UNIVERSITY OF FORT HARE

MNU 111

DEGREE EXAMINATIONS

June 2023

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Time: 3 HOURS

Subject: INTRODUCTION TO NUMERICAL MATHEMATICS

Marks:100

This question paper consists of 3 pages

Internal examiner(s)

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Instructions

Answer All questions.
Symbols have the usual meanings.

Question One

1.1. Prove the following DE Morgan's Law for sets:

$(A \cup B)^c = A^c \cap B^c$ (8)

1.2. Show that the bi-conditional proposition of p and q is logically equivalent to the conjunction of the conditional proposition of p and q and its converse. (8)

1.3. Let p: "The election is decided" and q: "The votes have been counted" be propositions. Express the compound proposition $\sim q \vee (\sim p \wedge q)$ informally.

(6)

1.4. Prove that the $\sqrt{2}$ is irrational. (8)

Question Two

2.1. Let $A = [-2; 5]$ and $B = (0; 10]$. With the aid of a number line, find the following intervals:

I. $A \cap B$

II. $A \cup B$

III. $A - B$

IV. $B - A$ (8)

2.2. Construct truth tables for the statements that follow: $P \Rightarrow Q, \sim P \vee Q$, and $\sim (P \wedge \sim Q)$. (8)

2.3. Prove that if $n > 0$, then any nonnegative integer less than n which is divisible by n must be 0.

(6)

2.4. Give a direct proof of the proposition, if m and n are both perfect squares and m and n are integers, then mn is also a perfect square.

(8)

Question Three

3.1. Write the contrapositive, converse, and inverse for the following propositions.

(a) If $x \geq 0$, then $x > 0$ or $x = 0$

(b) If $P$is a square, then P is a triangle (6)

3.2. A family has two children of different ages, and we want to know their gender. (FM means the female child is older than the male child).

a) What is the probability of having no male children given that one child is female.

b) What is the probability of having two female children given that the eldest is female.

(6)

3.3. Construct the truth table of the proposition $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$. Determine if this proposition is tautology. (8)

3.4. Consider the experiment of tossing a fair coin four times and let:

$A = \{all\ four\ results\ are\ the\ same\}, B = \{exactly\ one\ Head\ occurs\}$ and

$C = \{at\ least\ two\ Heads\ occur\}$, Show that $P(A) + P(B) + P(C) = \frac{17}{16}$ and explain why

$P(A) + P(B) + P(C) > 1$. (8)

Question Four

4.1. Consider the proposition "$\forall n \in \mathbb{N}$, If n is any prime number greater than 2, then $n + 1$ is even". Write the converse of the proposition and give a counter example showing that the converse is false.

(4)

4.2. A committee consisting of 2 mathematicians and 3 statisticians must be formed from 5 mathematicians and 7 statisticians. In how many ways can this be done if:

(a) Any mathematician and statistician can be included.

(b) One statistician must be on the committee.

(c) Two mathematicians cannot be on the committee. (8)

END