention of tangrams for Grade 4 learners to help them understand tessellations. Van Hiele (1999) stated that instruction needs to be at the learners' level of thought. The learners in Grades 4 and 5 at the research school do none of the above activities to develop their understanding of shapes. Although the Geometry module done in Grade 6 does contain construction activities for the learners, there are no other practical activities such as the ones discussed above.

4.4 Learner profiles

I asked each learner for information about their home language, the sports they play and the Academies they had attended during the year to develop a "thick" description (Henning *et al.*, 2004:85) of each learner. As the literature I had read indicated the importance of spatial activities in developing geometrical understanding (van Hiele, 1986; van der Walle, 2004; Way, 2007), I asked the respondents about their prior experience with activities involving spatial skills such as jigsaw puzzles, tangrams and building blocks.

The information about the learners is summarized in Table 4.2 on the next page. To indicate that a learner attended the Mathematics Academy, for example, I placed a cross in the relevant cell. No cross indicates that they do not participate in a particular activity. The table also includes the result each learner obtained for Part 2 of the Geometry assessment at the end of term 4.

Learner	Gender	Home language	Math Acad	Art Acad	Computer Acad	Chess	Sports	Jigsaw	Tangram	Lego ®	Geo Assess
A	F	Eng	X	X			3	500		X	95
B	M	Xhosa					0	25	X	X	95
C	M	Eng	X		X	X	3	1000	X	X	95
D	F	Eng					3	100		X	30
E	F	Eng			X		1	25	X	X	60
F	F	Eng	X			X	3	1000	X		100
G	F	Xhosa					3	100	X		100
H	M	Eng	X			X	3	500	X	X	75
I	M	Eng	X	X	X		2	100	X	X	95
J	M	E/Creole	X			X	2	1000	X	X	/
K	F	Xho/Zulu		X			2	50	X	X	80
L	F	Eng	X		X		3	500		X	50
M	M	Xhosa					0			X	50
N	M	Xhosa					2	20		X	75

Table 4.2 Summary of learner information.

From the table, it can be seen that seven of the learners were girls and seven were boys. Eight of the fourteen children spoke only English at home. Four spoke Xhosa, one spoke Xhosa with a little English and one child, who was born in Mauritius, spoke English and Creole at home. Seven of the children were in the school's Mathematics Academy. Five learners, of whom four were English Second Language (ESL) learners, did not attend any of the Academies.

Four of the learners, three boys and one girl, had played chess when they were in the Foundation Phase. I was interested in whether the learners had played with jigsaw puzzles as Maxfield (1975) suggested that the use of jigsaw puzzles improves geometrical understanding among younger children. Five of the learners had done jigsaw puzzles recently with 500 or more pieces. All five of these learners were also in the Mathematics Academy. One learner had never done a jigsaw puzzle to his knowledge and two of the learners had only completed 20- to 25- piece puzzles. One learner had completed a 50-piece puzzle. None of these four learners was in the Mathematics Academy.

All the learners had either very little or no experience with tangram pieces. One of the learners remembered using the tangram shapes at the Grade 6 Leadership Camp, earlier that year. They had been used in one of the team building exercises on the camp. None of the learners had used the tangrams during their Intermediate Phase schooling. Three had never seen them before and another three learners had played with them in Grade 1. The RNCS (South Africa, 2002) specifically mentions that Grade 4 learners should use tangrams to make shapes and to investigate tessellations. In the literature, many authors including van Hiele (1999) also suggest the use of activities such as playing with tangrams to help develop children's geometrical thinking. Van der Walle (2004) said, "A good way to explore shapes at Level 1 is to use smaller shapes or [tangrams] to create larger shapes" (van der Walle, 2004:315). Way (2007) suggested the use of tangrams in a structured programme to help children to, "progress from simply recognizing some shapes, to being able to discuss the shapes in terms of specific geometric properties and perhaps make some comparisons between shapes" (Way, 2007:5). These authors reinforce the requirement of the RNCS (South Africa, 2002) to use tangrams or cut-out shapes in the Foundation and Intermediate Phases to help children develop their understanding of the properties of two-dimensional shapes.

I asked the learners whether they had played with Lego® blocks because some of the literature suggests that playing with building blocks such as Lego® may improve spatial perception. Goldenberg and Cuoco suggested that,

> Certain visualization skills correlate well with, and plausibly contribute to, better mathematical achievement in the upper grades and college. In turn these skills are themselves plausibly honed through informal experiences at home like building models, manipulating structured visual materials like blocks or Legos and taking things apart and putting them back together.
>
> (Goldenberg and Cuoco, 1998:6)

Nine of the learners had played with Lego® blocks, building cars (learners A, C and I), rockets and space ships (learner C), houses and garages (learner I) and, self-designed buildings (learner J). Three of the remaining learners had played with the younger child's version of Lego®, called Duplo®. Of the seven learners in the Mathematics Academy, six had played with Lego®. The one exception was the girl, who came first in the grade in Mathematics, winning the Mathematics prize at the annual prize giving. She had never played with Lego®. She had an older brother who played with it but told me that she has "no interest" in it.

4.5 Learner responses to questions

During the interviews, I used an interview response sheet (Appendix E) and a tape recorder to record the learner responses. I provided diagrams, plastic shapes, a picture of a house and K'NEX® which were used by the respondents during the interviews. The responses to selected questions are discussed in detail below. For some of the questions, I have summarized the learners' responses into a table for easy reference and for comparison of responses. In other questions, clear trends emerged and so I discussed the trends among the learners' responses without using a table.

4.5.1 Naming Shapes

The respondents were shown five plastic shapes and asked to name them. The shapes were a triangle, square; rectangle, parallelogram and hexagon. The purpose of this question was simply to ascertain whether the respondents were able to identify shapes. Van der Walle says that, at van Hiele Level 1; "Students should recognize and name figures based on the global, visual characteristics of the figure (shape)" (van der Walle, 2004:309).

All of the learners could identify the triangle, square and rectangle with no difficulty. A few of the learners struggled to name the hexagon but, once they had counted the sides, they quickly remembered the correct name. Only two of the learners could correctly identify the parallelogram. Five learners named it a rhombus. One of the five said it was "a rhombus or something", which could indicate that she was not certain that it was a rhombus but that it was similar to a rhombus. The RNCS (South Africa, 2002) requires that learners in Grade 6, "Recognize, visualize and name two-dimensional and three-dimensional objects in natural and cultural forms and geometric settings including those previously dealt with and focusing on ... similarities and differences between rectangles and parallelograms" (RNCS, 2002:49) It should be noted that there is a sketch of a parallelogram in the Geometry module (Appendix G), next to a sketch of a rectangle. There is also a sketch of a rhombus next to a square. Possibly this is done to give the learners the visual impression of the similarity between a rectangle and parallelogram, a square and a rhombus. I suspect that not much time is spent on developing the concept of parallelograms and this may be the reason the learners were unable to identify them.

4.5.2 Identifying triangles

The respondents were shown the diagrams represented below and asked to identify which of the diagrams were triangles.

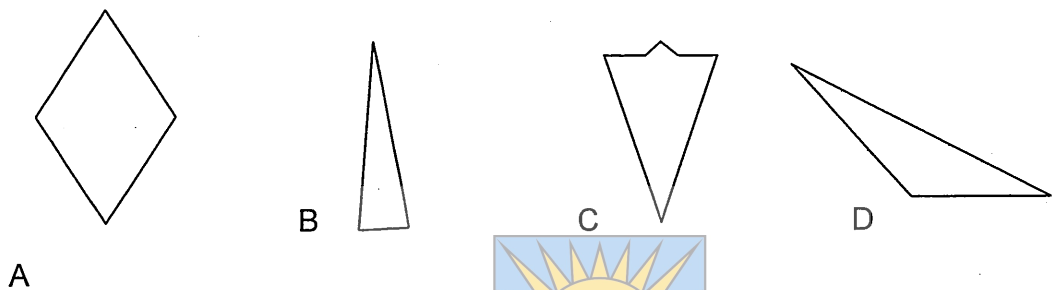


Figure 4.1 Question 1 "Which are triangles?"

The respondents' responses are summarized in Table 4.3 below:

Learner	Shapes chosen	Comment
A	B,D	
B	A,B,C,D	All have the same shape
C	B,D	
D	B,D,	A is two triangles
E	B,D	A is sort of. Not sure
F	B,D	
G	B	
H	B,D	
I	B,D,	
J	D	B is not a triangle
K	B,D	
L	B,D,	
M	B	C could be. D looked strange
N	B,C,D	

Table 4:3 Learner responses to question on identifying triangles.

All the learners, except learner J, correctly identified the acute-angled triangle (B) from Figure 4.1 as a triangle. Learner J was not sure whether the shape labelled B in figure 4.1 was a triangle as can be seen from the following extract from his interview:

Interviewer: In these pictures, can you point out which ones are triangles?

Learner J: This one here (points to D).

Interviewer: Ok.

Learner J: Must I look for more?

Interviewer: Do you think there might not be any more?

Learner J: I'm not sure about this one (indicates B).

Interviewer: Why are you not sure about this one?

Learner J: It may not add up to 180 degrees because..... no, it is not a triangle.

Learner J appeared to think that a shape can only be a triangle if the sum of the interior angles is 180°. This type of reasoning was also demonstrated by learner D as is illustrated by the extract from her interview below:

Interviewer: Tell me the letters of the ones you think are triangles.

Learner D: (Points to A). This is two triangles (Points to B). This could be a triangle. It is not equilateral. It is isosceles. C is not a triangle and this one could be (points to obtuse-angled triangle D). I just need a thingamabobby to measure it.

Interviewer: What would you measure to tell you whether it was a triangle?

Learner D: The degrees.

Interviewer: And what about the degrees?

Learner D: All the corners of the degrees should add up to 180.

Interviewer: So what would you do then to see if that was true?

Learner D: Um. I would put the ... (whispers to herself) what is it called again?

Interviewer: Is it that little instrument that measures angles?

Learner D: Yes. I can't think of the name. (Gets very agitated)

Interviewer: It's a protractor.

Learner D: A protractor. Yes. I would put it over the thingamabobby there and measure. Say, I would say it would be 100 and that would be 90. No. Say 50, say and that would be 30 and it would have to add up to 180 or it would not be a triangle.

It is interesting to note that, when learner D was suggesting possible values for the size of the angles inside a triangle, she corrected her estimate of ninety degrees for the second angle inside the triangle. She already had an angle of one hundred degrees in size and she realized that, if her second angle was ninety degrees, then the sum of the angles inside the triangle would be greater than 180 degrees.

Both learner D and learner J were in Mr L.'s class and it may be possible that he emphasized the sum of the interior angles of a triangle to his learners. However, these learners seem to have missed the fact that all triangles have three sides. Instead, they seem to be operating at a level that states: "if the sum of the interior angles is 180 degrees, then the figure must be a triangle." At van Hiele's level of informal deduction (Level 3), learners begin to develop "if .... then" reasoning (van der Walle, 2004:310). The correct statement would be that "if a figure is a triangle, then the sum of its interior angles is 180 degrees". I don't think that learners D and J are operating at van Hiele's Level 3 as they did not seem to understand all the properties of a triangle. The fact that learner D thought that shape A in Figure 4.1 was two triangles and that learner J was not sure whether shape B was a triangle at all indicates that they were probably operating at Level 1 of van Hiele's hierarchy. These learners in Mr L's class need to be given different triangular shapes to investigate so that they can develop an understanding of the properties a triangle has that make it different from other two-dimensional shapes.

Learner D seemed to be having difficulty finding the vocabulary she needed to express her ideas about shapes. She used the word "thingamabobby" twice in place of the term that she needed. She also used the phrase "the corners of the degrees" when she wanted to talk about the size of the angles inside a triangle. She was an English mother-tongue speaker which suggests that it is not only ESL learners who have problems with the language used in Geometry. Roux (2005) warned that a lack of proficiency in English comprehension and vocabulary are major stumbling blocks in learning Geometry. To overcome this problem, Roux (2005) recommended that teachers give their learners many opportunities in class to practice mathematical language.

Two of the learners did not identify the obtuse-angled triangle D as a triangle. Wu (2005) found that among the Grades 1 to 6 learners he tested for recognition of shapes in Taiwan, 34,15 % were unable to identify obtuse-angled triangles and 56,15% of the same group of learners were unable to identify obtuse-angled quadrilaterals such as parallelograms. He concluded that, in his study, "extremely obtuse figures are the most difficult (for learners)" (Wu and Ma, 2005:335).

Learner M said that shape D, (the obtuse-angled triangle) in Figure 4.1 "looked strange". It must be noted that there were no sketches of obtuse-angled triangles in the Geometry module that the Grade 6 learners at the research school studied. In the module, learners are given exercises to construct scalene, isosceles and equilateral triangles but none of the constructions involves an obtuse-angled triangle. In fact, in one of the lessons I observed, a boy asked Mr L whether there was such a thing as an obtuse-angled isosceles triangle and he was told categorically that there was not. Van der Walle suggested that teachers "include many different and varied examples of shapes so that irrelevant features do not become important" (van der Walle, 2004:311). The obtuse angle would be an irrelevant feature in an isosceles triangle. A child (or teacher) who fully understands the concept that an isosceles triangle has two sides of equal length would be able to see that the obtuse angle is an irrelevant feature. Katagiri recommended that when children are developing a concept of a shape, they be shown many different examples of the shape so that they can "elicit commonalities ... to clarify shared properties" (Katagiri, 2004:23). It appears that there is a need for more experiences with different triangles in this Grade 6 class so that they can realize that any three-sided shape is a triangle, whether it has an obtuse angle or not.

Learner B thought that all four of the shapes were triangles because, "They are made in the shape of triangles." According to the van Hiele hierarchy, learner B was operating at Level 1, the visual level. In fact, he may have been operating at what Clements and Battista (2001) describe as the pre-visual level, where a learner focuses on the overall appearance of the shapes rather than being able to identify specific shapes. Learner B's choice of the rhombus as being a triangle reinforces this conclusion

If one considers the responses of the fourteen learners to this question, only seven of the fourteen correctly chose only shapes B and D as triangles. Two of the learners, D and E who chose shapes B and D were unsure about whether to include shape A as a triangle. Shape A is a kite. Learners D and E were not confident that all triangles have the property of three sides. This would indicate that these two learners had not yet reached van Hiele's Level 2 where shapes are considered to be "bearers of their properties" (van Hiele, 1984:246). They appeared to be experiencing a "crisis of thinking" (van Hiele, 1986:46) during the interview because they were hesitant and indecisive about whether shape A could be considered a triangle or not. The remaining five learners seemed to be operating at van Hiele's Level 1, the visual level (or even the pre-recognition level in the case of learner B) as their responses clearly indicated that they had not yet realised that a triangle has three sides.

4.5.3 Identifying squares

The respondents were shown four diagrams of quadrilaterals and they had to identify which diagrams represented squares. The diagrams the learners were shown are represented below:

Which are squares?

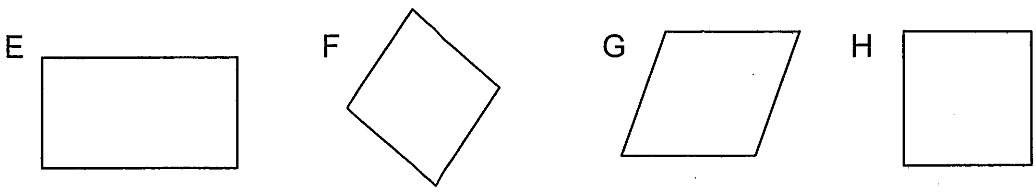


Figure 4.2 Question 2 “which shapes are squares?”

The responses from the learners are summarized in Table 4.4 below.

Learner	Shape	Comment
A	H	Maybe F
B	F,G,H	G is a skew square
C	H	Checked F very carefully
D	E,F,H	E is two squares
E	F,G,H	
F	G,H	Maybe F
G	F,H,	
H	H	
I	F,H	
J	H	
K	H,F	F only if turned back
L	H	
M	H	Definitely only H
N	F,H	

Table 4:4 Learner responses to question on identifying squares

All the learners correctly identified quadrilateral H in Figure 4.2 as a square. Nine of the learners considered shape F very carefully. Seven of the nine learners decided that F was a square that had been rotated. One of the nine learners, learner C, examined shape F closely, turned the page and decided it was not a square.

Learners G and M were adamant that only H was a square and they would not discuss the possibility that F could be a square. This could indicate that they are still at the visual level of geometric reasoning (Level 1). At this level, the learner focuses on the appearance of a shape and if a shape has been rotated it becomes a different shape altogether (van der Walle, 2004).

Three of the learners claimed that G was also a square. Whereas there is an element of doubt about shape F as the sides look equal in length and the angles all appear to be ninety degrees, there is nothing about shape G to suggest that it could be a square. Both learners B and E described figure G as a "skew square". Learner F also identified shape G as a square. When asked why, she replied, "F and G are just turned to the side so they turn at an angle".

I have heard mathematics teachers describe a rhombus as a "skew square" or "a square that has been pushed over". For a child who has not had the opportunity to discover the properties of rhombi and squares for himself, this could lead to confusion of concepts. If a child has heard that a rhombus is a skew square, the child might think that a rhombus is still a square. How does a child differentiate the fact that a square rotated 45° is still a square but a square "pushed over" is not a square? This understanding should come from playing with square shapes of different sizes and rotating or reflecting them. This could be the reason the RNCS (South Africa, 2002) requires that learners be taught transformations from Grades 4 to 6. Once a child understands what transformations are, he is better able to understand that a square retains its properties if it is simply moved from one place to another, reflected about a line or rotated through 45°. However, if a figure does not have four right angles and four sides of equal length, then it cannot be a square.

4.5.4 Identifying rectangles

The learners were shown the three shapes found in Figure 4.3 and asked to identify which of the shapes are rectangles. Their responses are summarized in Table 4.5.

Which are rectangles?

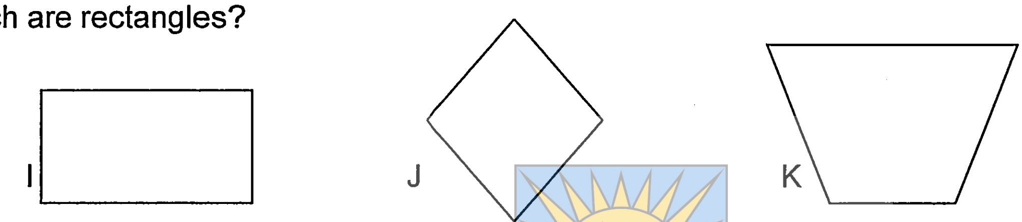


Figure 4.3 Question 3 "Which shapes are rectangles?"

Learner	Shape I	Shape J	Shape K
A	X		
B	X		
C	X		
D	X		
E	X		
F	X		
G	X		
H	X		
I	X		
J	X		
K	X		
L	X		
M		X	
N	X		

Table 4.5 Learner responses to question identifying rectangles

Every learner correctly identified shape I in Figure 4.3 as a rectangle. Learner N hesitated over whether shape K was a rectangle but decided that it couldn't be as the opposite sides were not the same length.

Learner A said that maybe shape J was also a rectangle and the discussion is recorded below:

Learner A:	Maybe J as well.
Interviewer:	Why do you say J?
Learner A:	Because those two could be longer than that and those two could be the same as those two and those could be different.
Interviewer:	How could you test that?
Learner A:	By measuring them.
Interviewer:	How would you do that?
Learner A:	Use a ruler.

It seems that learner A thought that, for a shape to be a rectangle the opposite sides have to be the same length and that one pair of sides has to be longer than the other pair of sides. This could be a concept that has developed from looking at the sketch of a rectangle in the Geometry module (Appendix G). Thus, if she had measured both pairs of sides and found them all to be the same length, it seems that she might not have considered the figure to be a rectangle. I did not have a ruler with me during the interviews as I did not consider that one was needed. I did notice, however, that on a few occasions, learners mentioned measuring the length of the sides of shapes. Sometimes, they picked up one of the other shapes and used it to help them compare the lengths of the sides of shapes. This suggests that the learners were operating at Piaget's concrete operational stage (Farrell and Farmer, 1980). Children at the concrete stage of cognitive development need to measure and compare physical examples of shapes in order to talk about their properties (Farrell and Farmer, 1980). In terms of van Hiele's levels of geometrical understanding, learners who want to measure the lengths of the sides of a shape are moving from judging the shape on the basis of its appearance (Level 1) to determining its properties (Level 2). Learner A did not mention that the four corners of the shape needed to be right angles for the shape to be a rectangle. I noticed frequently that the learners were more comfortable discussing the length of the sides of the shapes rather than the size of the angles.

From the responses of the learners interviewed in this case study, it appeared that they had understood the concept of a rectangle and could identify one from a sketch. The learners seemed to have understood that the opposite sides of rectangles have equal length. This could be a result of the construction of a rectangle they did in the Grade 6 Geometry module (Appendix G). This reinforces what the literature says about practical activities involving shapes helping learners to develop an understanding of the properties of the shapes (Farrell and Farmer, 1980; van Hiele, 1999; van der Walle, 2004; Way 2005).

4.5.5 Identifying parallelograms

The learners were shown three shapes and asked to identify which of them were parallelograms. In fact, all three of the shapes are parallelograms. Having covered the learner responses to the question on identifying rectangles, I will discuss, in more detail, the learner responses to the question asking them to identify parallelograms. The reason for this is that, in my experience, parallelograms are not easily understood by learners. The diagrams that the learners had to use are pictured below:

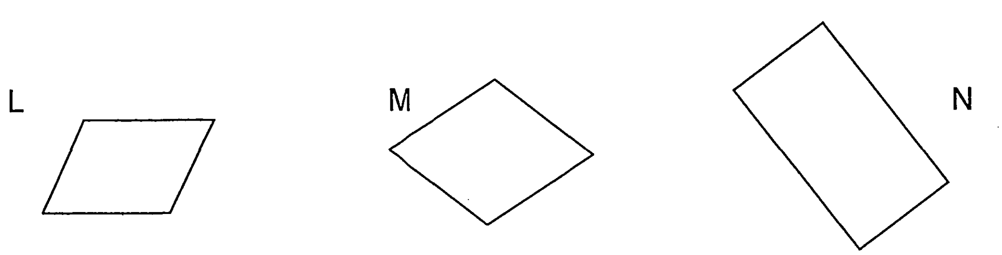


Figure 4.4 Question 4 "Which shapes are parallelograms?"

The learner responses have been summarized in Table 4.6.

Learner	Shape	Comment
A	L,M,N	
B	L,M,N	
C	L,M,N	All parallel
D	L	It is diagonal but not quite
E	L,M	
F	L,M	N is a rectangle not a parallelogram
G	L	It is a square pushed to the side
H	L,M,N	
I	L,M	Because of the angle of the lines
J	L	
K	L,N	Lines same distance apart
L	L,M,N	
M	L,M,N	Opposite sides are parallel
N	L	

Table 4.6 Learner responses to question on identifying parallelograms.

I had checked that the learners knew what parallel lines were earlier in the interview (see Section 4.5.6). By focusing their attention on parallel lines, it may have helped some of them to identify which ones could be parallelograms as the extract below indicates:

Interviewer: This shape you couldn't remember the name of is called a parallelogram. Can you remember what parallel lines are?

Learner B: Yes. The lines that are the same distance apart.

Interviewer: Can you see any parallel lines in this room?

Learner B: Yes.

Interviewer: Where can you see them?

Learner B: That line to this one here (*points to the two vertical sides of the window*).

Interviewer: You mean from here to here? These two are parallel?

Learner B: Yes.

Interviewer: Can you see any parallel lines in this shape here? (*interviewer shows the learner the plastic shape of a parallelogram).*

Learner B: There and there. (*indicates a pair of opposite sides*).

Interviewer: That's right and that is why it is called a parallelogram. It says here "Which ones are parallelograms?"

Learner B: Er. This one.

Interviewer: Yes.

Learner B: This one is parallel to that one and this one to this one.

Interviewer: So you think they are all parallelograms?

Learner B: Yes.

It could be that the discussion we had had about parallel lines immediately prior to Learner B's looking at the three shapes in Question 4, had focused his attention on the property that parallelograms have parallel sides. This had enabled him to identify all three as parallelograms. The conversation I had with learner B could be considered to be the first two instructional "phases" (van Hiele, 1984) which help children to develop a higher level of geometrical understanding. The first phase, "information" (van Hiele, 1984) involves gathering information about a shape. During the interview discussed above, the learner was shown a shape and told it was a parallelogram. During the "guided orientation" phase (van Hiele, 1984) the learner explored parallel lines in the room and in the parallelogram shape. This could have enabled the learner to realize that all three shapes had the property of opposite parallel sides and so they were all parallelograms.

Nine of the fourteen learners correctly identified figures L and M as parallelograms. Not all the learners had a discussion about parallel lines immediately prior to this question. Some of them were very confident that they knew exactly what a parallelogram was. The extract from the interview with learner C indicates this:

Interviewer: Which ones over there are parallelograms?

Learner C: All of them.

Interviewer: Why do you say all of them - that was very quick?

Learner C: Because all of them are parallel. That line is parallel to that one. That line is parallel to that one and that one, that one and that one is parallel.

Learner C had a good understanding of what makes a shape a parallelogram and knew that two pairs of opposite sides must be parallel. He also had a good understanding of what parallel lines are and so there was no need for me to discuss parallel lines with him.

Four of the learners were certain that shape L was the only parallelogram. Shape L resembles the diagram they were given in their Grade 6 Geometry modules, showing what a parallelogram looks like. The parallelogram represented in their Geometry module is slanted, at an angle. There is no mention in the module that the rectangle next to it is also a parallelogram. It could be that they remembered the diagram in the module they had done during class and this enabled them to identify it easily.

Extracts from the interviews with three learners illustrate the confusion surrounding the concept of parallelograms.

Interviewer: Look at these pictures. Which ones are parallelograms?

Learner D: L. I think that is a rectangle (*it isn't*), and M could be if you turned it that way, (*whispers*) like you turned it that way.

Interviewer: Why do you think these are parallelograms? For example, why do you think L is a parallelogram?

Learner D: Because it is a diagonal almost but not quite.

Learner I had a similar notion about what makes a figure a parallelogram as this extract from his interview illustrates:

Interviewer: Now which ones here are parallelograms?

Learner I: This one

Interviewer: Just L or are any of the others parallelograms?

Learner I: And M.

Interviewer: And why are L and M parallelograms?

Learner I: Because of the angle of the lines.

Interviewer: What about that one? (*Interviewer indicates N).* Is that one not a parallelogram?

Learner I: No.

Both learners (D and I) had the concept that a parallelogram has to have a "diagonal" or slanted shape. These two learners appeared to be operating at van Hiele's visual level (Level 1) when they had to identify parallelograms. At Level 2 (van Hiele's descriptive level), learners should be able to describe the properties of shapes (van der Walle, 2004). However, learners D and I had not understood that in parallelograms the opposite sides are parallel and therefore they were not operating at van Hiele's Level 2 of geometrical understanding.

A third learner who experienced difficulties with the concept of parallelograms was learner F as can be seen from the extract from her interview:

Interviewer: Look at the next one. Which ones there are parallelograms?

Learner F: L and M.

Interviewer: Why do you think these are parallelograms?

Learner F: Umm. I don't know.

Interviewer: You seem very certain that they are parallelograms.

Learner F: Ja. I'm not too sure but I just thought they were.

Interviewer: What made you think L and M are parallelograms but not N?

Learner F: N is a rectangle instead.

Learner F also thought that a rectangle cannot be a parallelogram. The idea that parallelograms have to be slanted was prevalent among a number of learners in the study group, despite the RNCS (South Africa, 2002) requiring that, by the end of Grade

6, learners should be familiar with "...the similarities and differences between rectangles and parallelograms" (RNCS, 2002:49). The intention of the RNCS (South Africa, 2002) is that learners should look at many examples of parallelograms and rectangles so that they can determine what properties these shapes have in common and how they differ from each other. This coincides with the recommendations made by van Hiele (1986) about how children develop an understanding of the properties of shapes. As mentioned previously, Katagiri (2004) recommended that when children are developing a concept of a shape, they should be shown many different examples of the shape so that they can "elicit commonalities ... to clarify shared properties" (Katagiri, 2004:23). It seems that these three Grade 6 learners (D, F and I) have not had sufficient experiences with different parallelograms to realize that squares, rhombi and rectangles are all parallelograms as they all have the property of opposite parallel sides.

During the interview with learner J, he expressed a very common misconception about parallel lines.

Interviewer: Which ones here are parallelograms?

Learner J: This one only.

Interviewer: Why is that one a parallelogram?

Learner J: Cos they are parallel lines. These two are parallel to each other and these two are parallel to each other.

Interviewer: Ok. And what about these ones? You say they are not parallelograms.

Learner J: No.

Interviewer: Why not?

Learner J: Wait. No. These are not parallel. These are all the same measurement.

Interviewer: So, if they are the same measurement, they cannot be parallel?

Learner J: They can be ..... But I think .... No. I am confused now. I don't think those ones are but I don't have an explanation for it.

Earlier, during his interview, learner J told me that, "When lines are parallel to each other, they go like this. One is on top and one is at the bottom and they never meet." Upon being questioned, this learner, who thought that he knew what parallel lines were, realized that he was not absolutely sure whether parallel lines are the same length or not. This learner became confused and agitated and experienced what van Hiele termed a "crisis in thinking" (van Hiele, 1986:45). This would have been the ideal time for learner J to investigate pairs of lines of different lengths, to measure their length and to measure the distance that they are apart so that he could have realised that parallel lines do not have to be the same length. Such experiences enable a child to construct his own understanding (Duckworth, 1996), which is far more valuable than giving learners ready-made definitions to learn (De Villiers, 1995).

In my experience at high school, learners often confuse the concept of parallel-lines with lines of equal length. For example, in Grade 9, when they are required to prove that two triangles are congruent, they know that they need to prove sides and angles in the two triangles equal to each other. However, many of them will use the fact that two lines are parallel to each other to prove that the lines must be equal. This example illustrates what van Hiele (1986) said about misconceptions developed in the lower grades having an influence on the child's ability to cope with formal Geometry higher up in the school.

4.5.6 Terminology: parallel and perpendicular lines

The learners were asked to explain what they understood by the terms "parallel lines" and "perpendicular lines". These questions were not on the original interview schedule but I realized during the interviews that it was important to establish the learners' understandings of these concepts. This was one of the reasons I used open, semi-structured interviews in my methodology. It enabled me to identify concepts that were problematic for the learners during the interviews and gave me the flexibility to add or change questions as needed. I had noted that Feza and Webb (2005) also used open and semi-structured interviews in their research into the level of geometrical thinking among Grade 7 learners. This enabled them to further investigate the answers given by their respondents and I wanted to be able to do this too.

I asked the question, "What are parallel lines?" Table 4.7 indicates the Mathematics teacher for each learner and the examples of parallel lines that the learners gave.

Learner	Teacher	Lines never meet but run the same way	The distance between them remains the same	Other comment	Example
A	C		X		Two vertical sides of the window frame
B	C		X		Two vertical sides of the window frame
C	C		X	Two lines that cross are not parallel	
E	C			When they are skew and when they are straight.	
F	C	X	X		
G	C			No definition but can identify a parallelogram.	
K	C		X		Two vertical sides of window
N	C		X		
D	L	X		Two lines that are constant and never meet	Two vertical sides of the window
H	L		X	Opposite, same distance apart and never touch	The vertical opposite sides of the window pane
I	L		X	Run same distance apart all the way and they carry on	No parallel lines seen in the room.
J	L	X		One at the top and one at the bottom and they never meet	Horizontal lines of window pane
L	L			No definition but could identify parallel lines in figures	
M	L	X			

Table 4.7 Answers to the question: "What are parallel lines?"

I have grouped the learner responses according to their teachers to try to identify any trends among the responses. Some of the learners could not describe parallel lines to me. Among the learners who could describe parallel lines, the two main responses were:

- "Parallel lines never meet but run the same way."
- "The distance between parallel lines remains the same."

The uniformity of the responses might be attributable to the introduction to the Grade 6 Geometry module (Appendix G) which has a photocopy of the instruction sheet found inside the tin that contains a Geometry set used in schools. The sheet gives definitions of shapes and useful symbols and units used in Mathematics. Its definition of parallel lines states that "These are lines drawn in the same direction which never meet and always remain the same distance apart."

Eight of the fourteen learners gave the definition that "parallel lines are the same distance apart" or "the distance between them remains the same". Six of these learners were in Mrs C's class. I heard her use this definition during one of the Geometry lessons I observed. The other two learners, E and G, from her class did not give a definition as I did not ask them to describe parallel lines. Thus, eight of the learners I interviewed were from Mrs C's class and all those who were asked about the meaning of the term "parallel lines" used the definition supplied by the teacher (part of which is found in the definition in the module). When I observed Mrs C in her classroom, she used repetition constantly as a means of helping learners to memorize the names of figures and geometrical terms. In fact, she started each lesson with a revision of the terms they were using in the module.

A number of researchers have emphasized the need for learners to understand the vocabulary used in Geometry (Nel, 2006; Roux, 2005; Rubenstein and Thompson, 2002). Roux (2005) stated that ESL learners are at high risk of not attaining the van Hiele Levels 1 to 3 in classrooms where the medium of instruction is English. Four of the five ESL learners that I interviewed were from Mrs C's class. The technique that Mrs

C used may be an effective way of helping learners to acquire the vocabulary needed in Geometry but does it lead to understanding of the concepts involved? The danger with learning definitions off by heart is that learners parrot sentences without having an understanding of what they are saying. Van Hiele (1984) said:

> The teacher reasons by means of a system of relations that he alone possesses. Starting with this system, he explains the mathematical relations that the students end up manipulating by rote. Or else the student learns by rote to operate with those relations he does not understand, and of which he has not seen the origin.
>
> (van Hiele, 1984:244)

This suggests that teachers need to steer clear of drilling definitions but should concentrate on giving learners sufficient concrete experiences with shapes and opportunities to talk about their experiences with each other so that the learners can develop their own understanding of the concepts involved.

Three of the six learners who were in Mr L's class gave the definition that parallel lines never meet but 'run the same way'. I heard Mr L use this definition while teaching his class. This may be his interpretation of the phrase "...lines drawn in the same direction, which never meet...." which was at the start of the module. I think that learners in Grade 6 would be able to manage the phrase "drawn in the same direction". The phrase "run the same way" could confuse certain learners. The word "way" has at least two meanings. It could mean "direction" but it could also mean "a method or manner of achieving an end"(Reader's Digest Universal Dictionary, 1987:1696). Rubenstein and Thompson (2002) warned against using language in the Mathematics classroom that could be unclear or ambiguous. This would be particularly important for ESL learners.

Two of Mr L's learners gave the same definition as Mrs C's learners. They may have learned the definition which appeared in the introduction to the module. However, in my experience, some learners do not learn the information in their modules and rely on what their teachers repeat in order to learn terms and definitions. De Villiers had this to say about teachers giving their learners definitions to learn:

> Another problem with just providing pupils with ready-made definitions is that the misconception is invariably created that there is only one correct definition for each concept.
>
> (De Villiers, 1995:3)

Learners who have been taught definitions by their teachers may think that the definition that they have been given is the only acceptable definition. This may encourage them to learn definitions by heart without understanding the concepts involved. To overcome this problem, de Villiers (1995) recommended that the teacher give learners the opportunity in class to construct their own definitions and to discuss them with one another. This is the method that Constructivists would use (De Villiers, 1995). This method may lead to a better understanding of the concepts involved.

The constructions of squares and rectangles in the worksheets in the Grade 6 Geometry module (Appendix G) start with the drawing of the base of the figure, given a particular length. At each end of the line segment that is used to form the base, the learners are instructed to construct a perpendicular line. They are told that these perpendicular lines are parallel to each other. These lines are vertical parallel lines.

Of the seven learners asked to identify parallel lines in the interview room, five of them pointed to the vertical lines formed by the sides of the window. One learner mentioned the horizontal lines on the window frame and one learner said he was unable to find any parallel lines in the room. During the lesson I observed in Mrs C's class, where she was revising what parallel lines were, she used the example of the two vertical sides of the window frame and then she asked the children to look around the classroom for parallel lines. I noticed that all of Mrs C's learners used the example of the two vertical sides of the window frame when I asked them to find an example of parallel lines in the interview room. This seems to indicate that the learners rely on examples used by their teachers in class when they are asked to identify parallel lines in a new situation.

Feza and Webb (2005) found that, when they asked Grade 7 learners for an example of parallel lines, all of the learners mentioned telegraph lines. Feza and Webb (2005) suggested that the Mathematics teachers of these learners had a limited understanding of parallel lines themselves and that the teachers possibly relied on an example in a textbook to illustrate parallel lines to their learners. This reinforces a finding in my case study, that children repeated an example that they heard their teachers use when they were asked to give an example of parallel lines.

Roux (2005), van der Walle (2004), van Hiele (1999) and Way (2005) all recommend that, in order to help children develop an understanding of a concept such as parallel lines, they need to have the opportunity to work with as many different examples of parallel lines as possible. Unless learners work with parallel lines of different lengths, in different orientations, in different figures, preferably in groups to give them the opportunity to verbalize their thoughts (Roux, 2005), they will not be able to construct their own understanding of parallel lines. Duckworth (1996) emphasized that learners who are able to construct their own mathematical experiences develop a far deeper understanding of the concepts involved than if they are merely given the subject matter to memorize.

On my interview schedule (Appendix D) there was a question asking whether the two diagonals inside a square are perpendicular to each other. I decided to check that the learners understood what perpendicular lines were before they attempted to answer this question. Of the fourteen learners, only four were able to give a satisfactory explanation of perpendicular lines.

Learner I said perpendicular lines would be equal in length. Learner D was unable to verbalize what she understood perpendicular lines to be but she demonstrated them by crossing her fingers over each other. Learner H said that, “if you take a triangle and cut it in half, the line down is perpendicular.” Learner N simply said that “they joined - they cross each other.” Two of the learners told me that they couldn’t remember what perpendicular lines were. Once again, these answers point towards a need in the classroom to talk about mathematical terms. Learners who do not know what these terms mean or who are unable to use them may be unable to follow explanations in the classroom that involve these terms.

4.5.7 Verbal Description of a Square

The respondents were shown a yellow plastic square shape and asked to describe the shape to a friend over the telephone. They were asked not to use the word square during the description. At van Hiele’s Level 2 learners should be able to focus on what it is that makes a square a square. Properties that could be mentioned simultaneously are four equal sides, both pairs of opposite sides parallel to each other, four right angles, diagonals of equal length which bisect the angles and diagonals that bisect each other at right angles. The problem posed to the learners required that they select the minimum of relevant information to describe the square so that the person they were speaking to on the phone would be able to know it was a square they were describing. Their responses are summarised in Table 4.8.

Learner	Description of square over the phone
A	It is yellow, has four right angles and both opposite sides are parallel
B	All four sides are equal.
C	It has four equal sides. Each vertex is 90° and they are all parallel.
D	It has four equal sides, each of 90° angle and all four lines are parallel.
E	It has four equal sides and it is yellow and all the measurements are the same.
F	It has four equal sides and the sides are parallel.
G	It looks like a block. It has four equal sides that are all equal.
H	It has four equal sides. Each angle is a right angle. The four angles inside add up to 360°. It can be bisected into two rectangles or two triangles.
I	It has four sides, each angle is 90°. It is solid and yellow.
J	It has four sides with the measurements the same.
K	It has four equal sides and is not a circle.
L	It has two parallel lines. It looks like some windows. It has four sides
M	It has four sides. It has this much cm. It has parallel sides and is yellow.
N	It is like a box. It has parallel lines that meet. They are perpendicular and at the top they also meet. It is the same at the top and the bottom.

Table 4.8 Learner descriptions of a square

Probably the most economical description that could be expected from learners in Grade 6 would be that the shape has four equal sides and four interior angles each measuring 90 degrees. Only three learners (C, D and H) managed to give this information about the shape, which would indicate that they were operating at van Hiele's Level 2 for the concept of squares.

Of the other learners, five stated that the four sides were equal and two mentioned that the angles were right angles. But none of those seven learners mentioned both properties together. The learners who stated that there were four equal sides did not realize that they could be describing a rhombus and so they needed to mention the right angles. The learners who only mentioned the right angles could have been describing a rectangle. The learners who mentioned the four equal sides or the four right angles have identified some of the properties of squares but not both properties that make a square different from other shapes. This suggests that they could be in transition between van Hiele's Level 1 towards Level 2.

It would appear that one of the learners was operating completely at van Hiele's visual level (Level 1) for this question. Learner N described the square as, "a box" and had great difficulty in describing any property of a square at all. He said that a square has "parallel lines that meet". Learner N's response to the question "What are parallel lines?" was that they are lines that are the same distance apart. Some learners used the phrase "They never meet." Thus the statement by learner N that the sides of a square are "parallel lines that meet" possibly indicates either confusion about what parallel lines are or a lack of vocabulary to express his ideas about the shape. He went on to say that, in a square, the parallel lines that meet are also perpendicular. It seemed to me that he was visualizing the construction of a square that occurred in his Geometry module (Appendix G) where they drew the base, constructed two perpendicular lines at either end of the base and were told that these two lines are parallel. Learner N was an ESL learner and it could be that, due to his lack of vocabulary, he had tried to memorize sections of the module. I noted in his interview that he often struggled to find the language to express his ideas. When he was asked whether he found English difficult, he said, "I struggle with the commas but I like creative writing".

Van Hiele (1986) suggested that if teachers use geometrical terms which the learners do not understand, during Geometry lessons, the learners will not be able to understand the concepts being discussed. This, in turn, may prevent the learners from progressing to the next level of geometrical understanding. It is possible that learner N's lack of proficiency in English has impeded his progress from van Hiele's Level 1 to Level 2. Thus learner N may be at risk of not being able to develop an understanding of the geometrical concepts discussed in class. Roux (2005) found that, for all children, one of the key factors that prevented them from achieving the higher levels of geometrical understanding was their lack of proficiency in language comprehension and general vocabulary in the medium of instruction. Roux recommended that:

> Mathematics teachers will have to give attention to use of language as

> well as specific subject language in the Geometry class. This includes correct use of language by the teacher, practising of geometric terminology, as well as monitoring of the use of language by the learners.
>
> (Roux, 2005:9)

This suggests that Geometry teachers need to pay careful attention to the language they use to introduce new concepts and to ensure that their learners acquire the vocabulary they need to talk about geometrical ideas. There is also a need for the learners to have opportunities in class to talk about geometrical concepts using their own words. Structuring lessons to include the above activities could help ESL learners like learner N to progress to a higher level of geometrical understanding.

4.5.8 Rotation of Square

The learners were shown the plastic square that was used in Section 4.5.7 which was then rotated through 45°. They were asked whether the shape was still a square. The diagram below illustrates the original orientation of the square and its appearance after it had been rotated through 45°.

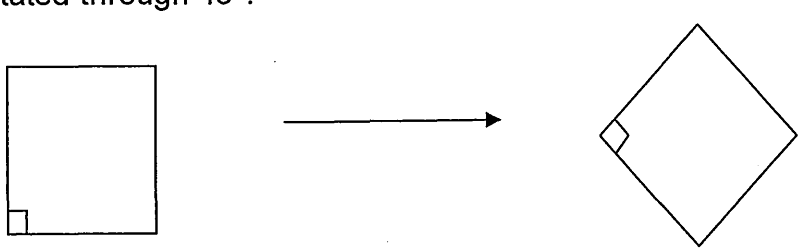


Figure 4.5 Square rotated through 45°.

According to van Hiele (1984), children who judge shapes on the basis of their appearance are operating at Level 1 (the visual level). Van Hiele said that, "At the [visual] level, figures are determined by their properties but someone thinking at this level is not aware of those properties" (van Hiele, 1984:246). This suggests that a child operating at the visual level would be able to recognize a square because it looks like a square. However, when the square is rotated through 45°, it may not look like a square any more as its orientation is different. The fact that the shape retains the properties of four equal sides and four right angles and thus is still a square is not apparent to a child operating at Level 1 of van Hiele's hierarchy.

Van der Walle (2004) suggested that children at the visual level would say that the shape was no longer a square because it did not look like one. Those who have progressed to Level 2 would be able to see that the rotated shape was still a square. Table 4.9 documents the learners' responses.

Learner	Is it still a square?	Explanation
A	Yes	The measurement is still the same-the lines and corners
B	Yes	The shape doesn't change. Just the angle
C	Yes	Even if you turn it the corners are still 90° and they are all the same length.
D	No	It is the shape of a diamond. It could be a square if you turned it.
E	Yes	Its been moved (or turned)
F	Yes	The sides are still equal but it has been turned at an angle
G	No	It is a diamond – it is not straight and the opposite sides are not equal
H	Yes and No	It could be a diamond or a rhombus
I	Yes	All the sides are still 90°
J	No	It is a diamond
K	Yes	The sides still stay the same
L	Yes	Still has 4 right angles and 4 sides the same length
M	No	It is the wrong shape. It is a diamond
N	Yes	It is still a square. It just got moved. It is a diamond.

Table 4.9 Responses to whether a square rotated 45° is still a square.

Nine of the fourteen learners interviewed in this study said that the shape was still a square. Four of the five remaining learners (D, G, J and M) said that the shape was no longer a square. It had become a diamond. Learners G, J and M were also unable to give an accurate description of a square over the phone in the previous question

(Section 4.5.7). They focused on only one of the properties of squares. It would appear that these three learners had not yet attained van Hiele's Level 2.

Learner D, who described the rotated square as a diamond, said it would be a square if you turned it back to its original position. It seems that learner D has focused on the horizontal/vertical orientation of the square as illustrated in the Grade 6 Geometry module. The horizontal/ vertical orientation is illustrated in Figure 4.6.

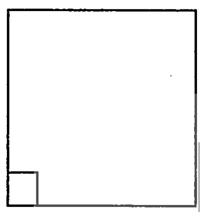


Figure 4.6 Horizontal/vertical orientation of a square.

Katagiri (2004) said that, when a teacher introduces a new geometrical concept to learners, he needs to show the learners as many examples of the concept as possible to prevent the learners from focusing on only one aspect of the concept. If learners are only shown examples of squares in the horizontal/vertical orientation, they may think that orientation is a necessary prerequisite for a square.

When learner H was asked whether the rotated shape was still a square, he said "yes and no". However, he went on to say it was now either a diamond or a rhombus. Neither a diamond nor a rhombus is a square. The term "diamond" is not a geometrical term. I think the children mean "kite" rather than "diamond". A kite is a quadrilateral with two pairs of adjacent equal sides as illustrated in Figure 4.7.

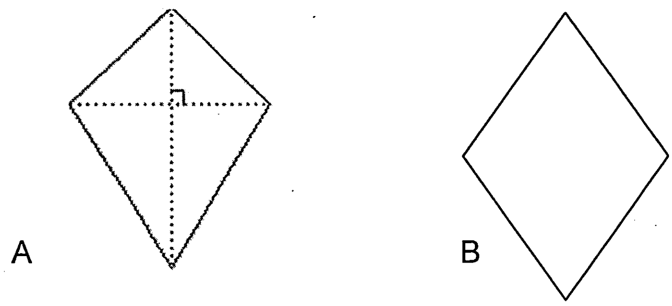


Figure 4.7 Kites.

Of the six children who used the term "diamond", four were in Mr L's class. Van Hiele (1999) recommended that teachers should help children to build concepts gradually and to learn the correct vocabulary for specific concepts. Roux (2005) also suggested that Mathematics teachers use correct terminology in the Geometry class and that they monitor the use of geometrical language by their learners. It may be that Mr L introduced the term "diamond" to his class during a class discussion and this is why two-thirds of the learners interviewed from his class used this term. However, this is not the correct terminology and may also lead to incorrect concept development. It would have been better to use the term "kite". In Figure 4.6 shape B looks like the "diamond" shape that appears in pictures in children's story books. However, in Geometry, shape B is a kite. It would have been good educational practice to have given the children different examples of kites so that they could have developed the correct concept of a kite's properties.

Interestingly, learner N, who was identified as being at the visual level (Level 1) in the previous question, demonstrated that he was probably still at the first level in this question. Although he said that the shape was still a square, he also said: "it just got moved. It is now a diamond".

4.5.9 Sorting of shapes into groups

The learners were given a selection of two-dimensional plastic shapes and asked to sort them into groups. They were told they could decide themselves on how to sort the groups. The shapes were parallelograms, rectangles, squares, an isosceles triangle, an equilateral triangle and a hexagon. Table 4.10 below indicates how many groups each learner made and what the groups contained. There were three types of responses:

- Some learners sorted the shapes into three groups according to the number of sides. So they had one group with three-sided shapes, one with four-sided shapes and one with six-sided shapes.
- Other learners sorted the shapes into five groups, having the groups mentioned above but separating the four-sided group into rectangles, squares and parallelograms.
- A few learners sorted the shapes into six groups, separating both the four-sided and the three-sided shapes. The isosceles and equilateral triangles were separated to make two groups of three-sided figures.

Learner	No of Groups	No of Sides	Different Shape	Divided Quadrilaterals
A	3	X		
B	5			X
C	3	X		
D	3	X		
E	6		X	
F	5			X
G	3	X		
H	3	X		
I	6		X	
J	5			X
K	3	X		
L	3	X		
M	5			X
N	6		X	

Table 4.10 Number of groups of two-dimensional shapes made by learners.

My interview with learner J presented some interesting insights. Learner J made four groups initially by dividing the shapes according to their colour. The shapes were different colours and this was the most striking feature for him. An extract from his interview follows:

Interviewer:	I am giving you these shapes and I want you to sort them into groups for me. You can choose how you want to sort them.
Learner J:	Um. Can you repeat the question?
Interviewer:	All right. Can you take these shapes and sort them into groups, as many groups as you like.
Learner J:	Do I have to make a shape with them?
Interviewer:	No you don't. You just need to think how could I sort these out? How could I separate them?
Learner J:	I would do that and that ... *(makes 4 groups).*
Interviewer:	How have you sorted them?
Learner J:	So I can see the difference between them – the colour.
Interviewer:	All right. So you have sorted them according to colour. Now what if I asked you to sort them according to shape, what would you do?
Learner J:	I would put all the squares together and all the rectangles together and all the triangles together.
Interviewer:	Let's quickly do that. How many groups do you have now?
Learner J:	Five.

I had asked learner J to sort the shapes into groups. I assumed he would sort them according to the number of sides or the shapes as I was thinking in terms of geometrical concepts. For learner J, the most striking feature of the shapes was their colour and he quite correctly sorted them into groups of different colour. Van Hiele (1984) said:

> "Two people who reason at two different levels cannot understand each other. Neither can manage to follow the thought process of the other and their dialogue can only proceed if the teacher tries to form for himself an idea of the student's thinking and to conform to it... A true dialogue must be established at the level of the students."
>
> (van Hiele, 1984:246)

Learner J was at a different level of geometrical understanding from me and thus we were thinking very differently from each other. When learner J explained to me the reasoning behind his sorting of the shapes, I understood why he had sorted the shapes the way he had. If he had not had the opportunity to explain his reasoning to me, I might have considered his answer to be incorrect. This demonstrates the importance of teacher/learner discussion in class. Teachers who make the opportunity to have discussions such as the one above with their learners in class will be able to affirm their learners or to correct misconceptions that their learners have developed.

Learner M did not sort the shapes into groups but made pictures with his shapes at first. He made a house by placing a triangle above a square. Then he put two triangles together to form a square. Learner M is an English second language (ESL) learner who had never played with tangrams or done jigsaw puzzles. He seemed to be playing with the shapes. This could possibly be because he had not had the opportunity at a younger age to play with two-dimensional shapes such as these.

The RNCS (South Africa, 2002) requires that learners in Grade 4 "Make two-dimensional shapes, three dimensional objects and patterns from geometric objects and shapes (e.g. tangrams) with a focus on tiling (tessellation) and line symmetry" (RNCS, 2002:51). Thus, even if children have not had the opportunity in their homes to play with shapes, the RNCS (South Africa, 2002) makes provision for them to do so during their Geometry lessons in Grade 4. As no Geometry was done at the research school in Grades 4 and 5, learner M had been denied the opportunity at school to explore shapes such as these. Pierre van Hiele (1984) said that children need to manipulate shapes to gather information about them during "a process of apprenticeship which leads to higher levels of thought" (van Hiele, 1984:247). I watched learner M play with the shapes and then asked him whether he could separate the shapes into different groups of shapes that had something in common. He then put the shapes into five groups, putting the triangles in a group, the hexagon on its own and the quadrilaterals into three different groups – rectangles, squares and parallelograms. When he was asked how he had separated them, he said he had put the shapes that look the same together. This would indicate that learner M was still operating at the visual level, where shapes "are judged by appearance" (van Hiele, 1984:245).

Learners E and N each started by stacking similar shapes vertically on top of each other from the largest at the bottom to the smallest on the top of the pile. They were comparing the shapes while they did this. According to Fuys, Geddes, Lovett and Tischler (1988) learners who are at van Hiele's Level 1 (the visual level) of geometrical understanding sort shapes on the basis of appearance as a whole. This would suggest that learners E and N were operating at Level 1 in this shape-sorting activity. Learner N put the equilateral and isosceles triangles in separate groups. Then he put them all together again, thought carefully and separated them, saying "I want them in two separate groups". He counted the number of groups that he had and found he had four groups. He had the parallelograms, rectangles and squares together and then separated them saying: "Some are isosceles and some are equilateral". This indicates he was confused about the terminology "isosceles" and "equilateral", using these terms to describe quadrilaterals as well as triangles. Learner N was an ESL learner. Feza and Webb (2005), who researched the geometrical understanding of Xhosa-speaking Grade 7 learners in rural schools in the Eastern Cape, found that:

> "…language competency in general is a barrier to the attainment of higher levels of understanding among this group of learners… This raises the issue of how to overcome language as a barrier for learners who speak English as a second language, a universal feature in multilingual societies."
>
> (Feza and Webb, 2005:45)

We live in a multicultural society in South Africa. Five of the fourteen learners interviewed for this research were ESL learners, speaking Xhosa or Zulu as their home language. As mentioned previously in this chapter, Roux (2005) recommended that teachers pay special attention to the use of language during Geometry lessons and that they give their learners the opportunity to practice the new terms in class. This is particularly important for ESL learners. If learner N had been given the opportunity to talk about "isosceles and equilateral" quadrilaterals in class, he might have discovered that he was using the terms incorrectly for quadrilaterals.

When the learners were questioned about how they had divided the shapes into groups, learners B, E, F, I, J, M and N all said they had put figures that had the "same shape" together. All the other learners had separated them according to the number of sides they had. According to van Hiele (1999) learners would tend to put shapes together because they look alike if they were operating at van Hiele's Level 1, the visual level. This would suggest that these seven learners were operating at the visual level in this activity.

4.5.10 Comparisons between rectangles, squares and parallelograms

In the RNCS (South Africa, 2002), one of the Assessment Standards for Grade 5, Space and Shape, is that learners should have investigated the similarities and differences between rectangles and squares. They are also required in Grade 6 to investigate the similarities and differences between rectangles and parallelograms.

I placed the plastic square and rectangular shapes into one group and asked the learners what the shapes had in common that would enable them to be placed in a group together, separately from the parallelogram. Their responses are recorded in Table 4.11.

Learner	4 Sides	Right angles	Opposite sides equal	Opposite sides parallel	Shape
A		X		X	
B	X				X
C		X			
D		X			
E				X	
F	X			X	
G	X			X	
H		X			
I	X	X			
J			X	X	
K					X
L		X			
M				X	
N	X			X	

Table 4.11 Reasons for placing rectangles and squares in the same group

Only six of the learners identified the unique property that the rectangle and square have in common; that they both have right angles. Van Hiele (1984) said that children who are at Level 2 are able to see that, "[Shapes] are bearers of their properties. That a [shape] is a rectangle means it has four right angles, the diagonals are equal and the opposite sides are equal" (van Hiele, 1984:245). As these six learners were able to say that both the square and rectangle contain right angles, they were possibly operating at van Hiele's Level 2 in this instance.

The properties that were mentioned by six of the other eight learners such as "opposite sides are equal" or "opposite sides are parallel" or "they all have four sides" are not properties that differentiate rectangles and squares from parallelograms. Van Hiele (1984) said that, at van Hiele Level 1: "[Shapes] are determined by their properties but someone thinking at this level is not aware of these properties" (van Hiele, 1984:246). The learners who mentioned these properties are beginning to be aware that shapes have properties but they are not able to identify the properties that are unique to a particular shape as yet. This may indicate that these six learners were in transition between van Hiele's Level 1 and 2.

The remaining two learners (B and K) both gave the response that the rectangle and square have a different shape from the parallelogram. Their use of the word "shape" indicated that they were focusing on the appearance of the rectangle and square which means that they were probably at van Hiele's Level 1, the visual level. Van Hiele said, "A child (at the visual level) recognizes a rectangle by its form and a rectangle is different from a square" (van Hiele, 1984:245). I have included an extract from the interviews with learners B and K to illustrate their reasoning.

Learner B: They are kind of the same ... Um ...Shape

Interviewer: In which way are they kind of the same?

Learner B: The square. If this wasn't that long, it would have been a square. (*points to the rectangle*)

Interviewer: What about it would make it a square if you made it shorter?

Learner B: If you put it into half.

Interviewer: Ok. You mean that if I cut it in half it might be a square. What is it that makes it look like a square, do you think?

Learner B: Uh. If you like cut it in half.

To learner B, a rectangle was just a "longer" square. The overall appearance of the two shapes was almost the same for him. In the interview with learner K, I took the parallelogram away from the group with rectangles and squares and asked her why it had been taken away.

Learner K: They all have four sides but it's like it has a different shape.

Interviewer: Why can we put the squares and rectangles together in the same group?

Learner K: They are a bit the same. They might be tall or short but long-short, but they are the same.

Both learners B and K were ESL learners. It appeared from what they were saying in these extracts that they both lacked the vocabulary to express what they wanted to say about the shapes. They said that the square and rectangle seem to have the same shape but did not seem able to say that it was because both shapes have right angles at their four corners. This limited ability to express their ideas may very well hinder them in moving from van Hiele's Level 1 (the visual level) to Level 2 (the descriptive level) (van Hiele, 1986).

However, there were learners who spoke English as their mother tongue, who also experienced problems with expressing in words what they are thinking. This is demonstrated in the following extract:

Interviewer: I am going to take that one out. Why do you think I would take that one out? (*Interviewer removes the parallelogram from the group containing the rectangle, parallelogram and square).*

Learner C: Because these are all, how can I say it? Ninety degrees. Ninety degrees going up while ... this one can't be ninety degrees because the lines can't be straight up.

Learner C also had difficulty in finding the correct geometrical terminology to describe what he was thinking. He tried to say that the parallelogram did not have right angles but the rectangle and square do. This underscores the need for all learners, regardless of whether they are ESL learners or not, to be helped to develop a mathematical vocabulary and to practice expressing their ideas aloud (Roux, 2005).

4.5.11 Placement of a rhombus into a group

I pushed two equilateral triangles together to make a rhombus. The learners were asked into which group of shapes they would place the rhombus. The options they had were as follows:

- Parallelograms
- Rectangles
- Squares
- Triangles

Table 4.12 indicates the groups they chose and their reasoning.

Learner	Group	Reason
A	parallelograms	Same kind of shape. Both lean to an angle
B	parallelograms	They have 4 sides and a sharp point
C	parallelograms	The angles are not 90° like a square or rectangle
D	parallelograms	They do not have right angles
E	parallelograms	If you cut it in half it would make a triangle
F	None	I was going with parallelograms but then decided that it needs its own group
G	Squares	If you push squares over they make a rhombus and all 4 sides are equal
H	parallelograms	The opposite sides are parallel
I	Squares	All the sides look like they have the same length
J	Squares	All the measurements are the same
K	parallelograms or triangles	They look the same
L	parallelograms	The opposite sides are parallel
M	parallelograms	They look the same but a parallelogram is longer
N	triangles	It is made of triangles

Table 4.12 Learners' choice of a group for a rhombus.

Nine of the fourteen learners placed the rhombus into the group with parallelograms. The learners gave various reasons for doing this. Two of the learners (C and D) said that the rhombus did not have right angles and for that reason the rhombus could not be placed with either the squares or the rectangles. Learners H and L identified that the opposite sides of a rhombus are parallel, which is the same property that parallelograms have. These four learners (C, D, H and L) were considering the properties of rhombi and parallelograms and trying to identify which properties these shapes had in common. Being aware that "figures are bearers of their properties" (van Hiele, 1984:246) is characteristic of van Hiele's Level 2.

The other five learners (A, B, E, K and M) placed the rhombus with the parallelograms and said it was because the shapes looked the same. They mentioned features such as "they both have a sharp point" or stated that "the parallelogram is longer." These learners were using the appearance of the shapes to compare them, which is characteristic of van Hiele's Level 1 (van Hiele, 1999).

Learners E, F, K and N did not place the rhombus with the parallelograms. The groups they chose, and their reasons for their choices, indicated that they also seemed to be focusing on the appearance of the rhombus rather than its properties. Learner K told me that the rhombus looked the same as a triangle or a parallelogram. Learner N said that he would place the rhombus in the group of triangles because it was made of triangles. These responses indicated that learners E, F, K and N were still operating at the visual level (van Hiele's Level 1) because they were looking at the shape of the rhombus and trying to find another shape that resembled it.

Three of the learners (G, I and J) placed the rhombus with the squares. I think this may be because of the preceding remarks I made in the interviews with these learners. In each case, I pointed out that I was putting two equilateral triangles together to form the rhombus (as I had no rhomboid shape). We discussed the fact that equilateral triangles have three equal sides. This may have focused the learners' attention onto the property that a rhombus has four equal sides. Having just discussed this property, it would be a simple step to look for one of the shapes that also has four equal sides. However, even though these three learners may have focused on the fact that a rhombus has four sides of equal length, they did demonstrate that they were able to put shapes together on the basis of a common property. This type of thinking indicates a move towards the type of thinking found at van Hiele's Level 2.

As discussed in Section 4.5.1, I noticed that, in the Grade 6 Geometry module (Appendix G), the only mention of a rhombus was a diagram, labelled "rhombus" which was placed next to a diagram of a square. When I measured the length of the sides of the figure labelled "rhombus" which appeared in the Geometry module, the sides were not equal in length. This may have caused confusion for some of the Grade 6 learners and could be one of the reasons why the learners I interviewed seemed to be experiencing difficulty with the concept of a rhombus.

4.5.12 Using K'NEX® to build a triangle and a square

I gave the learners a set of K'NEX® Education Elementary Math and Geometry. The set consists of differently coloured rods of different lengths and connectors which are of three types. There are connectors which could be used to make an acute angle. Others make a right angle and others make angles of a variable size as they are adjustable. I demonstrated the various angle connectors and assembly and deconstruction techniques to each learner before asking them to do this activity. I did this to ensure that they would know how to use the materials. I asked the learners to build a triangle and a square from the rods and connectors. Table 4.13 summarizes how the learners carried out this activity.

Learner	Response to K'NEX®
A	Quickly worked out how to make the figures and did so accurately
B	Struggled to make figures. Triangle took 2,5 minutes and square 1,5 minutes.
C	Has this set at home. Rapidly made the shapes.
D	Struggled to use the connectors but, once mastered, quickly made shapes.
E	Made the shapes easily and fairly quickly
F	Struggled to make a square, used wrong angle size for corners of square.
G	Struggled. Could not place third side of triangle in place.
H	Took set eagerly and made shapes easily
I	Made shapes with ease
J	Pleased to use the set and made shapes easily
K	Could not make shapes. Took a long time and needed help
L	Made shapes quickly and easily
M	Struggled to make a triangle. Could only put 2 sides together.
N	Took 3 minutes with triangle and 4 minutes for square. Struggled with angles.

Table 4.13 Summary of learners' use of K'NEX® to make shapes.

Seven of the learners were delighted to be using the rods and connectors to build a triangle and a square. These learners were very adept at putting the rods and connectors together and rapidly discovered how to manipulate them. Learner C, one of the seven learners, told me that he had a set of K'NEX® at home and often used them to build three-dimensional objects. He used the rods and connectors correctly and made the required shapes rapidly. The ease with which he made a triangle and a square could partly be as a result of his prior experience with K'NEX® but it also indicated that he was aware of the properties of these two shapes. Van Hiele (1999) said that children who are able to work with properties of shapes are at Level 2 on the van Hiele hierarchy of geometrical thinking. Therefore a child at van Hiele's Level 2, who was asked to make a square would be able to select the connectors that made 90° angles for the four corners and four rods of the same length to make the sides of the square. The seven learners discussed above were all operating at van Hiele's Level 2 for this activity.

It is interesting to note that, of the seven learners I have discussed, six attend the Mathematics Academy and the seventh attends the Computer Academy. All seven learners had previous experiences with jigsaw puzzles, six of them doing puzzles of between 500 and 1 000 pieces. Five of the learners had played with tangrams and all seven learners had played with construction sets such as Lego® and Duplo®. This suggests that the exposure to toys and activities that involve spatial experiences may have helped these learners to use the unfamiliar apparatus in a geometrical activity. It may have also made them feel confident about their manipulative skills.

One of the seven learners discussed above (learner E) said "Oh no!" in dismay when given the sticks and asked to make a triangle with the sticks. Learner E was the one learner discussed above who had not had much prior experience with jigsaw puzzles (she had done 16-25 piece puzzles) and she had only played with Duplo®. However, once she had been shown how to use the rods and connectors, she quickly and accurately made the required shapes. She had the understanding of the properties of the triangle and square but lacked the confidence to use unfamiliar apparatus. Building confidence and familiarity with shapes is a good reason for exposing children to a wide variety of activities involving shapes during the Foundation and Intermediate Phases of schooling.

The remaining seven learners struggled to use the rods and connectors to build a triangle and a square. One learner took seven minutes to build the two shapes. It appeared that the main problem for these learners was trying to choose connectors for the corners that would complete their shapes. Some of them tried to make a triangle with two right-angled connectors, not realizing that they would not get the triangle to close. In terms of van Hiele's levels of geometrical understanding, the learners who tried to make a triangle by using two right-angled connectors were probably operating at van Hiele's Level 1 (the visual level). They had not understood that there can only be one right or obtuse angle in a triangle. This showed that they were not operating with the properties of the shape yet.

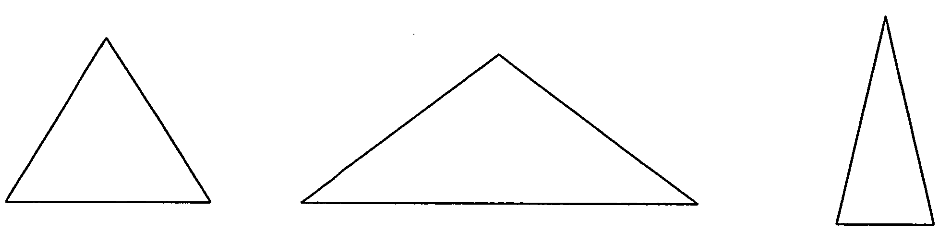
Van der Walle (2004) recommended that teachers give learners, "... lots of physical models that can be manipulated " (van der Walle, 2004:311) to encourage children who are at operating at van Hiele's Level 1 to develop to the next level. He also said that,

> The Geometry curriculum in Grades [1] to 8 should provide an opportunity to experience shapes in as many different forms as possible. These should include shapes built with blocks, sticks or tiles; shapes drawn on paper or with a computer; and shapes observed in art, nature and architecture. Hands-on reflective and interactive experiences are at the heart of good geometry activities at the [Foundation and Intermediate Phases].
>
> (van der Walle, 2004:306).

Van der Walle (2004) seemed to emphasize as many different experiences with shapes as possible for learners in the Intermediate Phase. This would also suggest that those learners who have played with toys involving spatial perception outside of the school environment have had extra opportunities to develop their geometrical understanding. Those learners who have not had such experiences at home need the opportunity to manipulate and reflect upon shapes at school. Unfortunately, if one studies the Geometry module (Appendix G) that the Grade 6 learners at the research school do, one notices that the only activities with shapes in the module are seven constructions of triangles, two squares and two rectangles. It appears that the constructions of shapes such as triangles and squares did not give some of the learners in this study the variety of experiences with shapes that they needed to develop an understanding of the properties of shapes. It also seems that if children only have the opportunity to construct shapes using pencil and paper, they may not be getting sufficient exposure to the type of activities that will help them to understand the properties of shapes. Teachers should try to expose their learners to a wide variety of experiences with physical shapes to build confidence and understanding of the properties of those shapes.

4.5.13 Question on Isosceles triangles

The learners were shown a printed page on which three triangles and a question on isosceles triangles was shown as illustrated in Figure 4.8.



An isosceles triangle is a triangle with 2 sides of equal length. Here are 3 isosceles triangles.

Which is true of every isosceles triangle?

a) The three sides have equal length.
b) One side is double the length of the other side.
c) There must be at least two angles of the same size.
d) 3 angles must be the same size.
e) None of (a) to (d) is true in every isosceles triangle.

Figure 4.8 Question on identifying properties of isosceles triangles.

This was a multiple choice question of the type that appears on the VGHT (University of Chicago, 2009). The learners were asked to select which one of the statements was true. The answers the learners chose are summarized in Table 4.14.

Learner	3 sides of equal length	One side double the length of the other side	At least two angles of same size	3 angles of same size	None is true
A					X
B			X		
C			X		
D			X		
E		X (In some)	X		
F			X (not sure. Maybe not all isosceles triangles)		X
G		X			
H			X		
I			X		
J			X		
K			X		
L			X		
M			X		
N			X		

Table 4.14 Learner responses about properties of isosceles triangles.

Most of the learners were able to identify the properties of isosceles triangles with no difficulty. What did cause problems for the learners, however, was my choice of the first diagram in this question. I had been alerted to the possibility that learners might be confused about the properties of isosceles and equilateral triangles during the pilot interview with my daughter (Appendix F) when she said that the first diagram was not an isosceles triangle. When I asked why not, she answered that it was an equilateral triangle. I decided not to change this diagram because the discussion I had with her during the pilot interview was so interesting, and I wanted to see what the other learners would say.

During the interviews, nine of the fourteen learners said that the first triangle was not isosceles. They insisted it was an equilateral triangle. When we discussed the properties of an equilateral triangle, they knew that an equilateral triangle has three equal sides. I asked the learners whether an equilateral triangle does in fact have two sides equal in length. They agreed that one of the properties of an isosceles triangle is that it has two sides of equal length. On reflection, seven of them accepted the fact that an equilateral triangle is a special type of isosceles triangle as it also has the properties of an isosceles triangle. Two of the learners remained unconvinced. This would indicate that they are unable to think logically about the properties of isosceles and equilateral triangles.

Van Hiele had this to say about the second level of geometrical thinking.

> "A pupil reaches the [second] stage of thinking as soon as he can manipulate the known characteristics of a pattern that is familiar to him. For example, he can associate the name 'isosceles triangle' with a specific triangle, knowing two sides are equal and to deduce that two angles are also equal."
>
> (van Hiele, 1984:238)

It would appear that the majority of the learners were able to operate at van Hiele's Level 2 in terms of understanding and manipulating the properties of isosceles and equilateral triangles. The learners who were unable to do this were probably still operating at van Hiele's Level 1.

4.5.14 Properties of Quadrilaterals

For this question, the learners were given the following shapes to look at:

- Rectangle
- Square
- Parallelogram

They were asked to read the following statements and say whether they were true or not.

(a) All rectangles are squares
(b) All parallelograms are squares
(c) All parallelograms are rectangles
(d) All rectangles are parallelograms

The learner responses are summarized in Table 4.15. A blank space indicates that the learner said the statement was false. The statements marked with a cross are the statements that the learners chose as being true.

Learner	(a) All rectangles are squares	(b) All parallelograms are squares	(c) All parallelograms are rectangles	(d) All rectangles are parallelograms	(e) None is true
A				X	
B	X				
C				X	
D			X		
E			X	X	
F			Could be true	Could be true	
G	X		X	X	
H				X	
I			X		
J			X	X	
K		X			
L				X	
M				X	
N					X

Table 4.15 Learner responses on properties of two-dimensional shapes

Two of the learners (learners C and H) read the statements aloud, confidently rejected options (a) to (c) and chose option (d) as the correct statement. Learner I was slightly more hesitant but also selected the correct option. Learners operating at van Hiele's Level 2 realize that each two-dimensional shape has properties that define it (van Hiele, 1984). However, van Hiele also stated that at Level 2, "Properties are not yet ordered and so a square is not necessarily identified as being a rectangle" (van Hiele, 1984:246). Van Hiele (1984) claimed that it is at Level 3, the level of informal deduction, that properties of shapes can be deduced from one another. These three learners were able to work with the properties of rectangles and squares and to realize that all squares must be rectangles as they have all the properties of rectangles. However, not all rectangles are squares as rectangles do not necessarily have all four sides of equal length. All three learners, learners (C, H and I) had attained Level 2 of the van Hiele hierarchy and were possibly operating at Level 3 for the properties of squares, rectangles and parallelograms.

The interviews with two of the learners demonstrated that they were experiencing problems with understanding the properties of quadrilaterals and the terminology used in talking about quadrilaterals. Both of the learners were ESL learners and spoke Xhosa at home. The first extract is from the interview with learner B and it was evident that he was operating at van Hiele's Level 1 (the visual level). He was unable to consider relationships between different classes of shapes and tended to focus on the appearance of the shapes.

Interviewer:	Have a look at the shapes and then decide whether the sentence is true.
Learner B:	All triangles are squares. Yes. *(Learner B misread the word "rectangle" in the first statement and called it a triangle. The statement is actually "all rectangles are squares")*
Interviewer:	You think all rectangles are squares. Why do you say that?
Learner B:	Because they have the same shape. Except they longer.
Interviewer:	All right. What about the next one.
Learner B:	All parallelograms are squares. No.
Interviewer:	So you think that is not true?
Learner B:	Wait! It could be a square if you squashed it. Um ...
Interviewer:	So what do you think?

Learner B:	I think it could be a square.
Interviewer:	All right. What about (c)?
Learner B:	All parallelograms are rectangles. No. That is not right.
Interviewer:	Why is it not right?
Learner B:	Because if you put it into a square, it's going to be a square, but it has to be a big one so it can be a rectangle and it is not.
Interviewer:	So you say that is not true.
Learner B:	Yes
Interviewer:	What about the last one?
Learner B:	All rectangles are parallelograms. Um… No
Interviewer:	Why do you say that?
Learner B:	Because all parallelograms are squares.

Learner B misread the word "rectangles" in the first statement, reading "triangles" instead. It could be that to an ESL learner, the words "rectangles" and "triangles" are so similar that they cause confusion. It could also just have been a genuine mistake. Learner B seemed to be confused about the shapes. He thought that all rectangles were "longer" squares. He also thought that if a square was "squashed" to make a parallelogram, it was still a square.

For an ESL learner, learning Geometry terms is often problematic, particularly if the teacher does not pay careful attention to the use of language during lessons (Roux, 2005). I have heard the Grade 6 Geometry teachers use the expressions: "A parallelogram is a pushed-over rectangle" or "A rhombus is a pushed-over square". Presenting concepts such as rhombi or parallelograms in this way can create conceptual problems for some learners, but particularly for those learners without a good command of the English language.

Learner K also seemed to operate at van Hiele's Level 1. This is illustrated by the extract from the interview with learner K below:

Learner K:	(*Reads the first statement*) All rectangles are squares - In a way but they are not all equal. All parallelograms are squares. Yes all parallelograms are squares because ... Well also about the sides 'cos when you look at it, it is like a square. That's ja. I think they are squares. It's just the ... Ja I can't say it. If the square was pushed over, it would be a para ... a paral .... Parallel... that word.
Interviewer:	Why is it still a parallelogram?
Learner K:	The same reason. Also the sides. If it had ...
Interviewer:	What about the sides? Is it the number of sides?
Learner K:	The number.
Interviewer:	Are all four sided figures parallelograms?
Learner K:	Yes.
Interviewer:	The next one?
Learner K:	All parallelograms are rectangles. Not all. I can't picture it. The rectangle is too long.
Interviewer:	What about the next one?
Learner K:	All rectangles are parallelograms. Yes and No.
Interviewer:	Why the yes?
Learner K:	Well...well...yes. Because. Well I don't know. Well yes. Ja I am sure of this one. If you change that and make it like a square (*points to figures of a rectangle and a parallelogram*), well it can also be a square but also a rectangle but if you do the same it will be a parallelogram but it won't be the same.

Not only was learner K very confused about the properties of shapes and about the differences between rectangles, squares and parallelograms, but it was a struggle for her to communicate what she was thinking. She seemed to consider that a parallelogram was a square but that it had just been "pushed over", a concept she may have developed from the diagrams of quadrilaterals in the Geometry module (Appendix G). As she was an ESL learner, it may have been difficult for her to express her ideas in English.

An important component of van Hiele's theory about how children progress to higher levels of geometrical understanding is that children must be taught Geometry using language and terminology that they can understand. No Geometry was done at the research school in Grades 4 and 5 of the Intermediate Phase (except for the activities done in the Mathematics Academy with a few selected learners). For the majority of the Grade 6 learners at the research school, the Geometry they did in Grade 6 was the first formal exposure to shapes that they had experienced in three years. As discussed in Section 2.6 of the literature review, van Hiele (1986) emphasized that teachers should use mathematical language in the classroom that can be understood by the children they are teaching. I noticed that Mrs C revised the terminology from the Grade 6 Geometry module (Appendix G) regularly in her Geometry classes. However, from the responses of learners B and K, it seems that they did not have a clear idea of the concepts represented by the words used in the Geometry module. In Section 2.6 I discussed Wellington and Osborne's (2001) assertion that children often answer questions in class without fully understanding what the words they are using actually mean. This may be happening in Mrs C's Geometry lessons. It is all very well to learn the word "parallelogram" but, unless learning this word is accompanied by a variety of practical activities where the children can construct their own understanding of what makes a parallelogram different from any other quadrilateral, it is a pointless exercise.

Constructivists claim that the ability to classify shapes on the basis of their properties is "developed gradually throughout [Piaget's] concrete operational stage as a result of interaction with sufficient experiences that require multiple classification" (Farrell and Farmer, 1980:58). Thus, all learners in the Intermediate Phase require many opportunities to manipulate and discuss shapes such as parallelograms, squares, rectangles and rhombi to help them develop an understanding of the properties of these shapes. I have mentioned many times in this chapter that Roux's research (2005) suggests that understanding in Geometry is closely linked to proficiency in the language of instruction. This implies that learners in the Intermediate Phase need opportunities to talk about their understanding of shapes during class, with each other, and with their teachers. The practice of learners listening to the teacher talk about shapes and parroting the names of the shapes does not lead to understanding or the attainment of higher levels of geometrical understanding in the van Hiele hierarchy.

4.5.15 Properties of Squares

The learners were shown a multiple choice question about the properties of squares, which is reproduced in Figure 4.9. They discussed each statement with me to help them decide which statement was the correct one.

PQRS is a square. Which relationship is true in all squares?

(a) PR and RS have the same length
(b) QS and PR are perpendicular
(c) PS and QR are perpendicular
(d) PS and QS have the same length
(e) Angle Q is larger than angle R

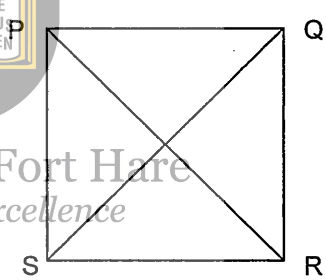


Figure 4.9 Question 12 on properties of squares.

The choices that the learners made are summarized in table 4.16 below.

Learner	PR and RS have the same length	QS and PR are perpendicular	PS and QR are perpendicular	PS and QS have the same length	Angle Q is larger than angle R
A		X	X		X
B			X	X	
C		X			
D				X	
E		X		X	
F	X				
G				X	
H			Yes and then No		
I		X			
J			X		
K		X			
L	X		X		
M		X		X	
N		X	X		

Table 4.16 Learner responses on the properties of squares.

The RNCS (South Africa, 2002:49) states that learners should investigate "the similarities and differences between squares and rectangles" by the end of Grade 5. This implies that learners in Grade 6 should have an understanding of the properties of squares (the four sides have equal length, the four corners are right angles and the diagonals have equal length). I included this question to determine the Grade 6 learners' understanding of the relationships between the lengths of the sides and the diagonals and the size of the angles inside squares.

The questions that most learners had problems with were (a) and (d). They are similar questions in that both questions ask the learners whether the length of a side of the square is the same as the length of the diagonal. I did check with the learners that they knew what a diagonal was so that they would not be prevented from answering because of a lack of vocabulary. We traced out with our fingers, for each question, where PR, RS, PS and QS were. Both (a) and (d) are testing the same concept and so they should both be false.

Eight learners said that (a) was false and six of the eight then told me that (d) was true, which indicated that they did not fully understand the idea that the diagonal must be longer than the side of the square. Four learners told me that (a) and (d) were true and only four learners (of the fourteen) realised that both (a) and (d) were false. Learners C, H and I all said that the diagonal had to be longer than a side and these are the three learners who indicated by their answers to the other questions during the interviews that they had probably attained Level 2 of the van Hiele levels of geometrical reasoning. Learner D, who also said that options (a) and (d) were false, had flawed reasoning. She said:

> "The area and the base would be very different".

I suspect that learner D was struggling with the correct terminology to express her reasoning. Learner D had demonstrated in a number of questions that she struggled to find the correct terminology to describe her ideas about shapes. Three of the learners reasoned that (a) and (d) had to be true as "in a square, all the lengths are the same". This may indicate that they have learned the definition that in a square, all the sides are the same length and this has led them to believe that any lengths in a square must be the same. Children who learn definitions without having constructed their own understanding of the concepts involved may draw faulty conclusions like this.

This extract is from the interview with learner F, the learner who came first in the grade for Mathematics and received the Mathematics prize at the end of year prize giving.

Interviewer: Where is PR on the diagram?

Learner F: Um. Here.

Interviewer: That's right. And RS? Where is RS? (*Learner points to the correct side*). That's right. Do you think they have the same length?

Learner F: Yes.

Interviewer: Why do you think they have the same length?

Learner F: Um...Wait. I don't know. (*long pause*)

Interviewer: So you still think they are the same length?

Learner F: I actually don't think they are the same length. One is a diagonal and ...(*whispers to herself*).

Interviewer: Which one is the diagonal?

Learner F: PR.

Interviewer: And the other one? RS?

Learner F: Is just a straight line.

Interviewer: So you have changed your mind. You think they are not the same length?

Learner F: I don't know ....

Later in the interview, when she looked at the statement (d) in the question reproduced above, she had this to say:

Interviewer: What do you think about (d)?

Learner F: Um... Yes I think they are. PS and QS? Oh they have the same length ..... Um. I think they do.

Interviewer: Why do you think they do?

Learner F: Um. Um. (*long pause*). 'Cos in a square isn't all the angles ... lines ...sides ... supposed to be the same and the diagonal lines the same?

Interviewer: You are quite right. In a square, the sides are all the same length and the diagonals do have the same length. Do you think the diagonals will be the same length as the sides?

Learner F: Yes, I do.

What is remarkable about this interchange is that I had witnessed a lesson in the Mathematics Academy a week earlier where this learner and her classmates had done a practical lesson on Pythagoras' Theorem. They had constructed the classical 3,4,5 right-angled Pythagorean triangle and then constructed a square on each of the three sides of the right-angled triangle to discover that the square on the hypotenuse is equal to the sum of the squares on the other two sides. Possibly an experience such as this should have embedded the idea that the diagonal in a square would be longer than the sides of a square but, in this case, it hadn't. This further reinforces the need for learners to have multiple experiences with shapes in order for them to establish relationships between the sides, diagonals and angles (van der Walle, 2004).

4.6 Analysis of Trends and Anomalies

An analysis of the responses to the interview questions given by the learners indicated some interesting trends and anomalies which are discussed here.

4.6.1 Language and vocabulary

Some of the learners seemed to struggle with the language they needed to express their ideas about shapes. This problem was evident among ESL as well as English mother tongue speakers. Learner D, an English mother-tongue speaker was unable to describe what perpendicular lines were, choosing to demonstrate them with her fingers instead. I could see from the way she placed her forefingers at right angles to each other that she knew what perpendicular lines were. She simply could not find the words she needed to express what she knew. Learner D used phrases during her interviews that demonstrated that she was struggling to master the terminology needed to talk about geometrical shapes. She described the size of the angles in a triangle as, "the corners of the degrees". She could not remember the word, "protractor", calling it a "thingamabobby" although she knew what the function of a protractor was. She described a square as having "four equal sides each of a ninety degree angle". She knew that the diagonals in a square must be longer than the sides of the square but she expressed this idea as, “The area and the base are different”. Thus learner D seemed to lack the vocabulary to talk about her understanding of two-dimensional shapes.

In fact, learner D demonstrated, with a number of her answers, that she was one of the more advanced respondents in terms of her geometrical understanding and seemed to be operating at van Hiele’s Level 2 for many of her responses. Yet she only achieved 30% for her end of year Geometry assessment. This was an anomaly. Research has shown that lack of language proficiency is a barrier to the attainment of higher levels of geometrical understanding (Roux, 2005; van Hiele, 1986) but this did not seem to be the case with learner D. It is possible that learner D has been able to develop an understanding of shapes despite her inability to express what she is thinking. This inability to express her ideas may be the reason she performs so poorly in a formal assessment. Rubenstein and Thompson (2002) recommended that Mathematics teachers should ensure that time is given during lessons to help learners to develop their Mathematical vocabulary. This would benefit learners such as learner D.

Another learner who would benefit from being able to spend time in class developing his ability to express his thinking about Geometry was learner N. Learner N was an ESL learner whose inability to express himself in English seemed to be preventing him from progressing beyond van Hiele’s Level 1. He said that a square was a “box” when he had to describe a square in one of the questions during the interview. He also said that perpendicular lines were lines that were “joined”. When he was shown a square that had been rotated through 45°, he said it had been “moved” and was no longer a square. He claimed it had become a diamond. Thus learner N appeared to be lacking the language he needed to talk about geometrical concepts and to help his geometrical understanding to progress to the higher van Hiele levels.

Although learner D experienced difficulty in talking about her understanding of shapes, she demonstrated geometrical thinking at van Hiele's Level 2 on a number of occasions. Learner N, however, operated consistently at Level 1 (the visual level). He placed a rhombus into a group with triangles because the shapes were the same and sorted shapes simply on the basis of their appearance. From Table 4.2, learner D had more experiences with spatial activities such as jigsaw puzzles and Lego® than Learner N. As discussed in Section 4.4, Maxfield (1975) suggested that if young children do activities such as jigsaw puzzles it may improve their geometrical understanding. If learner N had more opportunities at school to manipulate shapes in Geometry classes and to talk about the shapes, it might help him to progress to van Hiele's Level 2.

4.6.2 Geometric concepts

Learner F seemed very unsure about geometrical concepts. When she was asked questions about shapes, she was hesitant about her answers and it seemed she was trying to remember work she had learned by heart. When she was asked what parallel lines were she said:

> "They remain the same distance apart? Um …they never meet."

I have placed the question mark at the end of the first sentence she said because the tone of her voice when she gave her response sounded as though she was seeking confirmation from me that she had given the correct answer.

The responses she gave when trying to decide whether the diagonal and side of a square have the same length also demonstrated a tendency to fall back on statements she had memorized. She asked me whether "all the angles … lines …sides … and the diagonal lines" should be the same length in a square. Thus, she had remembered that all the sides in a square have the same length and that the diagonals in a square also have the same length. However, she did not really understand the properties of a square and did not seem to have internalized the fact that the diagonals must be longer than the sides. Even though she was presented with a diagram where she could compare the lengths visually (and some learners did do this, even using fingers to measure the lengths), she still did not realize that the lengths differ.

I received copies of the learners' scripts for the end-of-year Geometry assessment and saw that learner F had achieved 100% for the assessment. She also received the Mathematics prize for being the top student in Grade 6 Mathematics at the school's annual prize-giving. This was an anomaly. When I discussed learner F's uncertainty about geometrical concepts with her Mathematics teacher, I was told that learner F was a very diligent student who learned her work well for tests and examinations. Hence her good marks.

The other learner who achieved 100% for the Geometry assessment was an ESL learner (learner K) whose answers for some of the questions in the interviews indicated that she too was operating between van Hiele's levels 1 and 2. Her answers were less hesitant than those given by learner F and, unlike learner F, she did not constantly seek reassurance that she had given the correct response. She demonstrated that she could sort quadrilaterals into groups on the basis of the number of sides. However, she placed rectangles and squares together because they had the same shape (Section 4.5.10). It seemed to me that learner K struggled to find the correct terminology to discuss her ideas about shapes. Van Hiele (1986) emphasized that language could be a barrier to attaining the higher levels of geometrical understanding. This could be what has happened to prevent learner K from fully attaining van Hiele's Level 2 by the end of her Grade 6 year.

A copy of the Geometry assessment that was used at the end of the year can be found in Appendix H and two of the learners' answer sheets have been appended in Appendix

I. It does seem that the Grade 6 Geometry assessment at the research school tested the learners' ability to memorize information and to perform simple skills. There were no questions that tested for conceptual development or that required geometrical insight. Van Hiele (1984) warned that, "Both examinations and test papers tend to push the pupil towards algorithmic insight instead of leading him to far more valuable higher forms of insight" (van Hiele, 1984:241). I am certain that, if questions requiring geometrical understanding had been included in the assessment, neither learners F or K would have achieved 100%. Neither learner had developed the ability to think independently about two-dimensional shapes. Learners F and K needed more opportunities to manipulate and discuss two-dimensional shapes in class to achieve the Assessment Standards for Learning Outcome 3 of the RNCS (South Africa, 2002) and to attain van Hiele's Level 2 of geometrical understanding.

4.6 Summary and Interpretation of Results

In this chapter, I have attempted to show how the requirements of the RNCS (South Africa, 2002) align with the recommendations of van Hiele (1986) to help learners to develop their geometrical understanding. If teachers follow the Assessment Standards as prescribed in the RNCS (South Africa, 2002), learners in each grade in the Intermediate Phase will have multiple opportunities to explore two-dimensional shapes, using tangrams, models made from drinking straws, shapes drawn on grid paper and tessellations (to name just a few of the activities mentioned in the RNCS). If these opportunities to explore shapes are coupled with appropriate instruction and language geared to the learners' level of understanding (van Hiele, 1984), the learners have the opportunity to progress to van Hiele's Level 2 by the end of the Intermediate Phase.

At the research school chosen for the case study, I discovered that no Geometry was done in Grades 4 and 5 and that all the Geometry in the Intermediate Phase was covered in one module (Appendix G) in the fourth term of Grade 6. An analysis of the content of the Geometry module showed that it did not fulfill all the requirements as stated by the RNCS (South Africa, 2002). The Geometry assessment done at the end of Grade 6 (Appendix H) tested memory recall and did not assess the conceptual development of the learners.

On the basis of the semi-structured interviews that I conducted with fourteen Grade 6 learners at the research school, I was able to explore the geometrical reasoning of the respondents. I found that it was difficult to conclude, with absolute certainty which of the van Hiele levels each learner had reached in terms of their geometrical thinking. The responses that most of the learners gave were not consistently at Level 1 or Level 2 but appeared to vary according to the two-dimensional shapes that were being discussed. Dina van Hiele-Geldof (1984) said that, for a child's reasoning about a shape to progress from simply recognizing the shape to understanding the properties of the shape, the child needs to experience five phases of instruction. These have been discussed in Section 2.4.2.2. It seemed that shapes such as triangles and rectangles were less problematic for some learners than parallelograms and rhombi. I found that:

- Three of the learners were operating at van Hiele's Level 2, the level they should have reached by the end of the Intermediate Phase. In fact these three learners were operating at van Hiele's Level 3 for some of the two-dimensional shapes.
- Seven of the learners seemed to be in transition between Levels 1 and 2 for two-dimensional shapes. These learners were operating at Level 1 for some shapes and at Level 2 for others. The questions dealt with different shapes. Some of the learners were operating at Level 2 for triangles because they were able to answer questions about the properties of isosceles, equilateral and scalene triangles. However, the same learners were unable to answer questions about parallelograms and so they were operating at Level 1 for parallelograms.
- Four of the learners operated consistently at van Hiele's Level 1 for all the two-dimensional shapes that should be familiar to learners by the end of Grade 6.

On the basis of my research, it would appear that not all the learners had attained van Hiele's Level 2 of geometrical understanding for all of the two-dimensional shapes prescribed for the Intermediate Phase in the RNCS (2002). It seems that the learners do not have sufficient experiences with two-dimensional shapes to be able to construct their own understanding of the properties of the shapes. This was particularly the case for quadrilaterals. Many of the learners appeared to have rote learned definitions that appeared in their Geometry modules or that they had heard their teachers repeat in class.

As there was no formal Geometry done in Grades 4 and 5, the Grade 6 learners in this case study did not have sufficient exposure to experiences with shapes over a sufficiently long period of time. It seems that the few weeks of Geometry they experienced were insufficient for these learners to develop an understanding of the two-dimensional shapes as required by the RNCS (South Africa, 2002). This confirms the research findings of Dina van Hiele-Geldof (1984) and Halat (2007).

It was apparent that the progress of some of the learners was hampered by their lack of geometrical vocabulary. This was despite the almost daily revision of geometrical terms that occurred in the Grade 6 classes at the research school. Van Hiele (1984) emphasized the importance of Geometry teachers establishing what the level of geometrical understanding of their learners is before beginning instruction. He claimed that teachers had to use the language in the classroom that was based at the level of the learners' understanding. My impression was that the teachers did try to use simple language and to drill into their learners the geometrical terms they would need for the two-dimensional shapes they were studying in Grade 6. However, these teaching strategies did not seem to help most of the Grade 6 learners in this study to talk about geometrical concepts nor to help their geometrical understanding to progress to the higher van Hiele levels.

Chapter 5 : Conclusions and Recommendations

5.1 Introduction

This chapter discusses the main findings I made during my investigation into the level of understanding of two-dimensional shapes among fourteen Grade 6 learners at a well-resourced former model C school in the Eastern Cape. It also presents my recommendations as a result of my research with regard to the teaching of Geometry in the Intermediate Phase and suggests future research into the level of geometrical understanding among Intermediate Phase teachers.

I reviewed three theories about the development of geometrical understanding in my literature review (Chapter 2). Current research indicates that the van Hiele theory of geometrical understanding is the most universally accepted theory for the development of geometrical understanding in children (Feza and Webb, 2005; Halat, 2007; Halat, 2008; Roux, 2005; van der Walle, 2004) and I selected the van Hiele theory as the theoretical framework for my research.

As discussed in Chapter 3, the research was a case study and the fourteen respondents were purposively selected to include learners of both sexes, who represented a range of achievement on a Geometry assessment. I interviewed the learners individually using open, flexible, semi-structured interviews to allow me to probe the responses the learners gave to the questions. I hoped that this would help me to gain a deeper insight into the level of geometrical understanding of the learners. The interview schedule can be found in Appendix D and an example of the data sheet I used to note the learner responses is in Appendix E.

The purpose of my research was to investigate the level of understanding of two-dimensional shapes that children have at the end of the Intermediate Phase. To this end, I analyzed the RNCS (South Africa, 2002) in Chapter 4 using the van Hiele theory of five levels of geometrical understanding to establish the level of understanding learners should have reached by the end of the Intermediate Phase (Section 4.3.4). It seems that the Assessment Standards for Space and Shape of the RNCS (South Africa, 2002) require that learners reach van Hiele's level 2 of geometrical understanding by the end of Grade 6, the final year of the Intermediate Phase.

I presented and analyzed the data I had collected during the interviews with the fourteen learners in Chapter 4. Two of the transcripts of the interviews are provided in Appendices J and K. Based on the investigation undertaken and the analysis of the trends found during the interviews and supportive materials, I present my conclusions and recommendations below.

5.2 Learners lack of experiences with shapes

In Chapter 2, I discussed the theories of Piaget (Section 2.4.1) and van Hiele (Section 2.4.2) on how children acquire concepts in Geometry. Piaget (Farrell and Farmer, 1980) and van Hiele (1999) recommended that children be given many different opportunities to manipulate shapes to help them to develop their understanding of the properties of shapes and the relationships between different shapes. Van Hiele (1999) also said that the development of geometrical thinking does not happen automatically as children mature, but that a particular sequence of instruction is required to help them progress through five levels of geometrical understanding, from the purely visual recognition of shapes to being able to operate with relationships between shapes. Van Hiele (1999) said:

"Instruction intended to foster development from one level to the next should include sequences of activities, beginning with an exploratory phase, gradually building concepts and related language, and culminating in summary activities that help students integrate what they have learned into what they already know."

(Van Hiele, 1999:311)

In the light of the recommendations by Piaget and van Hiele mentioned above, I met with the teacher in charge of Grade 6 Mathematics and discovered that no formal Geometry was covered in Grades 4 and 5 at the research school. The only exposure to formal Geometry the Grade 6 learners had at the research school was in a module (Appendix G) that the learners did during the fourth term of their Grade 6 year. In Chapter 4, I analyzed the Grade 6 Geometry module (Section 4.3.2) and found that, apart from constructing two-dimensional shapes such as triangles, squares and rectangles, no other practical activities involving shapes were done during Grades 4 to 6 of the Intermediate Phase at the research school.

In Section 4.5.2, I discussed learner responses to a question asking them to identify the triangles from a group of four shapes. The responses from three of the learners indicated that they were confused about what makes a triangle different from other shapes. They had not understood the concept that triangles have three sides. Two other learners ignored the obtuse-angled triangle, not realizing it was a triangle. These responses illustrated that, for these five learners, more concrete experiences with triangles were required to help them develop the correct concept of a triangle.

As discussed in Section 4.5.5, when the learners were asked to identify the parallelograms from a set of three shapes (all of which were parallelograms in different orientations), four of the fourteen learners chose the parallelogram that resembled the one illustrated in their Geometry module. One learner told me that a rectangle is not a parallelogram. There was only one sketch of a parallelogram and no practical activities involving parallelograms in their Geometry module (Appendix G). These four learners were unsure about the concept of a parallelogram and had not had the opportunity to explore parallelograms of different sizes and with differently-sized angles inside them.

When I rotated a plastic square shape through 45° and asked the learners whether it was still a square, five of the learners told me it was no longer a square (Section 4.5.8). They said it had become a diamond. This response indicated that these five learners had not yet realized that a square retains the properties of a square even when it is in a different orientation. This understanding develops through activities involving square shapes, which the learners in this case study had not experienced. (van der Walle, 2004).

From the responses I received to the questions about triangles and quadrilaterals that I have discussed in this section, it seems that the learners in this case study have not had sufficient opportunities to manipulate and investigate shapes, particularly four-sided shapes. Although these Grade 6 learners had seen pictures of two-dimensional shapes in their Geometry module and had constructed triangles and rectangles they lacked instruction of the type that van Hiele (1999) recommended. A carefully planned sequence of instruction of the type recommended by Dina van Hiele-Geldof (1984) might help these Grade 6 learners to develop their understanding of shapes from purely visual (Level 1) to descriptive (Level 2), where they would have an understanding of the properties of triangles and quadrilaterals.

5.3 The link between geometrical understanding and prior spatial experiences

Van Hiele (1999) said: "For children, geometry begins with play" (van Hiele, 1999:310) and suggested that teachers and parents could help children to develop geometrical thinking through activities that involve play. Maxfield (1975) suggested that playing with toys such as jigsaw puzzles that require spatial understanding, as a young child, may improve geometrical understanding later in life. The RNCS (South Africa, 2002) stipulates that learners in the Foundation Phase should use building blocks and construction sets to develop their understanding of two-dimensional shapes (L.O. 3). As discussed in Section 4.4, I asked the learners at the start of their interviews whether they had played with jigsaw puzzles, Lego®, tangrams or chess. It was noteworthy that, of the three learners who had attained van Hiele's Level 2 (and were progressing towards Level 3), two played chess and all of them had played with jigsaw puzzles of 500-1000 pieces, tangrams and Lego®. This indicates that learners who have had the opportunity to play with activities like the ones mentioned above may develop improved geometrical understanding as a result.

5.4 Learners struggle to talk about geometrical concepts

In Chapter 2, I discussed the role that language plays in the acquisition of the van Hiele levels of geometrical understanding (Section 2.6). Van Hiele (1986) recommended that teachers use language and geometrical terminology that their learners can understand during Geometry lessons. Van Hiele (1986) maintained that, if children did not understand the geometrical terms used by their teachers, the children would learn definitions by rote to help themselves pass assessments.

During the Grade 6 Geometry lessons that I observed at the research school (Section 4.5.6), I noticed that the Geometry teachers used repetition and drilling of geometrical terms at the start of the Geometry lesson to try to help learners acquire the geometrical terminology that appeared in the Geometry module (Appendix G). However, I found that all of the ESL learners and some of the English mother-tongue speakers in the case study experienced great difficulty in finding the correct geometrical language to express their ideas about shapes.

Two of the learners (one of them an ESL learner) could not describe what parallel lines were and yet they could both point out the parallel lines in a parallelogram (Section 4.5.6), which indicated to me that they knew what parallel lines were. Another learner (an English mother-tongue speaker) could not describe what perpendicular lines were (as discussed in Section 4.6) but demonstrated perpendicular lines by using her fingers. These three learners knew what the concepts parallel and perpendicular meant but could not express their understanding verbally.

Learner D, an English mother-tongue speaker, who was discussed in detail in Section 4.6, demonstrated a number of times that she was lacking the correct geometrical terminology to express her ideas. She used the phrase "the corners of the degrees" when she wanted to talk about the angles in a triangle. She knew that the sum of the angles in a triangle is 180° but she had great difficulty in expressing this concept. Even learner C, also an English mother-tongue speaker, who showed good insight into the properties of two-dimensional shapes, had problems expressing his thoughts on the similarities between a rectangle and a square (Section 4.5.10). He was unable to say that the rectangle and square have right angles and described the shapes as "having ninety degrees straight up while [a parallelogram] can't be ninety degrees because the lines can't be straight up". Learner K, an ESL learner, also experienced difficulty with expressing her thoughts about the similarities between rectangles and squares (Section 4.5.10) and said that rectangles and squares are, "… a bit the same. They might be tall or short but long-short". The learners in these examples understood the concepts involved but struggled to put their thoughts into words.

I noticed that some of the learners had difficulty using the correct language to describe the size of angles. In Section 4.5.15, where the learners were asked whether angle Q in the square PQRS was larger than angle R, these learners replied that the angles were the same length. This indicated confusion between the concepts “length of sides” and “size of angles”. I think the learners did know that all the angles in a square are equal but they lacked the language skills to express this concept clearly.

These examples highlight the difficulties the learners in the case study seemed to experience in describing their geometrical reasoning to me. It is noteworthy that these difficulties were experienced by both the ESL learners and the learners who were English mother-tongue speakers. This reinforces Roux's (2005) recommendation that Geometry teachers should give learners opportunities in class to practise communicating geometrical concepts to each other during group work activities.

5.5 Geometry instruction in the Intermediate Phase

In Chapter 2, I established the important role the teacher plays in helping children to develop their geometrical understanding. Van Hiele (1986) said that a teacher needs to provide instruction at a higher level of geometrical understanding than his learners have attained so that his learners can be stimulated into progressing to the higher levels of understanding (Section 2.4.2.2). I wanted to investigate the understanding of two-dimensional shapes among the Grade 6 learners chosen for the case study but I decided that I also needed to know what type of instruction the learners had received in Geometry during the Intermediate Phase to give me insight into their learning experiences and hence their answers to my questions.

As discussed in Section 5.2, I discovered that no Geometry was taught at the research school during Grades 4 and 5. The only Geometry taught in the Intermediate Phase was contained in the Grade 6 Geometry module (Appendix G). I analyzed this module in Section 4.3.2 and found that the focus of the module was the learning of definitions and the practising of skills such as constructing triangles. The Geometry module seemed to follow Gagne's Instructional theory (Farrell and Farmer, 1980) because the learners started with lower order skills such as drawing a line segment, or using a protractor to draw an angle, and worked up to more complex tasks such as constructing a rectangle or triangle. The only activities involving shapes in the Geometry module were constructions of triangles, rectangles and squares. I noted that neither the exercises in the module nor the questions in the end of year Geometry assessment (Appendix H) required any higher order thinking skills (Section 4.3.3).

The lessons I observed and the responses I received to some of the interview questions indicated to me that there was an emphasis on rote-learning rather than giving learners practical experiences with shapes among the Grade 6 teachers in the research school. I had noticed that the Geometry teachers started each Geometry lesson by drilling the terminology from the Grade 6 Geometry module (Appendix G). In Chapter 4 I discussed an observation I had made while visiting Mr L's Geometry class (Section 4.5.2). Mr L was teaching a lesson on constructing triangles. He had drawn an isosceles triangle on the board and told the learners that, in an isosceles triangle, the angles opposite the equal sides were equal to each other. A learner asked how one would measure the angles in an obtuse-angled isosceles triangle. Mr L replied, "We don't get obtuse-angled isosceles triangles. They (obtuse-angled triangles) are all scalene." Apart from the fact that the information given to the learner was obviously incorrect, the opportunity to have the learners investigate different obtuse-angled isosceles triangles was missed. As a result, the learners did not experience what van Hiele (1986:43) termed "a crisis in thinking", which could have led to a higher level of understanding of isosceles triangles.

The emphasis on rote-learning could be seen in the questions asked in the end-of-year Geometry assessment (Appendix H), which tested for memory recall rather than conceptual development. In Section 4.6, I discussed anomalies I had found in my research and mentioned learner F, the recipient of the Grade 6 Mathematics prize at the school's annual prize giving, who had achieved 100% for the end-of-year Geometry assessment. I discovered during her interview that she was hesitant about her answers and seemed to be trying to remember properties she had rote-learned. During the interview, I asked her to identify which shapes were parallelograms from a set of three shapes, which were all parallelograms. She said that the rectangle in the set was not a parallelogram but when she was asked to describe parallel lines, she repeated the precise definition from the module, indicating that she had rote-learned it. However, she did not apply this definition to the opposite sides of the rectangle when trying to decide whether it was a parallelogram. From this answer and her answers to other questions, it seemed to me that she was in transition between van Hiele's Level 1 and Level 2.

I discovered during the interviews with the other thirteen learners that there were at least three of the learners who had greater insight into the properties of two-dimensional shapes and were at a higher level of geometrical understanding than learner F. When I discussed learner F with her Geometry teacher, I was told that the reason she performed so well in Mathematics was that she learned her work well. De Villiers (1995) and van Hiele (1986) warned that rote-learning in Geometry does not lead to the development of the higher levels of geometrical thinking. Katagiri (2004) and van Hiele (1986) suggested that it is the development of geometrical thinking and problem-solving that should be the focus of good Geometry teaching. While the rote-learning of geometrical terminology and structures may help learners to achieve good results in the short term, it does not help them to develop the understanding of shapes that learners need for the study of Geometry in higher grades (van Hiele, 1986).

5.6 Levels of understanding of the Grade 6 learners in the study

The children at the end of Grade 6 at the research school were not all at the same level of geometrical understanding according to the van Hiele hierarchy. Four of the learners were operating at van Hiele's Level 1 (the visual level) for all the two-dimensional shapes they were shown during the interviews. In all their comments about the shapes, these four learners focused on global, visual characteristics of the shapes (van Hiele, 1986). They used terms such as "they look the same" to explain their reasoning, which indicated that they were not considering the properties of the shapes but the appearance instead.

Three of the learners consistently gave answers that indicated that they were reasoning about the properties of the two-dimensional shapes they were shown. This type of reasoning is characteristic of van Hiele's Level 2 (van Hiele, 1986). These three learners, all of whom attend the Mathematics Academy, were also able to consider relationships between squares, rectangles and parallelograms as discussed in Section 4.5.14. This indicated that they were operating at Level 3 (informal deduction) in terms of their reasoning. They were able to reason that, if rectangles have opposite parallel sides, then all rectangles must be parallelograms. However, as not all parallelograms contain right angles, then not all parallelograms are rectangles. I mention that they all attended the Mathematics Academy because it is only those learners who attended the Mathematics Academy, who had investigated two-dimensional shapes during Grades 4 and 5. This experience may have assisted these learners to develop their understanding about quadrilaterals.

The other seven learners appeared to be in transition between Levels 1 and 2. Learner K was able to sort a variety of shapes into groups on the basis of the number of sides the shapes had. She did not say she had sorted them by putting those with the same shape together as some of the other learners did. This indicated that she was focusing on the property "number of sides" for the shapes, rather than their appearance, a Level 2 characteristic. She said that a square rotated through 45° remained a square, another indication that she was operating at Level 2 (van Hiele, 1986). However, she chose to place a rhombus with a parallelogram because "they look the same". This response indicated that learner K was operating at the visual level (level 1) in this instance. Learner K had attained van Hiele's Level 1 and was in the process of realising that "figures are bearers of their properties" (van Hiele, 1984:245). With careful instruction and exposure to many examples of shapes, learner K should be able to progress to Level 2 (van Hiele, 1986).

Van Hiele (1986) said that, unless Geometry instruction is geared to the level of understanding of the learners in a class, the learners will not be able to progress to the next level of geometrical understanding. In the Grade 6 classes in the case study, the learners had attained different levels of geometrical understanding. It seems reasonable to assume that this may be the case in many Geometry classrooms which poses a challenge to Geometry teachers. Van der Walle (2004) suggested that Geometry teachers should assess the level of understanding of their learners before teaching geometrical concepts. If teachers find that their learners are at different levels of geometrical understanding similar to the Grade 6 learners in this case study, they should consider their teaching strategy very carefully. Meng (2009) found that a combination of carefully selected geometrical activities, and appropriate guidance from the teacher, was a good teaching strategy for classes of learners at different van Hiele levels. Geometry lessons designed on the basis that one approach suits all learners are not going to meet the needs of learners who are at different levels of geometrical understanding.

5.7 Recommendations

On the basis of my research into the level of understanding about two-dimensional shapes among Grade 6 learners at a well-resourced former model C school, and the conclusions discussed in Sections 5.2 to 5.6, I make the following recommendations to address the challenges identified as a result of my research.

5.7.1 The Assessment Standards in the RNCS should be followed

The Assessment Standards in the RNCS (South Africa, 2002) stipulate activities involving shapes, transformations and models which, with careful instruction, could lead to learners developing a good understanding of the properties of two-dimensional shapes over the three years of the Intermediate Phase. The school I selected for this case study did no Geometry in Grades 4 and 5 and did only one term of Geometry at the end of Grade 6. I think that schools should follow the RNCS (South Africa, 2002) and do some Geometry in each grade in the Intermediate Phase. The Geometry module that is done at the end of Grade 6 at the research school (Appendix G) does not cover all the Assessment Standards as stipulated by the RNCS (South Africa, 2002). If the teachers at a school decide to develop their own modules in Geometry, they should adhere to the Assessment Standards in the RNCS (South Africa, 2002).

5.7.2 Intermediate Phase learners need concrete experiences with shapes

Research by van Hiele (1984) and Piaget (Farrell and Farmer, 1980) indicates that children need to have many opportunities to manipulate shapes to help them develop an understanding of shapes and their properties. I found in my research that the learners at the research school did not have sufficient exposure and experiences with shapes during their Geometry lessons. I suggest that Geometry teachers in the Intermediate Phase collect many different physical examples of two-dimensional shapes such as plastic or cardboard shapes, pictures of shapes and construction sticks or drinking straws that can be used to make shapes. They should use this equipment in their classrooms during carefully structured lessons to help the children develop an understanding of the properties of shapes.

5.7.3 Intermediate Phase learners need to practice geometrical language

Van Hiele (1986) emphasized the importance of language in the learning of Geometry and the development of geometrical understanding. In my research I found that a number of the learners in the case study were hampered by a lack of geometrical vocabulary and an inability to express their ideas about shapes. To address this problem, Geometry teachers should plan activities during their Geometry lessons which will encourage the development of a geometrical vocabulary among their learners. Simply drilling the terminology is not effective in helping learners to talk about their understanding of shapes. Roux (2005) recommended that teachers give their learners opportunities in class to talk with each other about geometrical concepts during group work. Learners should do more than sit quietly working on exercises to practice geometrical skills during Geometry lessons. The learners in the Intermediate Phase need to be given activities that involve working together to discover the properties of shapes during group work and to develop definitions of shapes that they can discuss with each other. Giving Intermediate Phase learners the opportunity to verbalise their ideas about shapes with each other and their teachers will help them to improve their geometrical vocabulary and understanding of geometrical concepts.

5.7.4 Geometry Assessments need to test for conceptual development

The Grade 6 end-of-year Geometry assessment in my case study focused completely on memory recall questions. The result of such an assessment is that those learners, who rote learn their work, are able to achieve good results. However these good results are no indication of the level of understanding the learners have about the concepts tested. Geometry teachers need to include questions in their assessments that test conceptual development and not just skills and terminology.

5.7.5 Research needed into the geometrical understanding of teachers.

During the Geometry lessons that I observed at the research school, I heard comments made by the teachers such as the one made by Mr L that "there is no such thing as an obtuse-angled isosceles triangle". Three of the Grade 6 Geometry teachers did not want me to observe their lessons at all which could indicate that they did not feel confident teaching Geometry in front of me. No Geometry teaching was done by the Grades 4 and 5 teachers which could indicate a further lack of confidence on their part in teaching geometrical concepts. These observations alerted me to the possibility that the level of geometrical understanding of the teachers in Grade 6 at the research school may not be higher than van Hiele's Level 2. Van Hiele (1986) warned that unless teachers have a higher level of understanding than their learners, they will be unable to provide the type of instruction that the learners require to progress to higher levels of understanding. It could be useful to assess the level of geometrical understanding of the teachers in the Intermediate Phase at school, and then provide workshops to help teachers improve their understanding of the Geometry concepts that need to be covered in the RNCS (South Africa, 2002).

5.8 Conclusion

This case study, the purpose of which was to investigate the understanding Grade 6 learners have about two-dimensional shapes in a well resourced former Model C school, has uncovered a number of issues around the teaching of Geometry in the Intermediate Phase. Although the results of a case study such as this one cannot be generalized to all schools, the performance of South African learners in Mathematics in international benchmark tests such as SACMEQ II (Southern African Consortium for Monitoring Educational Quality) (Kotze and Strauss, 2007) and TIMMS'99 (Repeat of the Third International Mathematics and Science Study in 1999) (Howie, 2004) suggest that Mathematics as a subject is problematic for learners throughout South Africa. It seems reasonable to assume that some of the issues around developing learner understanding in Geometry that have emerged from this study may apply to other classrooms in South Africa too. It is my hope that studies such as this one may help to improve the quality of Geometry instruction to learners in the Intermediate Phase throughout South Africa so that learners may develop an understanding of the Geometry concepts required by the Assessment Standards of L.O. 3 of the RNCS (South Africa, 2002). This would enable them to progress to van Hiele's Level 2 of geometrical understanding by the end of Grade 6 which would benefit them in their study of Geometry in Grade 7 and at high school.

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Appendix A

Letter to Department of Education requesting Permission

PO Box 15686
Beacon Bay
5205

11 October 2009

Chief Director
Strategic Planning
Eastern Cape Department of Education

Attention : Mr Greg McMaster

Dear Sir

Request for permission to do research at [redacted] Primary School

I am a student at the University of Fort Hare in East London, working towards a Masters degree in Education. My area of research is the development of geometrical thinking in learners in the intermediate phase. For my research, I would like to interview twelve learners from grade 6 at [redacted] Primary School to determine what their understanding of geometry is at the end of their grade 6 year. I would need to do the interviews during the last week of the academic year when they have completed their end of year assessments.

I request permission to work at the above school and to interview twelve of their grade 6 learners. If I am granted permission, I undertake to obtain written consent from the parents and the learners to be interviewed for my research. I will also respect the right of the learners not to participate or to withdraw at any stage, should they feel uncomfortable. I also undertake to maintain the dignity of the learners and the school by observing confidentiality of the results.

Yours faithfully

Caroline Selkirk (Mrs)

Educator : [redacted] High School

Appendix B

Copy of Letter of Permission to Parents (November 2009)

Dear parent

I am currently doing research for my M. Ed. into the way grade 6 learners understand geometrical concepts. In order to investigate this, I need to interview grade 6 learners individually during the week 30 November to 4 December. Equipment such as pictures, shapes and puzzles will be used in the interviews to help the children talk about what they understand more easily.

The interviews will be conducted at Hudson Primary in a non threatening environment. You are assured of confidentiality as no learner names will be published in my thesis. I have observed grade 6 geometry classes at [redacted] School and request you permission to interview your child. Each interview should last roughly 30 minutes.

If you agree to my interviewing your child, please would you complete the reply slip below and return it with your child to his class teacher on Monday.

Yours faithfully

Caroline Selkirk

I, ______________________, parent of ____________________________ in grade 6 ___ at [redacted] Primary School, give permission for my son/daughter to be interviewed by Mrs Caroline Selkirk for the purpose of research into the geometrical understanding of grade 6 learners.

Signed: ________________________ Date: ______________________

Appendix C

Examples of Permission letters from parents

[redacted] [redacted]

26 November 2009

Dear parent

I am currently doing research for my M. Ed. into the way grade 6 learners understand geometrical concepts. In order to investigate this, I need to interview grade 6 learners individually during the week 30 November to 4 December. Equipment such as pictures, shapes and puzzles will be used in the interviews to help the children talk about what they understand more easily.

The interviews will be conducted at [ ] in a non threatening environment. You are assured of confidentiality as no learner names will be published in my thesis. I have observed grade 6 geometry classes at Hudson Primary School and request you permission to interview your child. Each interview should last roughly 30 minutes.

If you agree to my interviewing your child, please would you complete the reply slip below and return it with your child to his class teacher on Monday.

Yours faithfully

Caroline Selkirk

I, [redacted], parent of [redacted] in grade 6 L at [ ] Primary School, give permission for my [redacted]/daughter to be interviewed by Mrs Caroline Selkirk for the purpose of research into the geometrical understanding of grade 6 learners.

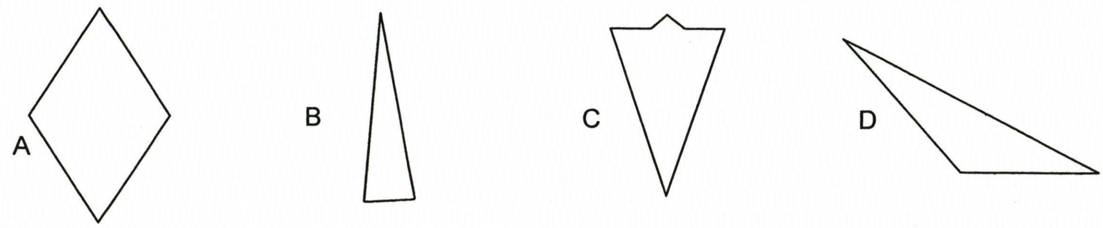
Signed: ____________ Date: 28/11/2009

Appendix D

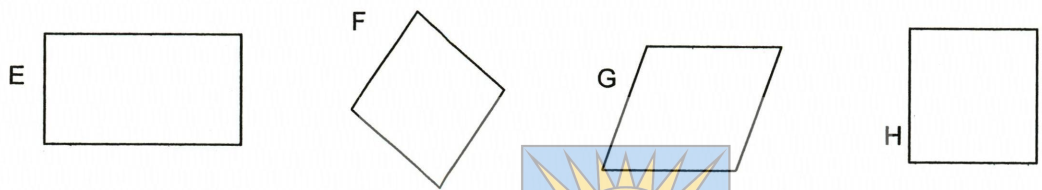
Research Instrument: Structured interview schedule and Data Sheet for learner responses

Picture for Question 1: Identify the shapes you see in this picture.

1. Which are triangles?

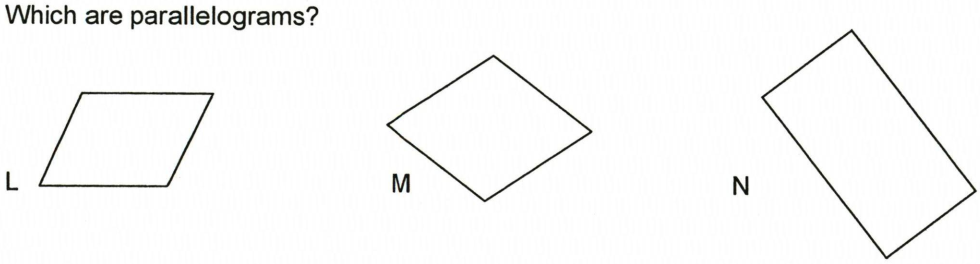


2. Which are squares?

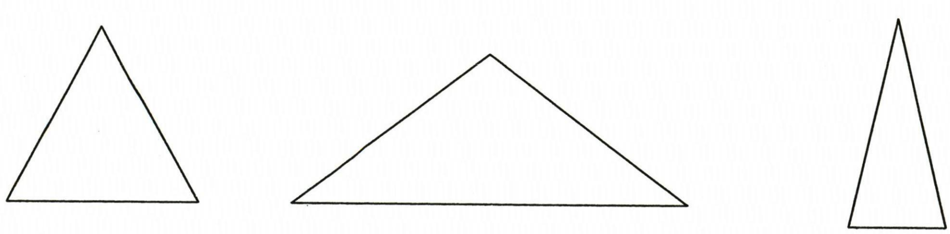


3. Which are rectangles?

4. Which are parallelograms?



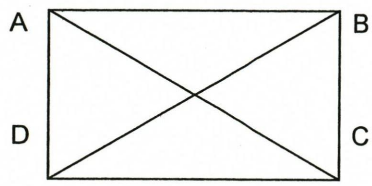
5. An isosceles triangle is a triangle with 2 sides of equal length. Here are 3 isosceles triangles.



Which is true of every isosceles triangle?

a) The three sides have equal length.
b) One side is double the length of the other side.
c) There must be at least two angles of the same size.
d) 3 angles must be the same size.
e) None of (a) to (d) is true in every isosceles triangle.

6. In the rectangle, AC and BD are diagonals. Which of the following is NOT true in rectangle ABCD?



a) There are four right angles
b) There are four sides
c) The diagonals have the same length.
d) All the sides have the same length

7. Which is true?
   (a) All rectangles are squares
   (b) All parallelograms are squares
   (c) All parallelograms are rectangles
   (d) All rectangles are parallelograms
   (e) None of the above

Appendix E

DATA SHEET FOR INTERVIEWS

Name of learner: ____________________ **Date:** __________

School: __________________ **Time:** __________

Gender: **Male** ☐ **Female** ☐

Home language: **English** ☐ **Afrikaans** ☐ Xhosa ☐ **Other (specify)** ☐

School activities: ____

Prior spatial experience: **Jigsaw puzzles** ☐ **Tangrams** ☐ **Lego** ☐

---

1. What shapes can you see in this picture? **[level 0]**

   Triangles **squares** **rectangles** **trapezium** **pentagon**

2. Here are some shapes *(triangles, squares, rectangles, parallelogram).* Can you name them? **[level 1]**

   Triangles squares rectangles parallelogram hexagon

   Questions 1 to 4 above

   Solutions: 1. ______ **2.**_______ **3.** ________ **4.** ________

3. (*Point out the square.*) If you were talking to your friend on the telephone, how would you describe this shape to her/ him. **[level 1 to 2]**

4 sides [4] right angles sides same length other _______________

4. (*Rotate another square)* Is this shape a square? Why? **[level 1]**

Y/N sides still same length angles the same size

5. (*Give the learner some construction sticks*). Can you make a triangle with these sticks? What can you say about your triangle? (it should be isosceles). **[level 1 - 2]**

Can make triangle ☐ Knows it is isosceles ☐

6. Can you make a square with the sticks? **[level 1]**

Y/N makes a rectangle

7. Can you sort the shapes into groups? (Which shapes would you like to put together?) **[level 2]**

Number of sides Types of quads Rectangles/squares separate 5 groups

8. Why did you put them into these groups? **[level 2]**

Number of sides Size of angles Other: ____________________

9. [If necessary] Can I put the squares and rectangles into the same group? What is the same about them? **[level 2]**

4 sides **right angles** **opp sides equal** **shape**

10. (*Place two triangles together to make a rhombus*) Into which group would you place this shape? Explain why. **[level 2]**

Group: _______________ **Sides** **Angles** **Shape**

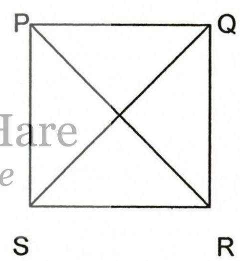
11. (*Put the rhombus, square, rectangle and parallelogram together*). Why do you think I put these together? **[level 2]**

4 sides **opp sides equal** **opp sides parallel** **opp angles equal**

Do questions 5 and 6 and 7 on diagram sheet:

Solutions: **5.** ____________ **6.** ____________ **7.** ____________

12. PQRS is a square. Which relationship is true in all squares?
(a) PR and RS have the same length
(b) QS and PR are perpendicular
(c) PS and QR are perpendicular
(d) PS and QS have the same length
(e) Angle Q is larger than angle R



Appendix F

Pilot Interview

On 29 November 2009, I performed a pilot interview on Sarah Selkirk at 16.15.

She is thirteen years old and has completed grade 7 at [redacted]. She attends the Maths, Computer and Art Academies at school and has played with jigsaw puzzles, tangrams and lego often as a child.

She was able to identify the shapes in the picture easily and identified extra ones that I had not included in my interview schedule. She named triangles, squares, rectangles, parallelograms and hexagons with ease. She was able to identify triangles, squares and rectangles from sets of shapes. **She did not consider that a rectangle was a parallelogram, though.**

She described a square as a 4 sided shape with 4 right angles and the sides the same length. If the square was rotated, she knew it was still a square because "the angles and sides have stayed the same size". She was able to construct a triangle and a square successfully using construction sticks and could talk about the properties of the triangle easily. She correctly identified it as an isosceles triangle.

When asked to sort the shapes into groups, she made 5 groups, placing triangles, squares, hexagons, rectangles and parallelograms into separate groups. She used the size of the angles and length and number of sides to group them, she told me. She said that we could place the rectangles and triangles into the same group as they both have right angles and their opposite sides were equal. (**She did not mention the opposite sides being parallel)**

When two triangles were placed together to make a rhombus, she decided to place this shape with the parallelograms as the opposite sides were parallel (**she did not identify the adjacent sides being equal and thus overlapping with the squares)**

On being shown the set of isosceles triangles, she did not like the first one being called isosceles. She insisted it was an equilateral triangle. When I asked whether it could also be an isosceles triangle, she thought about it and then realised that it could as two of its sides were equal.

She correctly answered the question about the properties of rectangles.

She became very agitated about the question on the properties of squares and which lines were equal in length or perpendicular to each other. "This question is too hard", she said. "It is just like being at the optician when he asks which circle is darker and you don't know the answer. It is confusing".

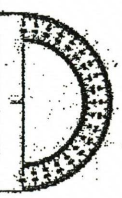
After the interview, I added more answer codes to Q 1 and 2 and added worksheet Q 7.

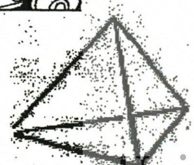
Appendix G

Geometry Module done in Grade 6

e: ______________________ Grade: ________

Geometry and Constructions





Geometry:

Point: A point has no size. It indicates position only. e.g Ȧ Ḃ. Points are named after capital letters.

The segment: A segment is when you join two points together. A segment has two end points. It has no width, but you can measure its length e.g we write it as

EF = 35 mm $\overline{FE}$ = 35 mm.

The ray: A ray has one end point. It cannot be measured. It is named after two points.

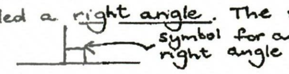
e.g. R→S   M⟶N   We write it as $\overrightarrow{RS}$ or $\overrightarrow{MN}$

The line: The line has no end point. It is named after two capital letters. e.g. O⟷P. We write it as $\overleftrightarrow{OP}$ or $\overleftrightarrow{PO}$.

erpendiculars:

Vhen two lines intersect (meet or cross) so that the ngle thus formed is a right angle (a 90° angle), we ay that these two lines are perpendicular to each other.

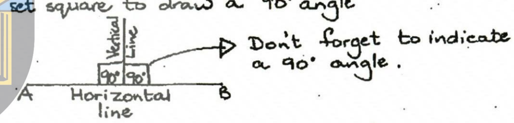
A 90° angle is called a right angle. The symbol is



low we construct a perpendicular:

tep 1: Draw a straight line or segment or ray.

tep 2: Use a set square to draw a 90° angle



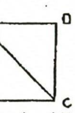
You can also construct a perpendicular line using your compass.

Geometric concepts

segment is a part of a line; it has a start point and an end point.

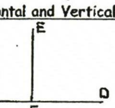
(ED is a line segment)

onal Lines



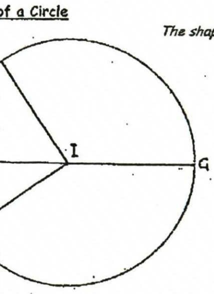
Line Segment AC is a *diagonal line* through square ABCD

ntal and Vertical Lines



Line CD is a horizontal line and line EF is a vertical line.
Line EF is also perpendicular to CD. This means it is at right angles to CD

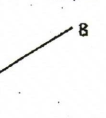
of a Circle



The shape shown here is a circle.

a) The perimeter of a circle is called the *circumference*
b) POINT I is the centre of the circle and it is *the same distance from all points on the circumference of the circle.*
c) LINE FG is the *diameter* of the circle. It is a line from one edge of the circle but it MUST PASS THROUGH THE CENTRE. It cuts the circle into two semi-circles.
d) HI is the radius of the circle; this is the distance from the centre to any point on the circumference.
e) HI and KI are the radii of the circle.
f) There are 360° in a circle

Diameter of Circle = 2 x Radius of the same circle.

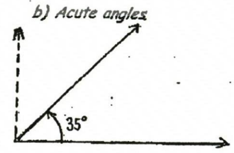


An *angle* is the opening between two lines, which meet at a point.
This point is known as the *vertex point.*

Angle

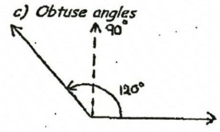
A right angle is when the two lines are measured to be 90° apart.

b) Acute angles



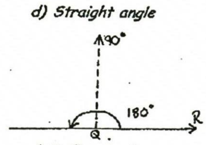
ABC is an *acute angle* because it is less than 90° (a right angle) in size.

c) Obtuse angles



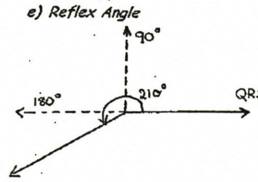
EFG is an *obtuse angle* because it is larger than 90° (a right angle) in size. It is less than 180°

d) Straight angle



PQR is a *straight angle* because it is equal to 180° It is also equal to two right angles put together. 2 x 90° = 180°

e) Reflex Angle

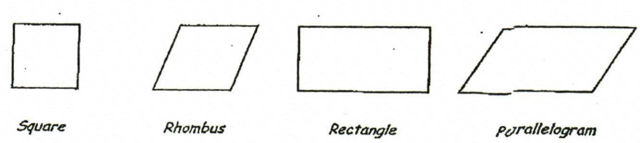


QRS is a *reflex angle* because it is more than 180° in size. It is less than 360°

6)

I. A protractor is used to measure angles. It is a semi-circular shape and is marked out with degrees 0° to 180°. It has an OUTER SCALE and an INNER SCALE.
II. The horizontal line of the protractor is called the *zero line.*

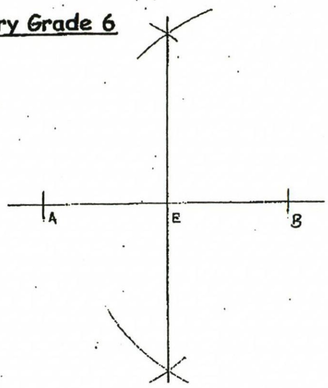
7) A *quadrilateral* is a four-sided shape with four angles inside of it.



Geometry Grade 6

Bisect a given line segment

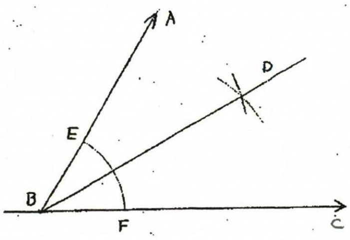
e. Mark *POINT A*, Mark 7cm on your
nd mark a new *POINT B* from *A*.
compass to more that half the way
OINT A and B.
rcs from *A* above and below the
e same from *POINT B*.
aight line through both arcs.
INE SEGMENT AB.



2) Bisect a given Angle
g an angle into two equal parts)

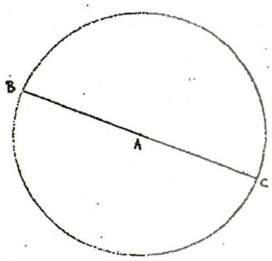
= 60° To bisect ABC you put
ss point on POINT A and draw a
between the two lines.
t where your semi circle meets
ark that F and the point where your
meets BA mark that E.
mpass on POINT E and draw an arc,
justing your compass, do the same
F.
from POINT B through the point
two arcs meet and this will be LINE

isects the ANGLE ABC and divides
to two 30° angles.



3) Bisect a Circle
a circle into two equal parts)

le with a radius of 3cm. BC is a
e segment through the centre of
ouching the circumference at B and
bisects the circle into two equal
s.
the *diameter or 2 radii = 6cm*

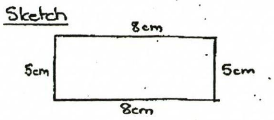


1

2.

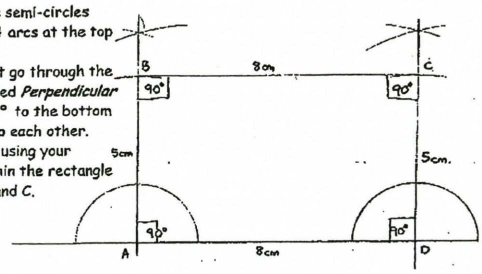
4) Construct a rectangle using a ruler, pencil and compass
(Length = 8cm; Breadth = 5cm)

Sketch



Begin by drawing a base line slightly longer than the 8cm you will need for the length of the rectangle.
Draw *LINE SEGMENT AD* (see note 1). At POINT A and D draw a semi circle using a compass.
Using the 4 points where the semi-circles meet the bottom line, draw 4 arcs at the top on either side.
Draw lines from A and D that go through the two arcs. These lines are called *Perpendicular lines*, because they are at 90° to the bottom lines. They are also parallel to each other.
Draw a mark 5cm up the line using your compass. All four angles within the rectangle will be 90° after you join B and C.

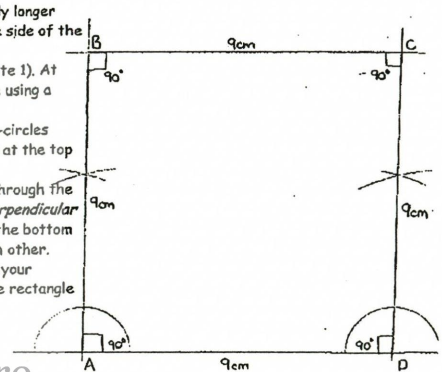
AD = BC and AB = CD



5) Construct a Square Using a ruler, pencil and compass.
(Length of Side = 9cm)

Begin by drawing a base line slightly longer than the 9cm you will need for the side of the square.
Draw *LINE SEGMENT AD* (see note 1). At POINT A and D draw a semi circle using a compass.
Using the 4 points where the semi-circles meet the bottom line, draw 4 arcs at the top on either side.
Draw lines from A and D that go through the two arcs. These lines are called *Perpendicular lines*, because they are at 90° to the bottom lines. They are also parallel to each other.
Draw a mark 5cm up the line using your compass. All four angles within the rectangle will be 90° after you join B and C.

AD = BC and AB = CD

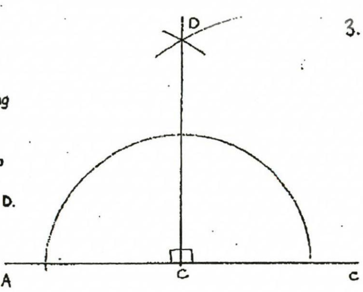


truct a Perpendicular Line on a
Horizontal Line Segment.

3.

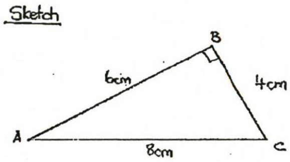
Segment AB. Mark a POINT C
n the line. Draw a semi-circle using
he centre of the circle.
ompass slightly and where the
meets Line Segment AB draw two
the line.
two arcs meet label that POINT D.

pendicular line from D to C.

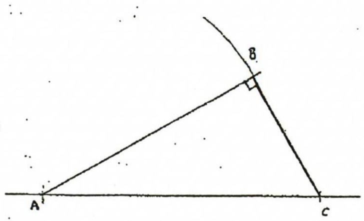


) Constructing a triangle

Sketch



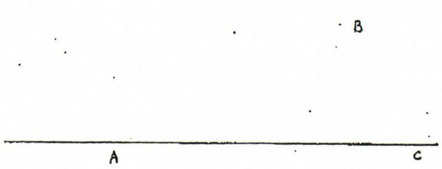
ABC with base AC = 8cm; AB =
= 4cm using a compass, pencil and



awing a base line slightly longer
n you will need for the base of the

and mark POINT A and using a
asure 8cm and mark POINT C.
T A measure 7cm on your compass
arc. From POINT C measure 4cm
pass and draw another arc meeting
e. Where they meet, label this B.
e a triangle.

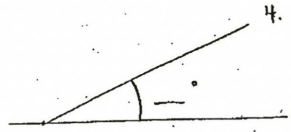
Try to construct ΔABC below

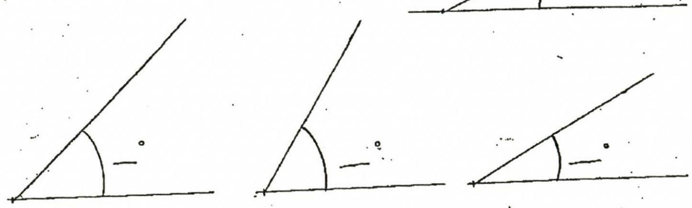


4.

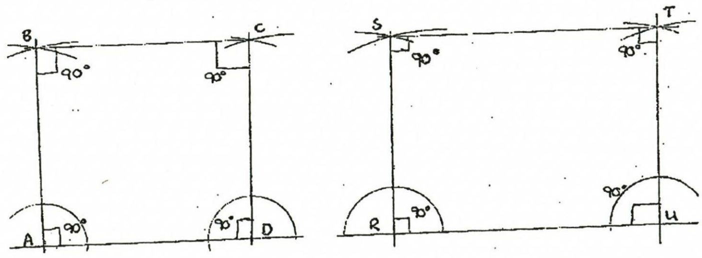
8) Using a protractor

Measure the following angles using your protractor.

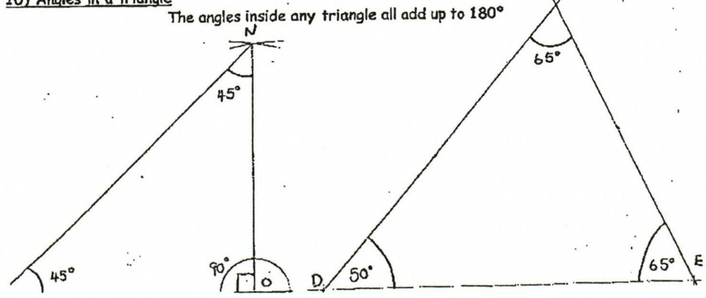




9) Angles in a Square and Rectangle
All four angles inside of a square and a rectangle add up to 360°



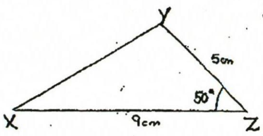
10) Angles in a triangle
The angles inside any triangle all add up to 180°



5.

struct △XYZ with XZ = = 50° and Line Segment ZY sing a ruler, protractor and compass.

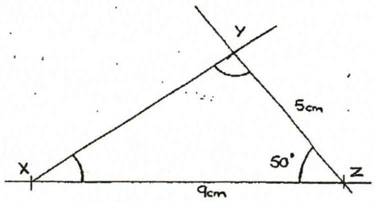
Sketch



awing a rough sketch of what ucted shape will look like. your base line, it is usually e your longest line as the base

awing a base line slightly the 9cm you will need for the triangle.
and mark POINT X and using measure 9cm and mark POINT

UTER scale draw a 50° at Z. n using your compass and DINT Y.
son 14 Measure YX̂Z and XŶZ $\overrightarrow{XY}$

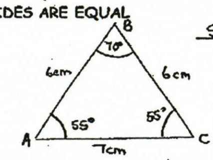


a) i) YX̂Z =
ii) XŶZ =
b) $\overline{XY}$ =

SCALENE TRIANGLE AS NO SIDES ARE EQUAL

struct ABC which is an OSCELES TRIANGLE

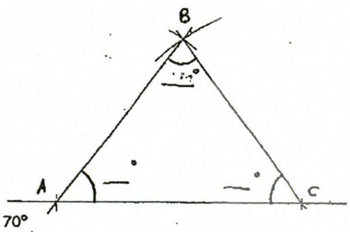
Sketch



awing a base line slightly the 7cm you will need for the triangle.

SEGMENT AC by measuring compass and mark an arc int on the line. Label your DO NOT ADJUST YOUR Put your compass on POINT C DINT A.
compass to 6cm and mark ove the line using POINTS A

wo arcs meet, label that oin POINT B to POINTS A wo straight lines.
AB = 6cm : AC 6cm
C = 55° ; BCA = 55° and ABC = 70°

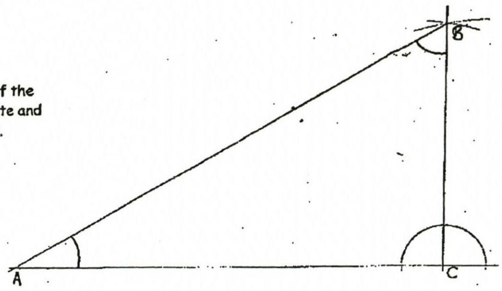


ELES△ the two angles on the base are the same size and the two opposite side of

6.

11) Angles

Measure the angles of the triangle shown opposite and fill in the sizes below.



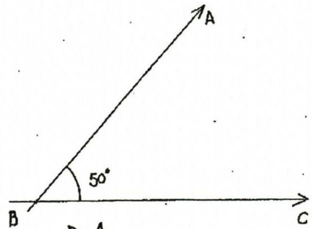
Angle AB̂C =
Angle BÂC =
Angle AĈB = ____
Total = ____

This triangle is called a ________________ because it has a ____° angle

12) Construct an angle of 50° using the Inner Scale

AB̂C = 50°

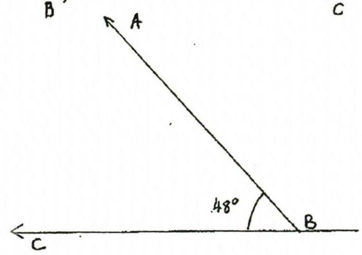
The two rays $\overrightarrow{BC}$ and $\overrightarrow{BA}$ have the same end POINT B which is called the *VERTEX.*



13) Construct an angle of 48° using the Outer Scale

AB̂C = 48°

The two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ have the same end POINT B which is called the *VERTEX*

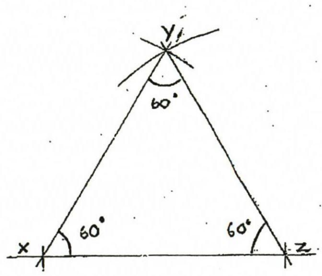


7.

onstruct △XYZ which is an Equilateral triangle

awing a base line slightly longer m you will need for the base of the

SEGMENT $\overline{XZ}$ by measuring 7cm pass and mark an arc from any line. Label your first arc Z. DO ST YOUR COMPASS. Put your POINT Z and mark POINT X. r compass measured to 7cm, draw ove the line and label their meeting

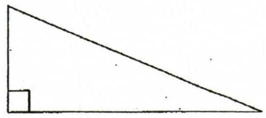


= 60°
= 60°
= 60°

In an Equilateral triangle all the sides are equal. Because of this the angles will all be 60°

ths Homework

Construct a **rectangle** *ABCD* with <u>*AD* = 75mm</u> and *AB* = 35mm.
Construct a **square** *EFGH* with segment <u>*EF* = 45mm.</u>
In the **square** that you have just drawn *EFGH*, there are two **equilateral triangles.** Draw them in two different colours.
Draw a **segment** *DE* that is 85mm in length in an open space. **Bisect the line.**
In the triangle below *ABC*, the angle at ***point A*** is 90 degrees and the angle at ***point B*** is 60 degrees. How big is the angle at ***Point C*** ?



8.

CONSTRUCTIONS

1. Construct △ OPQ (using a ruler and compass) $\overline{OQ}$ = 90mm, $\overline{OP}$ = 50 mm and $\overline{PQ}$ = 75mm
1.1 ∠ OPQ =
1.2 ∠ POQ =
1.3 ∠ PQO =

2. Construct△RST (using protractor, compass and ruler)
$\overline{RT}$ = 65mm, ∠TRS = 35°, $\overline{TS}$ = 70 mm
2.1 $\overline{RS}$ =
2.2 ∠RTS =
2.3 ∠TSR =

3. Construct△EFG (using a ruler and protractor) $\overline{EG}$ = 100mm ∠GEF = 55°, ∠EGF = 55°
3.1 Measure $\overline{EF}$ =
3.2 Measure $\overline{FG}$ =
3.3 ∠ EFG =
3.4 What kind of triangle is this?

4. Construct ☐ ABCD, (using a compass, ruler) $\overline{AD}$ = 80 mm and $\overline{AB}$ = 40mm
4.1 Join point D TO B and point C to A. Where they cross mark this point E.
4.2 Measure $\overline{AE}$ =
$\overline{BE}$ =
$\overline{CE}$ =
$\overline{DE}$ =
4.3 Measure $\overline{BD}$ =
$\overline{AC}$ =
4.4 What kind of triangles are △ ADB and △ ABC?
4.5 Measure ∠ ADB =
4.6 What kind of triangles are △ AEB and △ CED?

ngles

ngles have 3 sides. They have
gles and if you add the angles
her they will add up to 180°.

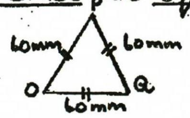
50° + 45° + 85° = 180°

ngle is always named after 3 points

t many different kinds of triangles.
are a few.

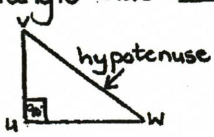
equilateral triangle:

three sides are equal in length



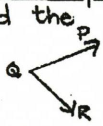
ight angled triangle:

triangle has one right angle (i.e 90° angle)



gles

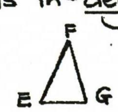
angle is formed when 2 rays have the ne end point. This common end point is ed the vertex of the angle.



angle is made up of $\overrightarrow{QP}$ and $\overrightarrow{QR}$. Q is vertex

ngle is always named after 3 points.
middle point must always be the vertex
e.g <PQR or <RQP or <Q
measure an angle with a protractor and
answer is in degrees e.g 30°

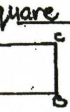
ngle



iangle has 3 angles e.g

G or <GFE
G OR < GEF
F OR < FGE

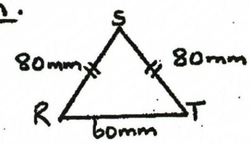
uare and rectangle have 4 angles



< ABC or < CBA
< BCD or < DCB
< ADC or < CDA
< BAD or < DAB

3. An isosolese triangle:

Two sides of this triangle are equal in length.



(Remember:)

The side opposite the right angle is the hypotenuse.

Construct Δ ABC. $\overline{AC}$ = 60mm, $\overline{AB}$ = 50mm, $\overline{BC}$ = 55mm

1. An acute angle is an angle that measures less than 90°.

e.g

2. A right angle

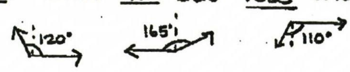
A right angle is an angle that measures 90°

e.g.

3. An obtuse angle

An obtuse angle is an angle that measures more than 90° but less than 180°

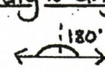
e.g



4. A straight angle

A straight angle is an angle that measures 180°

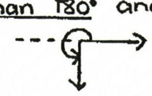
e.g



5. Reflex angle

A reflex angle is an angle that measures more than 180° and less than 360°.

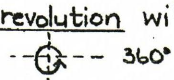
e.g



6. A full revolution

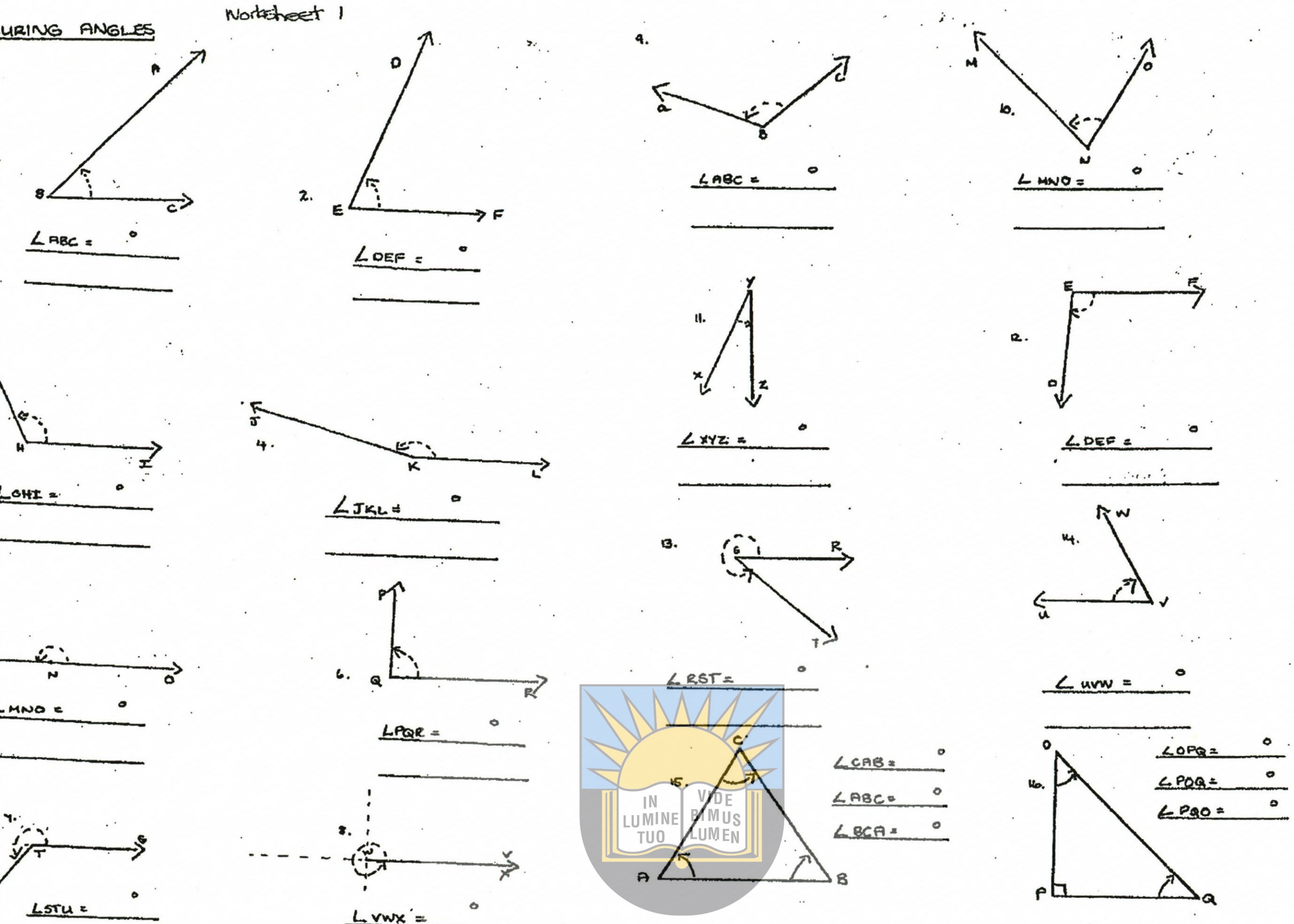
A full revolution will measure 360°

e.g



URING ANGLES

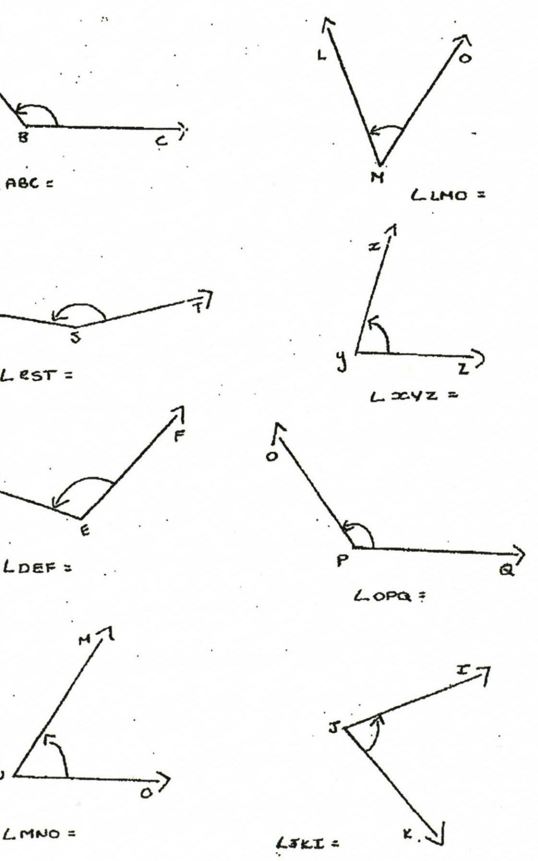
Worksheet 1



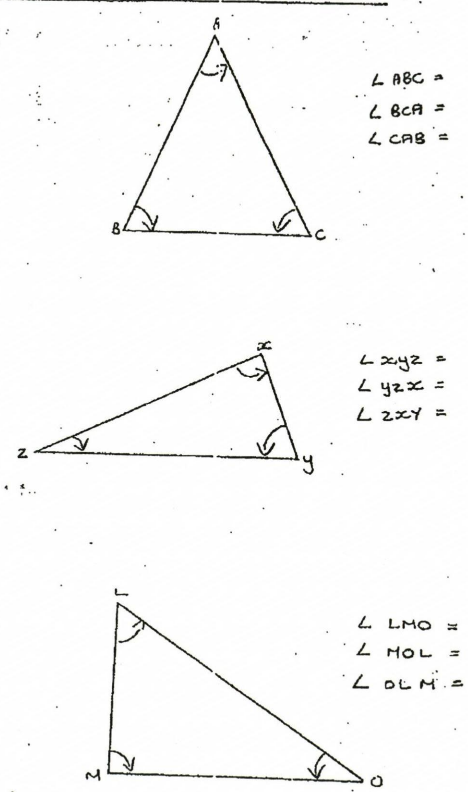
The 3 angles of a triangle must add up to ______

Worksheet 2

MEASURE THE ANGLES

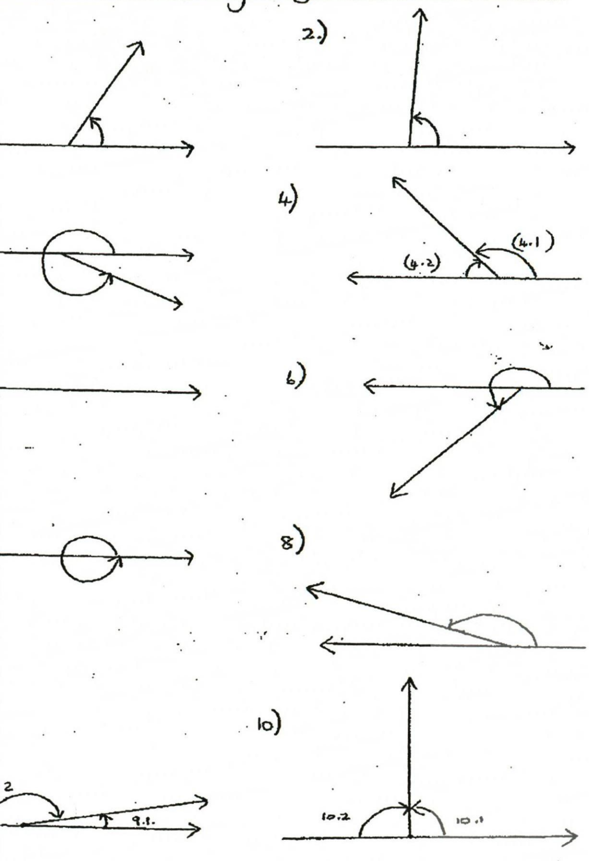


MEASURE THE ∠'s OF THE TRIANGLES



Worksheet 3.

asure the following angles and name them:



Appendix H

Geometry Assessment done at the end of Grade 6

RADE 6 L
OVEMBER 2009

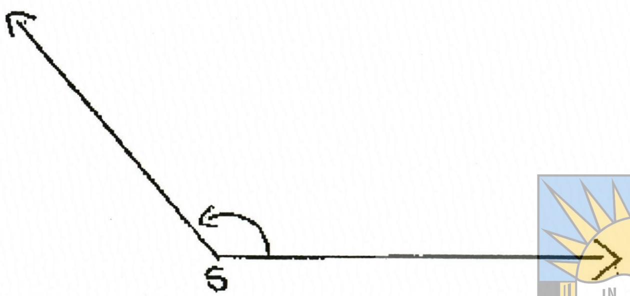
MATHEMATICS PROBLEM SOLVING
PART 2

EOMETRY

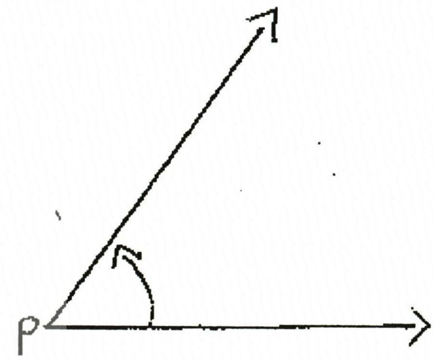
.B. Make sure your pencils are sharpened.
Remember, accuracy is essential, so work carefully.
Label all constructions.

Measure the following angles accurately.

1



1.2



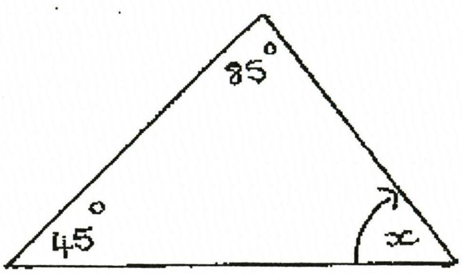
∠S = 130° ✓

∠P = 56°

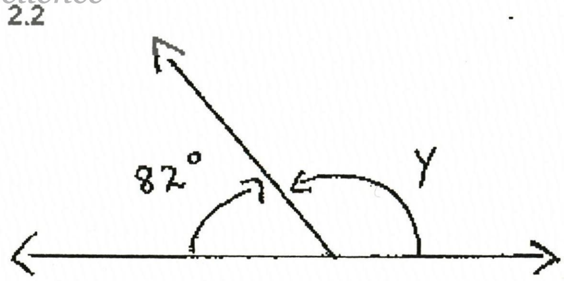
(2)

What will the size be of the following angles?

1



2.2



∠ x = 30°

∠y = 98° ✓

(2)

Draw ∠T = 120°. Bisect the angle using a compass. (2)

Construct a circle with the radius of 40mm. (1)

Draw in the diameter with its measurement.

Construct square RSTU. $\overline{RU}$ = 50mm. Draw a diagonal from R to T and from S to U.
Mark the point where they cross V. (4)

Measure $\overline{RT}$ (1)

Measure $\overline{RV}$ (1)

6. Using a ruler and compass construct triangle ABC. $\overline{AC} = 6cm$, $\overline{AB} = 7cm$ and $\overline{CB} = 7cm$.

6.1 Measure ∠B. (1)

6.2 What type of triangle is this? (1)

7. Using a ruler and protractor, construct triangle DEF.

$\overline{DF} = 75mm$, $\angle D = 60^{0}$ and $\angle F = 45^{0}$. (2)

TOTAL PART 2: [20]

Appendix I

Geometry Tests of two learners in Grade 6 at the research school

2009·11·26
Constructions

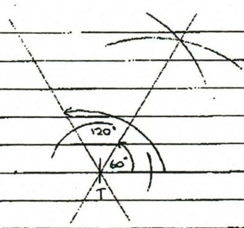
20/20

Fantastic !

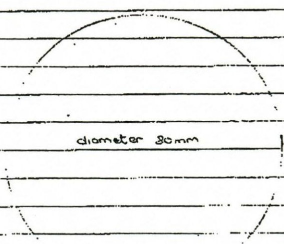
[redacted]
Grade 6c

∠S = 130° ✓
∠P = 55° ✓
∠x = 50° ✓
∠Y = 98° ✓

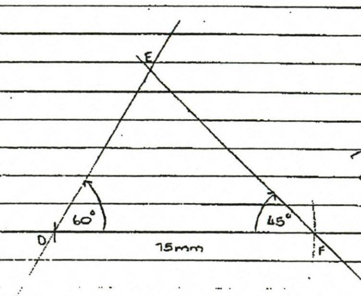
4

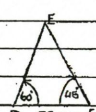


2



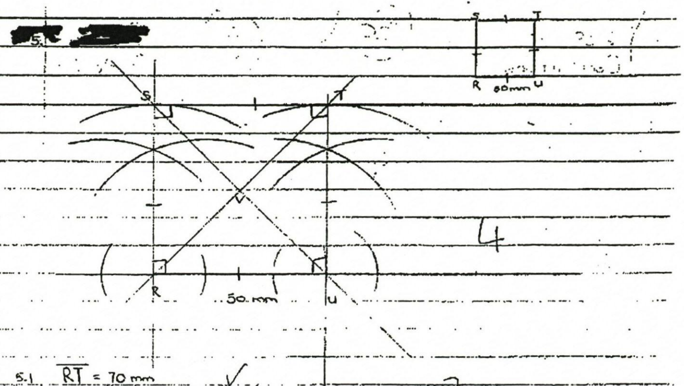
2





2

5. [redacted]

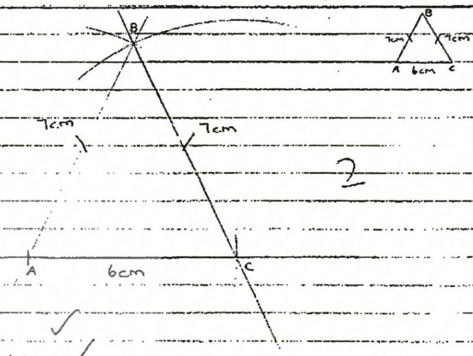


4

5.1 $\overline{RT}$ = 70 mm ✓
5.2 $\overline{RV}$ = 35 mm ✓

2

6.



2

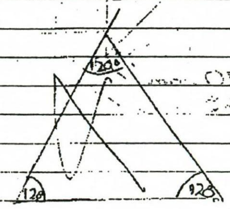
6.1 ∠B = 51° ✓
6.2 Isosceles triangle ✓

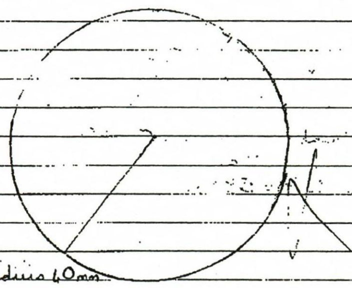
2

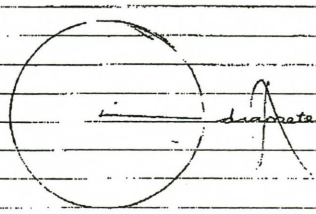
-11-2004 [redacted] fm 6h

per 2 - Part [illegible]

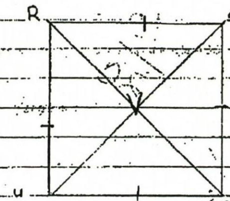
10/20







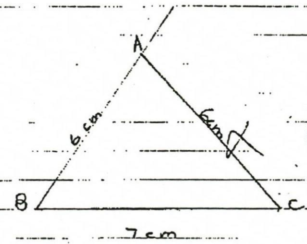
5.



5,1 Measure RT = 70 mm ✓

5,2 Measure RV = 35 mm ✓

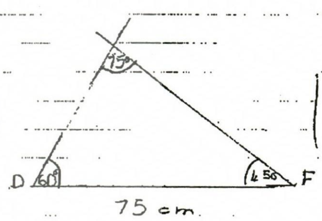
6.



6,1 Measure ∠B = 58° ✗

6,2 Isoscelise ✓

7.



Appendix J

Interview with Learner B

Date of Interview: 1 December 2009 Time: 11 a.m.

Age: 13,5 years Home Language Xhosa

Sports played: Summer: Athletics running

Winter: Rugby

Academies: None

Jigsaw puzzles: Seen: said not Played with: When: In grade 3

Lego: Seen: yes Played with: made a house When: last year at home

Tan gram: Seen: yes Played with: lego When: at aftercare in grade 4

Interviewer: This is a picture. Can you look at that picture and tell me the names of any shapes you can see.

Answer: Yes. Triangle

Interviewer: Show me where you can see the triangle.

Answer: Here.

Interviewer: Ok

Answer: And a square and rectangles.

Interviewer: Where is the square

Answer: Here

Interviewer: And the rectangle.

Answer: Here

Interviewer: Ok. Can you see anything else?

Answer: Er. The chimney.

Interviewer: What shape is the chimney do you think?

Answer: A rectangle.

Interviewer: Yes. It looks like a rectangle to me. You are quite right. Anything else you can see? Or do you think we have seen everything?

Answer: (pause) Yes. We have.

Interviewer: All right. I am going to give you these shapes. Can you name these shapes?

Answer: This is a triangle. Square, rectangle and this is a .... Hexagon.

Interviewer: Well done. Do you know what that one is there?

Answer: No.

Interviewer: You have done well. Now I want to show you some more pictures. There. Which ones there are triangles?

Answer: This one here and this one and this one.

Interviewer: Yes. What about this one here?

Answer: Oh sorry. I skipped that one there.

Interviewer: Do you think it is a triangle?

Answer: Yes.

Interviewer: Why do you think they are all triangles?

Answer: Because they are made in the shape of triangles.

Interviewer: And what is the shape? What shape do triangles ahave?

Answer: I can't explain it.

Interviewer: All right. But you know what it looks like.

Answer: Yes.

Interviewer: Which ones here do you think are squares?

Answer: This one and this one and this one and this one.

Interviewer: So you think there are three squares here. You say it is F, G and H. Why do you say they are all squares?

Answer: Cos if you turn this one around and it's a square and this one is a kind of a skew square.

Interviewer: All right. So you say it is a skew square. Which ones here are rectangles?

Answer: This one.

Interviewer: You think there is only one rectangle.

Answer: Yes.

Interviewer: Why do you think these are not rectangles?

Answer: Cos they don't look like the shape of rectangles.

Interviewer: Ok. So the shape is wrong to be a rectangle, hey?

Answer: Yes.

Interviewer: And this last one here. This shape you couldn't remember is called a parallelogram. Can you remember what parallel lines are?

Answer: Yes the lines that are the same distance apart.

Interviewer: Lines that are the same distance apart. Can you see any parallel lines in this room?

Answer: Yes

Interviewer: Where can you see them?

Answer: That line to this one here.

Interviewer: You mean from here to here? These two are parallel.

Answer: Yes.

Interviewer: Can you see any parallel lines in this shape here?

Answer: There and there.

Interviewer: That's right and that is why it is called a parallelogram. It says here "which ones here are parallelograms?"

Answer: Er. This one

Interviewer: Yes

Answer: This one is parallel to that one and this one to this one.

Interviewer: So you think they are all parallelograms?

Answer: Yes.

Interviewer: Well done. They are. Now I am taking this here. What shape is this?

Answer: A square.

Interviewer: That's right and if I do this to it (turns it through 90 degrees), is it still a square?

Answer: Yes.

Interviewer: Why?

Answer: The shape doesn't change. It is just the angle that changes.

Interviewer: Now pretend you are talking to your friend on the phone and your friend couldn't see that shape. But you want to tell your friend about the shape without saying the word "square"

Answer: All four equal sides.

Interviewer: Anything else

Answer: They have four sides

Interviewer: Yes

Answer: And …… ja.

Interviewer: And they are all equal, you say? You think that is enough and he would know exactly what you were describing?

Answer: Yes.

Interviewer: All right. I am giving you some construction sticks which have different lengths and these are the little corners. You can slide them in and out like that.

Answer: Yes

Interviewer: You will have to use a connector there to make the angle.

Answer: Is this also a connector

Interviewer: Yes and the nice thing about that is you can make it smaller or larger. (long pause) Yes it is not so easy to get them out. Let me help you. We can just slide it out. (pause) Do you think if we used a different connector it would help?

Answer: Yes.

Interviewer: Right. Is it going to fit? It is a little bit bent, hey? (took 2 minutes and 10 seconds to make the triangle) What sort of triangle have you made there?

Answer: A equilateral triangle

Interviewer: Why is it an equilateral triangle

Answer: Cos all 4 sides are equal

Interviewer: 4 sides?

Answer: Oh. I mean 3 sides are equal.

Interviewer: Good. Let's take these apart. Can you make me a square with these sticks? Remember you have lots of different sized sticks.

Answer: (took roughly 90 seconds to make the square). He starts with the grey connector which would not make a 90 degree angle. He battled to make the right angled corners))

Interviewer: Why have you chosen them all the same colour.

Answer: Cos they are all the same.

Interviewer: Could you make a square with two red ones and two yellow ones?

Answer: No cause these ones are too big. They are too long.

Interviewer: So you couldn't make a square with the red ones at all?

Answer: You could make a square with them but not next to the yellow ones.

Interviewer: Why would that be?

Answer: Cos these ones are smaller and these ones are longer.

Interviewer: So what are you telling me about the sides in a square?

Answer: Um. The sides. Um the length isn't the square.

Interviewer: Ok. You obviously know about a square. Can you take all those shapes there and sort them into any groups that you would like.

Answer: Must the squares all be in the same place.

Interviewer: You are in charge. Noone is telling you what to do. You are deciding. Ok so how many groups have you got?

Answer: One, two, three four five.

Interviewer: You have five groups. Ok. Why did you put them into these groups.

Answer: They are all like the same shapes with the group.

Interviewer: Why are those ones different from those ones and these different from these ones?

Answer: Cos these ones are shorter than these ones and um. These ones don't have three sides.

Interviewer: So you sorted them because of their sides and because of their shape. Anything else?

Answer: No.

Interviewer: What if I wanted to put these (the squares and rectangles) together in the same group. Why would I do that?

Answer: Because they have four sides.

Interviewer: But look, how many sides does this one have?

Answer: Four.

Interviewer: But I don't want that one with those. I am leaving that one out. So why can I put those two together but leave that one out?

Answer: They are kind of the same. Er… Shape.

Interviewer: In which way are they kind of the same?

Answer: The square. If this wasn't that long, it would have been a square. (points to the rectangle)

Interviewer: That's right. What do you think about it that would make it a square if you made it shorter?

Answer: If you put it into half.

Interviewer: Ok. You mean if I cut it in half, it might be a square. What is it that makes it look like a square do you think?

Answer: Uh. If you like cut it in half.

Interviewer: All right. Let's take two of these triangles and put them together. How many sides does this shape have?

Answer: Er. Three – no four.

Interviewer: That's fine. Which group would you put it in?

Answer: I would put into this group.

Interviewer: Why?

Answer: Because it Is almost the same as this one.

Interviewer: So you put it with the parallelograms. Why? What is it about the group that makes it almost the same?

Answer: Because the sharp point here.

Interviewer: Is there anything else that puts them into the same group?

Answer: The sides. They also have four sides.

Interviewer: Anything else?

Answer: (sighs) No.

Interviewer: This shape is called a rhombus. If I put it with this one and this one and this one, why can I do that?

Answer: Because these two are the same. And these two are the same (puts the rectangle and square together and then the rhombus and parallelogram)

Interviewer: But if they can all be put together? Is there anything about them all that is the same?

Answer: Yes. They all have 4 sides.

Interviewer: Is there anything else that is the same?

Answer: No. Oh if you cut them in half like this they can be the same as these ones.

Interviewer: Oh, so if you cut this one in half, what would you get?

Answer: A triangle.

Interviewer: That's right and would it be the same with this one?

Answer: Yes

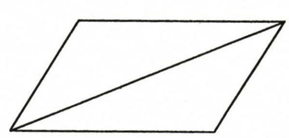
Interviewer: And this one? If you had a pair of scissors, could you cut it?

Answer: Yes. You could cut it like this.

Interviewer: Yes. Could you cut it in another direction to make two triangles? You could cut it like this. Could you cut it like this?

Answer: No.

Interviewer: So you could cut it but not like this



Answer: Yes

Interviewer: Do you know what an isosceles triangle is?

Answer: Yes. It is when two sides are equal.

Interviewer: That's right. It says here that an isosceles triangle is a triangle with two sides of equal length. Here are three triangles. Do you agree that they are all isosceles?

Answer: Yes.

Interviewer: We need to decide which is true for every isosceles triangle.

Answer:	The three sides have equal length. No.
Interviewer:	So you don't think that is true. What about the next one?
Answer:	One side is double the length of the other side. No.
Interviewer:	You don't think it is true in all isosceles triangles. Could it be true in one isosceles triangle?
Answer:	Yes.
Interviewer:	Why?
Answer:	Because they could be double and then they could be equal.
Interviewer:	They could be double the third one?
Answer:	Yes.
Interviewer:	So it could be two very long ones with a short one and then this one could be double that one?
Answer:	Yes.
Interviewer:	It could be the case in some. But not in all of them?
Answer:	No.
Interviewer:	What about c?
Answer:	No. That is an equilateral one.
Interviewer:	What does c say?
Answer:	It says that three angles must be the same.
Interviewer:	You are right. What about d?
Answer:	There must be at least two angles of the same size. Yes.
Interviewer:	You like that one? What about the last one?
Answer:	None of a to d is true in every isosceles. Um.
Interviewer:	If it says none is true, it means that none of these is true. What do you think? You liked c? So you think c is right? (this was not the one he had chosen. The interviewer made a mistake but he did not correct her).
Answer:	Yes.
Interviewer:	All right. Look at number 6. What shape is this?
Answer:	If they cut.
Interviewer:	What is this whole big shape?
Answer:	Er a rectangle.

Interviewer: That's right. It says "In the rectangle ABCD" Are you familiar in class with naming lines. What would we call this?

Answer: The length.

Interviewer: Yes and what would we name it? Your teacher's name is Mrs. C. What would be the name of this side?

Answer: Oh. BC.

Interviewer: That's right and what would be the name of this side?

Answer: AC

Interviewer: That's right. It says, "in the rectangle, there are diagonals, BD and AC". A diagonal is like that line you said we could draw that we could cut the shape before.

Answer: Yes.

Interviewer: In the rectangle, diagonal BD and AC cut the rectangle in half, into two triangles. Which of the following is not true in the rectangle?

Answer: There are four right angles. In a what? In a triangle?

Interviewer: No. In a rectangle. Are there four right angles?

Answer: Yes.

Interviewer: What is a right angle?

Answer: Like an angle that forms 90 degrees.

Interviewer: Perfect. So do you think there are four 90 degree angles in a rectangle?

Answer: Yes.

Interviewer: What about b.

Answer: There are 4 sides.

Interviewer: Is that true?

Answer: Yes.

Interviewer: So you think a and b are true. We are looking for the one that is not true. What about c?

Answer: The diagonals have the same length. That is true.

Interviewer: Why do you say that?

Answer: Because they start from this point and that one also starts from that point.

Interviewer: So you can see they are the same length. What about the last one? D?

Answer: All the sides have the same length. No.

Interviewer: So you think that is the one that is not true. Why do you say that?

Answer: Because not all the sides are the same.

Interviewer: Which sides are the same or are none of the sides the same?

Answer: Not all of them. The ones that are opposite each other are the same.

Interviewer: Well done! So you chose d there. This is the last one and thee is no picture here. It says "which one do you think is true?" So I want you to read out the sentence. Here are the shapes to help you. Take a square, a rectangle and a parallelogram. Have a look at the shape and decide whether the sentence is true.

Answer: All triangles are squares. Er Yes.

Interviewer: You think all rectangles are squares. Why do you say that?

Answer: Because they have the same shape. Except they longer.

Interviewer: All right. What about the next one.

Answer: All parallelograms are squares. No.

Interviewer: So you think that is not true.

Answer: Wait. It could be a square if you like squashed it. Um.

Interviewer: So what do you think?

Answer: I think it could be a square.

Interviewer: Right. What about c.

Answer: No.

Interviewer: What does it say?

Answer: All parallelograms are rectangles. No. That is not right.

Interviewer: Why is it not right?

Answer: Because if you put it into a square, it's going to be a square but it has to be a big one so it can be a rectangle and it is not

Interviewer: So you say that is not true. It can't be.

Answer: Yes.

Interviewer: And what about the last one

Answer: All rectangles are parallelograms. Um. No

Interviewer: Why do you say that?

Answer: Because all parallelograms are squares.

Interviewer: So you think the first one is the right one?

Answer: Yes.

Interviewer: Ok. We have one more question. There is a shape, PQRS and they say it is a square. It says which relationship is true in all squares. Do you know what perpendicular is, by the way?

Answer: Yes.

Interviewer: What is perpendicular?

Answer: Lines that meet.

Interviewer: Ok. So if I draw two lines like this, are they perpendicular?

Answer: Yes. Lines that form a right angle.

Interviewer: That's right. So they look like that and form a right angle. Read out the statement. Find PR.

Answer: PR and RS have the same length.

Interviewer: Where is PR?

Answer: There

Interviewer: That's right. What is the other side?

Answer: RS.

Interviewer: Where is that?

Answer: Here.

Interviewer: Do you think they have the same length?

Answer: No.

Interviewer: So you think that one is not true. And the next one?

Answer: QS and PR are perpendicular. QS and PR. No

Interviewer: Where are they?

Answer: QS and PR. Ja they are perpendicular?

Interviewer: Why do you think they are perpendicular?

Answer: They cross to form a right angle.

Interviewer: Where do they cross?

Answer: Here. Q and S. QS.

Interviewer: And PR?

Answer: PR. Does it mean where they cross?

Interviewer: Yes.

Answer: No.

Interviewer: So you don't think that one is correct. What about C?

Answer: PS and QR are perpendicular.

Interviewer: Where is PS? And QR?

Answer:	Yes they are.
Interviewer:	Why do you think they are perpendicular?
Answer:	Because they cross … to form a right angle.
Interviewer:	Are you sure they cross? Find PS.
Answer:	There
Interviewer:	And QR?
Answer:	There.
Interviewer:	Are they going to cross?
Answer:	No. They are parallel.
Interviewer:	That's right. Are they ever going to be perpendicular?
Answer:	No. They won't touch each other.
Interviewer:	That's right. Let's look at d. What does it say?
Answer:	PS and QS have the same length.
Interviewer:	There's PS and there's QS. Do you think they have the same length?
Answer:	Long pause.
Interviewer:	There's PS and QS. Do you think they have the same length?
Answer:	Yes.
Interviewer:	You think they do. All right. And the last one?
Answer:	Angle Q is larger than angle R
Interviewer:	Where is angle Q?
Answer:	Here
Interviewer:	And angle R?
Answer:	Here
Interviewer:	Do you think angle Q is bigger than angle R?
Answer:	No
Interviewer:	Why not?
Answer:	Because they are the same.
Interviewer:	Why do you say that?
Answer:	Because this side is equal to this side.
Interviewer:	Ok and so the angles will be the same size. You have done very well. Thank you. I am turning off the recorder.

Appendix K

Interview with Learner E

Date of interview:	1/12		Time:	11:35		
Age: 12	Home Language		:	English		
Sports played:	Summer:	Swimming				
	Winter:	swimming at home				
Academies:	Computer					
Extra curricular activities:		Library				
Jigsaw puzzles:	Seen: yes	Played with:	20 – 25 piece	When:	5 - 7 years old	
Lego:	Seen: yes	Played with:	lego and duplo with sister	When:	grade 0	
Tan gram:	Not remembered	Played with:	used to make a house	When:	grade 1	

Interviewer: What shapes can you see in this picture?

Answer: Um – a rectangle, a square

Interviewer: Where is the square?

Answer: Up top there. There is the other side of the square on that side (points to roof)………… and a triangle ………. And another rectangle

Interviewer: Where is the rectangle?

Answer: Over there. The long one.

Interviewer: Ok. So it is a rectangle on its side sloping up the side of the roof?

Answer: And then…………there's another square.

Interviewer: Do you think that is a square or a rectangle?

Answer: A rectangle

Interviewer: So you have seen 3 rectangles so far

Answer: And then there's a rhombus (points to bottom of picture where the shapes are indistinct and run into each other)

Interviewer: Ok. So you have seen a rhombus.

Answer: And there is a half of a triangle. A skew one like that.

Interviewer: So there are lots of funny little shapes at the bottom as well. I'm going to give you some shapes now. Do you know the names of those shapes?

Answer:	A hexagon, a square, I think that's a rhombus there and then a triangle and then a rectangle
Interviewer:	Good. I am going to show you some questions on a page and want you to decide what the correct answer is. Which ones are triangles? Just tell me the letters for the ones you think are triangles.
Answer:	D,….. B, and that is sort of a triangle (points to A, the kite)
Interviewer:	What makes a figure a triangle?
Answer:	A figure a triangle?
Interviewer:	Yes, how do you know if a shape is a triangle?
Answer:	Because it looks like a kite. It looks like half of a kite.
Interviewer:	Have you ever flown a kite?
Answer:	Not those diamond ones. I've flown those animal ones.
Interviewer:	So it looks like half of a picture of a kite in a book? Is that what you are saying?
Answer:	Yes.
Interviewer:	Which of those pictures are squares?
Answer:	This one. H and G (G is a parallelogram).
Interviewer:	Why do you say that G is a square?
Answer:	Because it is a skew square.
Interviewer:	None of the others are squares?
Answer:	(shakes head)
Interviewer:	All right. What about number 3.
Answer:	That one (points suddenly to F, a kite, among the shapes for Question 2)
Interviewer:	You think F is a square. Why?
Answer:	Because it is sort of on its side.
Interviewer:	What has happened to the square?
Answer:	On its side, it's been bumped. (laughs)
Interviewer:	Ok so it is on its side. Its been bumped.
Answer:	If you push it up it is a square.
Interviewer:	Now which ones are rectangles?
Answer:	Um. I and ….. just I
Interviewer:	Just I. Why do you think the other ones aren't
Answer:	Because that could be a rectangle (K) but those ones are not equal.

Interviewer:	So you think that could be a rectangle but those ones are not equal?
Answer:	No that one is not equal to those ones (the sides). That one has to equal that one and that one has to equal that one.
Interviewer:	Ok. Now lets look at the last one on here. You do know that this one here is called a parallelogram.
Answer:	Oh. (laughs)
Interviewer:	Do you remember what parallel lines are?
Answer:	Mmm. When they are skew. And also when they are straight.
Interviewer:	That's fine. Do you remember that parallel lines run opposite each other an equal distance apart for ever and ever. They never meet. They are like a railway track. Have you ever seen a railway track?
Answer:	No
Interviewer:	Or like the sides of this table here.
Answer:	A train, a train.
Interviewer:	That's right. The trains wheels are like this and so the train track has to go like this otherwise where would the train's wheels go? And the sides of the table. They are also parallel. Are the two sides ever going to meet?
Answer:	Not unless you cut them off ……..
Interviewer:	That's true. So that is what we mean by parallel. So this is called a parallelogram. Why do you think these shapes are called parallelograms?
Answer:	Because those ones are equal and those ones are equal.
Interviewer:	All right. Or parallel. So now what do you think? Which ones are parallelograms?
Answer:	M and L.
Interviewer:	Why do you choose those two?
Answer:	Because those ones are parallel and those ones are parallel. (ignores the rectangle)
Interviewer:	Right. I am giving you this shape here. You know what it is don't you? What is it.
Answer:	A square.
Interviewer:	And if I did this to it is it still a square?
Answer:	Yes
Interviewer:	Why is it still a square?
Answer:	It's just been moved.
Interviewer:	How have I moved it? Did I move it (translates the shape) or have I turned it?
Answer:	Turned it.

Interviewer: I want you to pretend you are talking to your friend on the phone and you want to describe this shape, but you may not use the word square. Your friend does not know what you have in front of you.

Answer: They have four equal sides and it is yellow.

Interviewer: Would that be enough for her to know what it was?

Answer: Yes. And all the measurements are the same

Interviewer: What measurements are the same?

Answer: Like 45 cm.

Interviewer: So all the lengths would be exactly the same. I don't know if you have ever played with these. If you play with your little sister you might have played with something like these. These are construction sticks. You take a stick and slide it into a connector. These are other connectors. Can you make a triangle with these?

Answer: Ok. Do they have to be all equal? The sides?

Interviewer: Any kind of triangle you like. You are in charge.

Answer: Oh no

Interviewer: You might find that is not the right kind of connector. It might be easier if you used a different kind of connector.

Answer: This is too long.

Interviewer: So what could you do? You have this one and those ones. What about a different length stick?

Answer: There. (one minute has passed)

Interviewer: That's a clever idea. I can see you are used to playing with a little sister. What kind of triangle have you made there?

Answer: A triangle.

Interviewer: It is a triangle but what sort of triangle do you think?

Answer: Scalene

Interviewer: And why do you say that?

Answer: Because two sides are equal.

Interviewer: Ok. Why do you know two sides are equal?

Answer: Because those are equal.

Interviewer: That's right. You chose two sticks that are equal. They are the same colour. Now can you make a square for me? You can take this apart or use other sticks if you like.

Answer: Mmmmm.

Interviewer: Now what colour stick would you have to use to finish off your square?

Answer: These ones.

Interviewer: You could use that one. You don't have to take it apart. You could put that one in the corner. Try it and see if it works. (pause. She has been working for 1 minute) Do you think it will work?

Answer: Here.

Interviewer: Well done. Now why did you make all of them red. Why did you not use red and yellow sticks?

Answer: Because then they won't be the same because of that size.

Interviewer: So what would you end up with if you used red and yellow?

Answer: one longer.

Interviewer: that's true. So that is why you chose them all the same colour. Well done. Now I am giving these shapes and you need to choose your own groups. You know how you sort your toys when you tidy your room. I want you to sort all these shapes into groups. You can choose how you sort them.

Answer: Like colours …….

Interviewer: Any way you like. (Waits for 45 seconds while the shapes are sorted). Now Jade how many groups have you made?

Answer: 6

Interviewer: You have 6 groups. Why did you put them into 6 groups

Answer: I wanted to put them in order. Like all the rectangles together and um…….

Interviewer: All right, you wanted to put the rectangles together and…… Is this a separate group?

Answer: Ya

Interviewer: Ok. And how did you decide. Ok you said these are rectangles and these are squares. What else did you do?

Answer: Ummm. I put those together.

Interviewer: Why

Answer: Because they are all triangles.

Interviewer: It looks as though there are two groups of triangles there.

Answer: That's a different kind of triangle

Interviewer: What sort of triangle is that?

Answer: A scalene … no isosceles.

Interviewer: Ok those are isosceles triangle there and over here. Why are these ones different?

Answer: Because those ones are all equal.

Interviewer: So you thought that even though the triangles all have 3 sides, there are different types of triangles.

Answer: Actually these are scalene.

Interviewer: Why? What is a scalene triangle/

Answer: Isn't a scalene triangle when 2 sides are equal?

Interviewer: No that is an isosceles triangle. In an isosceles triangle two sides are equal

Answer: So these are isosceles.

Interviewer: But you are able to see clearly that those have two sides equal and those have all sides equal. Now. I want to put these in the same group. (puts squares and rectangles into same group). What is the same about them?

Answer: That they have 2 equal sides. Those ones there.

Interviewer: And the rectangles do they have two equal sides as well?

Answer: Yes

Interviewer: Which two sides?

Answer: Those two.

Interviewer: Is this one equal to this one?

Answer: No that one equals that one and that one is equal to that one.

Interviewer: That's right. And is it the same with these ones?

Answer: Not really

Interviewer: Is this one not equal to this one?

Answer: Yes it is.

Interviewer: Is this one here equal to this one? Oh so they could be put into the group because of the length of the sides. Is there anything else you think is the same about them.

Answer: Mmmmmm. Oh and if you cut them in half they can be a square. Like that. They can be a square.

Interviewer: All right, I am taking these two triangles which you called equilateral triangles and, if I put them together I get a shape, called a rhombus. Which group of your 6 groups would you put this rhombus into?

Answer: Into any group?

Interviewer: Yes. Which group would you choose? It must go into a group.

Answer: (long pause) This one

Interviewer: Why did you put it into that group

Answer: Because if you cut it in half, it would be a triangle.

Interviewer: Is it the same as this one. If you cut it in half it makes triangles?

Answer: Yes

Interviewer: Ok. If I put the rhombus, this and this and this together (places rhombus, square, rectangle and parallelogram together), why can I put them all together in one group? Is there anything about them that is the same?

Answer: Hmmmmmm. (long pause) Two of their sides are equal.

Interviewer: Do all of them have two sides equal. And which sides are equal?

Answer: That one and that one. That one and that one. That one and that one. (whispers) and that one and that one.

Interviewer: And do they all only have one pair of sides equal. You said two sides are equal. What about the other sides?

Answer: They are also.

Interviewer: So that one is equal to that one and that one to that one …….. All right. So you say it is because the opposite sides are equal. Is that the only reason that you think they could be put together?

Answer: Also if you put this square, if you halve it then you could get another one and if you take the triangle like that and its sort of that it could be a smaller rhombus.

Interviewer: All right and then how would that connect those two with those two (pointing to the rectangle and parallelogram)

Answer: Not much. But they equal. Equal sides.

Interviewer: All right. We are going to look at some pictures. You know what an isosceles triangle is, hey? You just got a little confused with the names. Now it says here that an isosceles triangle is a triangle with two sides that have the same length. We know that. Can you see the three triangles? Do you agree that those are all isosceles.

Answer: No

Interviewer: Why not?

Answer: Because that one, they all equal (points to the first triangle)

Interviewer: So what must the first one be?

Answer: An equilateral

Interviewer: That's right but ………

Answer: Or an equal right angle

Interviewer: If it is an equilateral triangle, in other words, it has those three sides equal could it not also be an isosceles triangle? Because an isosceles triangle has two sides equal?

Answer: It could be but then those two are equal

Interviewer: That's true. We will look at those two isosceles triangles. It says which is true of every isosceles triangle. I want you to read these out and then think whether it is correct. Read the first one and see what you think.

Answer: The 3 sides have equal length

Interviewer: Is that true in all isosceles triangles?

Answer: No

Interviewer: Then let's look at the next one

Answer: One side is double the length of the other side. It could be.

Interviewer: You are right. But would it be in all isosceles tiangles

Answer: No

Interviewer: Maybe in some. What about the next one.

Answer: There must be at least two angles of the same size. Yes

Interviewer: So you think that might be the true one. Lets read d.

Answer: 3 angles must be the same size. No

Interviewer: Ok what about e?

Answer: None of a to d is true in every isosceles triangle. I don't get that.

Interviewer: It means that none of them would be true. Do you think that any one is true?

Answer: c

Interviewer: Ok and you are right. Let's look at the rectangle. Don't worry about what is inside the rectangle. Can you see the rectangle shape? ABCD. And you are used to us calling lines……

Answer: Look at the hour glass the sand comes down

Interviewer: Yes. You mean this looks a bit like an hour glass? (points to where the diagonals cross)

Answer: Ja when the sand comes down and those are the two sides.

Interviewer: That's right. We use one when we play 30 seconds. Have you ever played 30 seconds?

Answer: And also when you have to do those words.

Interviewer: Yes Boggle. It does look a bit like an hour glass. We are looking at the shape on the outside. It is this (draws over the shape with finger) and it is called a rectangle. The rectangle, ABCD. We know that, if I joined B to D.. In other words if I picked up my ruler (picks up a shape). Pretend this is a ruler and I drew a line there, that would be called a diagonal that line and if I took my ruler and I put it there, that line would also be called a diagonal. So it says that AC and BD are the diagonals. We have to look through here and decide which one is NOT true. Ok. So look at a and read it out.

Answer: There are 4 right angles.

Interviewer: Do you think that is true in a rectangle that there are 4 right angles or don't you think that is true?

Answer: Shakes head

Interviewer: So you don't think a is true? Then what about b

Answer: There are 4 sides. Yes that is true.

Interviewer: You think that is true. What about c?

Answer: The diagonals have the same length. That's not true.

Interviewer: You also think that is not true? Ok and what about d?

Answer: That I'm not sure. All the sides have the same length. That is not true.

Interviewer: So that one is also not true?

Answer: There is only one true one.

Interviewer: There is only one true one about all rectangles having four sides. So everything else is wrong, you think. (She nods) Ok. Lets look at the next one. There is no picture so you will have to think here. It says which one do you think is true? Read a and tell me if you think it is true. To help you, it talks about rectangles and squares and parallelograms. There you are, if you are not too sure you can look and see the shapes and think about it. Let's read the first one.

Answer: All rectangles are squares. That is true becausehalf of that is a square.

Interviewer: So you think a is true.

Answer: Look there

Interviewer: But now, if I cut this rectangle in half, would I get a square?

Answer: Yes

Interviewer: Do you think that would be a square?

Answer: On that side

Interviewer: So I could cut it in half you say and I would get two squares.

Answer: Mmm. No

Interviewer: You don't think so. What would I get if I cut it in half?

Answer: Half a rectangle.

Interviewer: Half a rectangle, that's right. It could be a square but…

Answer: a little bit smaller

Interviewer: that's true so maybe when it says all rectangles are squares, do you think that is true?

Answer: Not really. Sometimes.

Interviewer: Sometimes it could be true. What about b?

Answer: ummmmm. All parallelograms are squares. Ummm. That's not true.

Interviewer: Why do you say that?

Answer: Because there is no square connected to this. So you can't have a square.

Interviewer: What about the next one?

Answer: This actually is quite a square. If you squish it together like that it would be a square.

Interviewer: That is true if you could squish it but you can't cos it is hard plastic. Do you think if you squished it you would get a square or...... Which sort of shape do you think you would get? Would you get that or that?

Answer: Mmmm. You would get that.

Interviewer: So it would be a rectangle rather than a square wouldn't it? So do you think b is right. That all parallelograms are squares?

Answer: No

Interviewer: You don't. What about c/

Answer: All parallelograms are rectangles. Yes

Interviewer: So you think c is correct. And d?

Answer: All rectangles are parallelograms.

Interviewer: There's a rectangle. Is it .... Do you think all rectangles are parallelograms?

Answer: Yes

Interviewer: You do? So you think there is another one there that is true. You are saying that all parallelograms are rectangles and all rectangles are parallelograms. Do you think that is true.

Answer: Yes.

Interviewer: What about the last one?

Answer: None of the above. That's not true.

Interviewer: Ja because you said which is true. So you think that there is a couple true. The last question is at the bottom of this page here. I drew a square for you and do you remember that when you were doing your geometry constructions with Mrs C. that if you wanted to name a side, it would be PQ or SR. So it tells you that PQRS is a square. P Q R S - that shape there is a square. And it says which one do you think is true in all squares?

Answer: Couldn't you start with that? It is the wrong way round.

Interviewer: You could if you wanted to. You could go SPQR if you wanted to.

Answer: Yes A B C D E F G H . That looks like a 5 (points to the capital letter S)

Interviewer I know that is the way my printer did it. It is a capital S. I went PQRS to do it alphabetically, but, if you wanted to you could call it SPQR. Now can you read out each one and we must say which one is going to be true. So only one of these is true. Read it out and then try and find those sides.

Answer: That says 5 rand (laughs, pointing to RS in option a) Um PR and RS have the same length.

Interviewer: Just find PR first of all. Where is PR?

Answer: PR is there.

Interviewer: Where is RS?

Answer: RS

Interviewer: Do you think they are the same length?

Answer: No

Interviewer: All right so you don't think that's true. What about that one?

Answer: Um. QS and PR are perpendicular.

Interviewer: Do you remember what perpendicular means?

Answer: Not really.

Interviewer: Do you remember that if 2 lines cross each other and they make a 90 degree angle, that would be perpendicular? Do you remember that? Ok there's two lines that cross each other. These two lines cross each other and they make a right angle so they are called perpendicular.

Answer: Oh those. We had to bisect them but we didn't know what they were called.

Interviewer: You probably drew perpendicular bisectors, didn't you? So now I want you to find Qs. Where is QS?

Answer: Um. QS

Interviewer: and PR

Answer: Yes, they equal.

Interviewer: You think they are equal. But we want to know whether they are perpendicular. In other words where they cross each other do you think they make a right angle?

Answer: It would be sort of like a cross.

Interviewer: So you don't think that one is true? Or you think it is true?

Answer: I don't know if crosses are perpendicular.

Interviewer: All right. Is there a cross here? There isn't a cross here.

Answer: That ladies holding a cross.

Interviewer: That's right. And also if you have a look here. This is a bit like a cross. So that would be perpendicular. Do you think that those two lines PR and SQ might be perpendicular? They might be. So that could be the true one. Have a look at c and read it out.

Answer: PS and QR are perpendicular.

Interviewer: Where is Ps.

Answer: PS. Yes

Interviewer: Where's QR? Do you think those two are going to be perpendicular?

Answer: Yes

Interviewer: Why do you think they are perpendicular?

Answer: Because on each side.

Interviewer: So you think they will cross and make 90 degrees.

Answer: No

Interviewer: Why not

Answer: Because if they come together they will be next to each other

Interviewer: Then it won't be a square, will it. It would be like a triangle. Because then they would cross. So you are going to say c is wrong?

Answer: Yes c is wrong.

Interviewer: Well what about d?

Answer: PS and QS have the same length. Yes they do.

Interviewer: Where are they? Just point them out.

Answer: Ps and then QS

Interviewer: Ok so you think they have the same length?

Answer: Yes.

Interviewer: All right and the last one?

Answer: Um. Angle Q is larger than angle R. It can't be because it is a square.

Interviewer: And what do we know about a square

Answer: All 4 sides are equal

Interviewer: Thank you.