UNIVERSITY OF FORT HARE

STM 502

HONOURS EXAMINATIONS

JUNE 2017

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Time: 3Hours

Subject: Multivariate Statistics

Mark: 100

This paper consists of 6 pages including the cover page

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INSTRUCTIONS

- Answer ALL questions
- Statistical Tables will be provided.

STM 502

Question 1

1.1 The following are five measurements on the variables x_1, x_2 and x_3.

x_1	9	2	6	5	8
x_2	12	8	6	4	10
x_3	3	4	0	2	1

Find the arrays $\overline{X}$, S_n, and R. [10]

1.2 Let $x' = \begin{bmatrix} 5 & 1 & 3 \end{bmatrix}$ and $y' = \begin{bmatrix} -1 & 3 & 1 \end{bmatrix}$.

1.2.1 Find the length of x. [2]

1.2.2 The angle between x and y. [2]

1.2.3 The projection of y on x. [2]

1.3 Let X have covariance matrix

$$\Sigma = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$$

1.3.1 Determine ρ and $V^{\frac{1}{2}}$. [6]

1.3.2 Find ρ_{13} .[2]

1.3.3 Find the correlation between X_1 and $\frac{1}{2}X_2 + \frac{1}{2}X_3$. [6]

....[30Marks]

Question 2

2.1 Let

$$A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$$

2.1.1 Show that A is positive definite. [2]

2.1.2 Determine the eigenvalues and eigenvectors of A. [5]

2.1.3 Write the spectral decomposition of A. [3]

2.2 Given the random vector $X' = [X_1, \ X_2, \ X_3, \ X_4]$ with mean vector $\mu'_X = [4, \ 3, \ 2, \ 1]$ and variance-covariance matrix

$$\Sigma_X = \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{bmatrix}$$

Partition X as

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} X^{(1)} \\ X^{(2)} \end{bmatrix}$$

Let

$$A = \begin{bmatrix} 1 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$$

And consider the linear combinations $AX^{(1)}$ and $BX^{(2)}$.

Find

2.2.1 $E(X^{(1)})$ [1]

2.2.2 $E(AX^{(1)})$ [2]

2.2.3 $Cov(X^{(1)})$ [2]

2.2.4 $Cov(AX^{(1)})$ [2]

2.2.5 $E(BX^{(2)})$ [2]

2.2.6 $Cov(BX^{(2)})$ [2]

2.2.7 $Cov(X^{(1)}, \quad X^{(2)})$ [2]

2.2.8 $Cov(AX^{(1)}, \quad BX^{(2)})$ [2]

..[25Marks]

Question 4

4.1 Define a 100 (1-α) % confidence region for the mean of a p -dimensional normal distribution. [4]

4.2 Evaluate T^2, for testing $H_0 : \mu' = [9,5]$, using the data

$$X = \begin{bmatrix} 6 & 9 \\ 10 & 6 \\ 8 & 3 \end{bmatrix}$$ [6]

4.2.1 Specify the sampling distribution of T^2 for the situation in (4.2) above. [8]

4.2.2 Using (4.2) and (4.2.1), test H_0 at the α=0.05 level . What conclusion do you reach? [2]

....[20Marks]