UNIVERSITY OF FORT HARE

University of Fort Hare
Together in Excellence

PHY 321

SUPPLEMENTARY EXAMINATIONS

DATE	:	*January/February 2019*
TIME	:	*3 HOURS*
SUBJECT	:	*PHYSICS 321 (Quantum Mechanics & Solid State)*
MARKS	:	*100*

INTERNAL EXAMINERS

V.A. Xuza (Dr)

EXTERNAL EXAMINERS

Prof. S. Chikwembani (WSU)

INSTRUCTIONS TO CANDIDATES:

Answer 5 Questions.

[Answer 4 (four) questions from Section A and 1 (one) question from Section B]

Section A

QUESTION 1 [20]

(a) The initial wave-function of a particle is given as $\psi(x,0) = C\exp\left(-\frac{|x|}{2}\right)$, where C is a constant.

(i) Sketch this function. (4)

(ii) Find C such that $\psi(x,0)$ is normalized. (6)

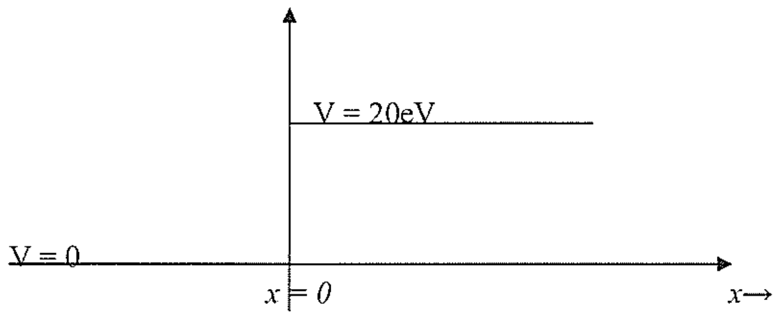
(b) The copper conducting wires are separated by an insulating oxide layer, (CuO). Modeling the oxide as a square barrier of height *10.0 eV*, it can be shown that the transmission coefficient of a barrier of length L is given by

$$T(E) = \left\{1 + \frac{1}{4}\left(\frac{V(x)}{E[V(x)-E]}\right)\sinh^2 \alpha L\right\}^{-1} \quad \text{Where } \alpha = \frac{\sqrt{2m(V(x)-E)}}{\hbar}$$

Estimate the transmission coefficient for the penetration by 7.0 eV electrons if the layer thickness is $L = 5.00nm$. (10)

QUESTION 2[20]

An electron with a kinetic energy of $10eV$ at $x = -\infty$ is moving from left to right along the *x-axis*. The potential energy is $V = 0$ for $x < 0$, and $V = 20\ eV$ for $x > 0$.



(a) Write the Schrödinger equation for $x < 0$ and $x > 0$. (5)

(b) What is the wavelength for $x < 0$? (5)

(c) What are the boundary conditions at $x = 0$? (5)

(d) What is the transmission coefficient $T = 1 - R$? (R= reflection coefficient) (5)

QUESTION 3[20]

A particle of mass, m, moving in 1-D, experiences the potential $V(x) = \frac{1}{2} m\omega^2 x^2$.

Its ground state function is $\psi_o = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\frac{\alpha x^2}{2}}$ where $\alpha = \frac{m\omega}{\hbar}, and, \omega = \sqrt{\frac{k}{m}}$

Determine,

(a) the energy eigenvalue E_o (4)

(b) the expectation values $\bar{x}$ and $\bar{p}$ for this ground state. (4)

(c) the expectation values $\overline{x^2}$ and $\overline{p^2}$, for the ground state. (8)

(d) that Heisenberg Uncertainty Principle, $\Delta x \Delta p = \frac{\hbar}{2}$ for the ground state if

$\Delta x = \sqrt{\left(\overline{x^2} - \bar{x}^2\right)}$ and $\Delta p = \sqrt{\left(\overline{p^2} - \bar{p}^2\right)}$. (4)

QUESTION 4[20]

(a) A 15-kg mass, attached to a massless spring whose force constant is 2500 N/m, has an amplitude of 4 cm. Assuming the energy is quantized, find the quantum number of the system, n, if $E_n = nhf$. (5)

(b) The wave function $\psi(x)$ of a particle confined to $0 \le x \le L$ is given by $\psi(x) = Ax$. $\psi(x) = 0$ for $x < 0$ and $x > L$. When the wave function is normalized, find the probability density at coordinate x. (5)

(c) Consider a particle of mass m, which can move freely along the x-axis anywhere from $x = -a/2$ to $x = +a/2$, but bound in an infinite square well potential. If the wave function for its lowest energy state is found to be:

$$\psi(x,t) = A\cos\left(\frac{\pi x}{a}\right)e^{-\frac{iEt}{\hbar}} \quad \text{for } |x| = \frac{a}{2}$$

and $\psi(x,t) = 0$, otherwise.

(i) Calculate the constant A. (4)

(ii) Find energy E from the time-dependent Schrödinger equation,

i.e. $-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = i\hbar\frac{\partial \psi}{\partial t}$ (6)

QUESTION 5[20]

Consider an infinite square well. The wave function of a particle trapped in an infinite square well potential of width $2a$ is found to be:

$\Psi(x) = C\ \{cos\left(\frac{\pi x}{2a}\right) + sin\left(\frac{3\pi x}{a}\right) + ¼\ cos\left(\frac{3\pi x}{2a}\right)\}$ for $-a \leq x \leq a$

$\Psi(x) = 0$ outside

a) Calculate the coefficient C. (8)
b) If the measurement of the total energy is made what are the possible results of such a measurement? (6)
c) What is the probability to measure each of them? (6)

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Section B

Question 6 [20]

(a) What is the angle between the [211] and [111] directions of a cubic lattice? (5)
(b) What lattice type has primitive lattice vectors,

$$a_1 = \tfrac{1}{2}a(\hat{y}+\hat{z}); \quad a_2 = \tfrac{1}{2}a(\hat{x}+\hat{z}); \quad a_3 = \tfrac{1}{2}a(\hat{x}+\hat{y}).$$

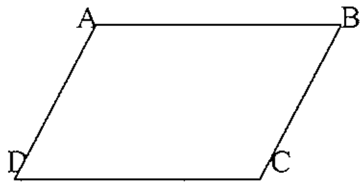
Determine the primitive translation vectors of the lattice *reciprocal* to this lattice. (10)
(c) From the definition of the primitive reciprocal lattice vectors $\mathbf{b}_j$, show that the equation $\mathbf{a}_i.\mathbf{b}_j = 2\pi\delta_{ij}$. (5)

Question 7 [20]

(a) Consider a set of planes (211) in a direct lattice that need not have orthogonal coordinate axes. Show that the reciprocal lattice vector $G = 2b_1 + b_2 + b_3$ is perpendicular to these planes. (8)
(b) A 2-*D* direct lattice is formed by repetition of the primitive unit cell ABCD shown in the figure below, in which $AB = DC = 5Å$ and $AD = BC = 3Å$.

Show that $V_r = \frac{8\pi^3}{V_u}$, where V_r = volume of the reciprocal lattice

V_u = volume of the unit cell. (12)



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CONSTANTS AND OTHER RELEVANT DATA

Speed of light in vacuum	c	=	2.998×10^{8} m-s^{-1}
Electronic charge	e	=	1.602×10^{-19} C
Boltzmann constant	k	=	1.381×10^{-23} JK^{-1}
		=	8.617×10^{-5} eV K^{-1}
Electron rest mass	m_e	=	9.11×10^{-34} kg
		=	0.511 MeV/c^2
Proton rest mass	m_p	=	938.3 MeV/c^2
Neutron rest mass	m_n	=	939.6 MeV/c^2
Mass unit	u	=	1.661×10^{-27} kg
		=	931.5 MeV/c^2
Planck's constant	h	=	4.136×10^{-15} eV-S
	hc	=	1.24×10^{4} eV-Å
	ħc	=	1973 eV-Å
Electron-volt	1 eV	=	1.602×10^{-19} J
Angstrom	1 Å	=	10^{-10} m
		=	10 nm
	1 nm	=	10 Å = 10^{-9} m
Avogadro Number	N_A	=	6.022×10^{23} mol^{-1}
		=	6.022×10^{26}(K-mol)$^{-1}$
Stefan-Boltzmann Constant	δ	=	5.670×10^{-8} Wm^{-2} K^{-4}
Bohr radius	a_0	=	5.292×10^{-11} m
		=	$\hbar^2/ke^2m_e$
		=	0.529 °A
Bohr magneton	u_B	=	5.788×10^{-5} eV/tesla
	1 tesla (T)	=	1 Wb – m^{-2}
		=	10^4gauss (G)
1 eV photon has wavelength	λ	=	1.243×10^{-6} m
$ke^2 = 1/4\pi\varepsilon_0 e^2$		=	14.40 eV·Å
	$K^2Me^4/2\hbar^2$	=	13.58 eV

USEFUL FORMULAE

Relativistic energy	E^2	=	$p^2c^2 + m_0^2c^4$
Compton relations	$\lambda' - \lambda$	=	$\frac{h}{m_e c}(1-\cos\theta)$

$$= \frac{h}{m_e c^2}(1-\cos\theta) \quad \text{Wien's Law}$$

$$\lambda_{max}T = 2.9 \times 10^{-3} \text{ mK}$$

Relativistic kinetic energy $\quad K = (\gamma - 1)\, m_0c^2$

Where $\quad \gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$

Total energy (relativistic) $\quad E = K + m_0c^2$

$$= \gamma E_0$$

Values of the integral $\quad I_n = \int_0^\infty x^n e^{-\lambda x^2} dx \quad$ for $n = 0$ to $n = 5$

n	I_n
0	$\frac{1}{2}\sqrt{\frac{\pi}{\lambda}}$
1	$\frac{1}{2}\lambda$
2	$\frac{1}{4}\sqrt{\frac{\pi}{\lambda^3}}$
3	$\frac{1}{2}\lambda^2$
4	$\frac{3}{8}\sqrt{\frac{\pi}{\lambda^5}}$
5	$1/\lambda^3$

If n is even $\quad \int_{-\infty}^{\infty} x^n e^{-\lambda x^2} dx = 2I_n$

If n is odd $\quad \int_{-\infty}^{\infty} x^n e^{-\lambda x^2} dx = 0$